Global and Local Cosmological Metrics

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Abstract

In this paper, it is proposed that to fully describe the Cosmology of the Universe, we need to consider two metrics together: the FRW metric and the internal Schwarzschild metric. In static spacetime, if you zoom into a local region of the spacetime, you get the Minkowski metric locally. In this case, where the spacetime is dynamic, if we zoom in to a specific time slice of the internal Schwarzschild metric, we get the Minkowski-equivalent FRW metric for flat space with a spatial scale factor that depends on the specific time slice at which we are looking. By solving for the unknowns in the internal Schwarzschild metric using cosmological data, we obtain values for the scale factor at different times which can then be used in the FRW metric to obtain energy densities of the Universe at various times. No cosmological constant is required because the internal Schwarzschild metric provides a scale factor that generates a slowing expansion for some time after the Big Bang followed by an accelerated expansion that ultimately ends in a Big Rip. The entire Schwarzschild metric in Kruskal-Sezekeres coordinates is examined and we see that it describes two CPT symmetric Universes moving in opposite directions in the time dimension. One Universe contains matter while the other contains antimatter. When these Universes meet at the singularity, they annihilate each other resulting in a single, massless, pure radiation Universe. This state regenerates the Big Bang conditions where the radiation decays into matter and antimatter pairs, resulting in a new cycle where the two Universes again fall in time.

The Case for Global and Local Cosmological Metrics

Let us consider a 2D shell of gas spherically symmetrically distributed in space. This shell will collapse according to the Schwarzschild metric where the shell falls toward the center of the gas shell. This metric is a vacuum solution because there is no matter at larger or smaller radii, the gas exists effectively at a specific radius at any given time.

Now suppose we have an observer in the gas. The observer is a 2D creature that can only see along the surface of the shell. The observer would expect the gas around them to become more dense over time as the matter and energy in the gas pulls the gas to higher density over time. However, over time, the observer would find that the matter is collapsing more quickly than expected because it is not taking into account the additional collapse that comes from the fact that the entire shell is falling in the Schwarzschild spacetime.

As will be shown, the same can be said to be true for Cosmology. In the Cosmological case, we can imagine that the matter and energy in the Universe is isotropically distributed throughout infinite space (3D space in this case), but exists only at the present time (time is the radius in this case). Another way to say this is that matter and energy in the past and future has no gravitational effects on the present. The curvature of the present spacetime can be understood completely using present data.

It is well established that in static, curved spacetimes, if one zooms in to a region of the spacetime, it would look locally like Minkowski spacetime. The FRW metric is essentially the Minkowski metric where the spatial dimension is scaled by a function of time. As will be demonstrated, the Universe can be modelled as 3D space falling in the *internal Schwarzschild metric*. If one zooms in to a specific time slice of this spacetime, it looks like the FRW metric (assuming zero spatial curvature) instead of the Minkowski metric because the scale factor of space depends on which slice of time we are zooming into. Thus, we can say the Universe has a 'Global Metric' which is the internal Schwarzschild metric when looking at the Universe over large cosmological time scales, and a 'Local Metric' which is the FRW metric when describing the Universe over negligible changes of cosmological time. When viewing the Universe as a whole as falling through time in the internal Schwarzschild metric, we get accurate predictions not only of accelerated expansion after a period of slowing expansion, but an eventual Big Rip after a finite, calculable time.

Going back to the collapsing shell scenario, note that the radii in the two metrics would mean very different things. To that observer, arc lengths given by the Scharzschild metric would have little meaning with regard to the observer's direct observations. The observer would use the local metric, centered on themselves to calculate arc lengths. To the observer, the radius from the Schwarzschild metric essentially acts as a measure of time that can be used to account for discrepancies in the collapse of the gas when calculated with only the local metric. In other words, the scale factor of the collapse in the local metric will be a function of the gas's present radius in the Schwarzschild metric. As will be shown, this is exactly what the internal Schwarzschild metric provides when used as the global Cosmological metric with the difference being that in the Cosmological case, the shell is now a 3D volume of space and the radial direction is the time dimension.

Therefore, we can say that the FRW metric allows us to predict energy distributions at given times in the Universe when provided the scale factor for the times in question from the global metric. As will be shown, the scale factor can be calculated exactly for any time by solving for the unknowns in the internal Schwarzschild metric using cosmological data (the transition redshift and the Hubble parameter). Thus, we can plug known values for the scale factor and its derivatives from the global metric into the FRW metric to predict the energy densities of the Universe at different times.

The Schwarzschild (Global) Metric

The Schwarzschild metric is the simplest solution to Einstein's field equations. It is a vacuum solution for the spacetime around a spherically-symmetric distribution of energy. The general form of the metric can be expressed as:

$$d\tau^2 = \frac{r}{u-r} dr^2 - \frac{u-r}{r} dt^2 - r^2 d\Omega^2 \tag{1}$$

Depending on the ratio $\frac{u}{r}$, we get three distinct descriptions of spacetime:

1. $