20211120

A Proof of Goldbach Conjecture

Zhi Li and Hua Li

(lizhi100678@sina.com, lihua2057@gmail.com)

Abstract: natural numbers can be divided into even numbers and odd numbers, and odd numbers can be divided into composite numbers, odd numbers and prime numbers. Any even number can be decomposed into the sum of a larger prime number and a smaller odd number.The Goldbach conjecture can be proved by calculating whether the cumulative probability that the small odd number is a prime number is much greater than 1 and determining whether the prime number in many small odd numbers is inevitable.In this paper, the Goldbach conjecture is proved by the method of probability and statistics, the Goldbach number theorem is further discovered, and a new method for the study of prime number distribution is created.

Key words: Goldbach conjecture probability statistics Goldbach number theorem

The Goldbach conjecture was proposed in 1742. The current strong Goldbach conjecture is described by Euler, and it has been nearly 300 years since then. Although people's research on this conjecture continues to make progress, it has never been fully proven. In this paper, probability and statistics are used to prove the conjecture.

1. Proof method

Natural numbers can be divided into even numbers and odd numbers, and odd numbers can be divided into composite odd numbers and prime numbers. Since there is no mathematical model for prime numbers that can be completely and accurately represented, prime numbers are randomly distributed on the number axis. The distribution of prime numbers is the same as the decay of radioactive atoms. The specific time of the decay of a single atom cannot be accurately expressed by a function. Only a certain number of atoms can be selected for observation to obtain the probability of decay. When the number of atoms exceeds a certain number, and the cumulative probability is far greater than 1, it can be determined that a decay event will occur in a certain period of time.

The prime numbers are randomly distributed. Humans cannot change it to conform to a certain function expression, so they can only use probability statistics to describe them. According to big data statistics, it is found that the appearance of prime numbers within a certain range has the following laws.

If the integer value is small, the number of prime numbers in the interval is more; if the value is large, the number of prime numbers in the same interval is less. When the integer value gradually increases, the increase rate of the number of prime numbers in the interval gradually slows down, and the ratio of the number of prime numbers to the number of the natural numbers in the interval gradually decreases. According to the prime number theorem, the function expression of the number of prime numbers in the interval can be approximately expressed as:

The number of prime numbers in the first x integers is approximately [1]:

$$\frac{x}{\ln x} \tag{1}$$

Any even number can be decomposed into the sum of a larger prime number and a smaller odd number[2], and the larger prime number is greater than one-half of the even number, the smaller odd number is less than one-half of the even number.

Calculate the cumulative probability that the smaller odd number is a prime number, that is, the sum of the probabilities that occur in different intervals. When the sum of the probabilities is n, much greater than 1, it indicates that the event has occurred n times, and it can be determined that the occurrence of prime numbers among many smaller odd numbers is inevitable.

Because the mantissa of any larger prime number cannot be the number 5, the mantissa of the corresponding smaller odd number can only be one of 1, 3, 7, and 9.

Let an even number is 2x, then the number of larger prime numbers between integers x and 2x is $\frac{2x}{\ln(2x)} - \frac{x}{\ln(x)}$, the cumulative probability that the smaller odd number between 1 and x corresponding to the larger prime number is a prime number is approximately:

$$\frac{5}{2} \cdot \frac{x}{\ln(x)^2} \cdot \frac{\ln(x) - \ln(2)}{\ln(x) + \ln(2)} \tag{2}$$

Because the discriminant function (2) presents a divergent shape similar to a straight line with a slight arc, as shown in Figure 1 (x is greater than 4), it gradually increases as x increases. When 2x is greater than 40, its value is much greater than 1.

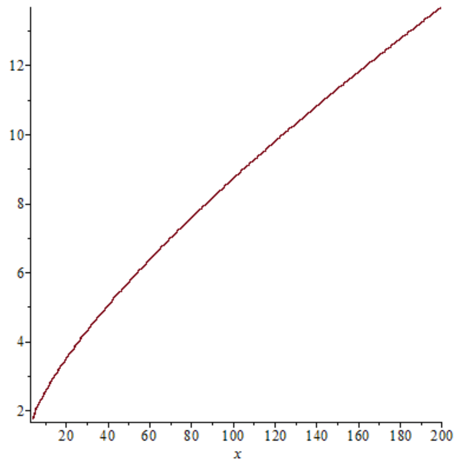

Fig.1 The discriminant function.

It can be proved that the first derivative of the discriminant function (2) is always positive and takes the x-axis as the asymptote, as shown in Figure 2, while the second derivative is always negative, as shown in Figure 3.

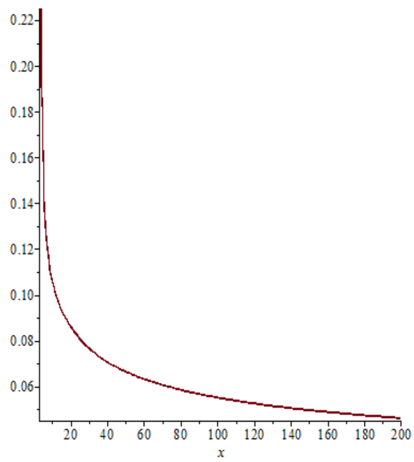

Fig.2 The first order derivative of the discriminant function.

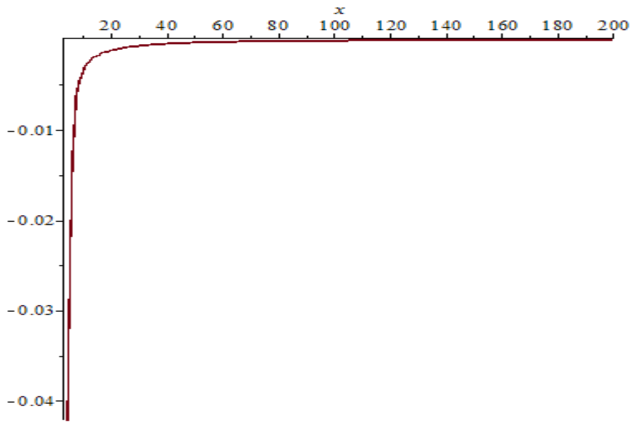

Fig.3 The second order derivative of the discriminant function.

Therefore, it is proved that the occurrence of prime numbers in smaller odd numbers is inevitable, thus, Goldbach's conjecture is proved.

2. Experimental verification

2.1 The cumulative probability value of the smaller odd number being a prime number

For even numbers 2x' s, the cumulative probability values of the smaller odd number being a prime number are shown in Table 1 for details.

Table 1 cumulative probability values of the smaller odd number being a prime number

Even numers （2x）	Number of prime in x ~ 2x	probability that the corresponding smaller odd number is a prime number	Number of actual prime numbers of corresponding small odd numbers
10	1	1.92	1
20	4	2.53	2
40	4	3.48	3
80	9	5.02	4
100	10	5.71	5
200	21	8.70	8
1000	73	25.87	28

It can be seen from Table 1 that as the even numer value 2x becomes larger, the cumulative probability value of the corresponding smaller odd number being prime also gradually increases, and the actual number of the prime numbers corresponding to smaller odd number also gradually increases, and the two are getting closer. When the even number 2x is any value greater than 40, the cumulative probability value of the smaller odd number being a prime number and the actual number are far greater than 1.

As the even value increases, although the growth rate of the probability value of the smaller odd number being a prime number gradually decreases, the cumulative probability value of the smaller odd number being a prime number and the absolute value of the actual number still gradually increase.

2.2 The number of predicted prime numbers and the actual number of prime numbers within any integer range

When the integer value x gradually increases from 1000 to 10000, the actual value of the number of prime numbers smaller than the integer x gradually approaches the predicted value, as shown in Table 2:

Table 2 The ratio of the actual number of prime numbers to the predicted number

Integer value (x)	The actual number of prime numbers less than x (y)	Predicted number of prime numbers less than x ($\frac{x}{\ln x}$)	Ratio Y/(x/lnx)
1000	168	145	1.16
2000	303	263	1.15
5000	669	587	1.14
10000	1229	1086	1.13

It can be seen from Table 2 that the actual value and the predicted value of the prime number gradually approach as the integer becomes larger, and the actual value is slightly larger than the predicted value. Studies have shown that when the integer approaches infinity, the actual value and the predicted value gradually close, the ratio limit is 1. It can be shown that the cumulative probability value of the smaller odd number calculated by applying the prime number theorem is a prime number is valid and reliable.

3. Conclusion

Since the cumulative probability, of the smaller odd number being a prime number corresponding to a larger prime number of any even number 2x greater than 40 in the range of the value x~2x, is far greater than 1, and the actual number of prime numbers verified by the experiment is very close to the predicted value, and the number is also far greater than 1, so it is inevitable that there are prime numbers among the smaller odd numbers corresponding to the larger prime numbers smaller than the even numbers. This proves that any sufficiently large even number can be expressed as the sum of two prime numbers, which also proves the strong Goldbach conjecture. The probability and statistics method used in this paper creates a new method for the study of the distribution of prime numbers.

4.Corollary

Goldbach's number theorem: Any sufficiently large even number 2x can be expressed as the sum of two prime numbers, and the number of combinations of the sum of two prime numbers is approximately

$$\frac{5}{2}\cdot\frac{x}{\ln(x)^2}\cdot\frac{\ln(x)-\ln(2)}{\ln(x)+\ln(2)} \quad (3)$$

5.References

[1] Baoyang Liu and Yawei Liu . On the distribution of twin prime numbers. Mathematics learning and research[J], 2011, (15): 87-88. (in Chinese)

[2] Jingrun Chen. Expressing big even number as the sum of a prime number and the product of no more than two prime numbers[J].Science Bulletin,1966(9):385-386. (in Chinese)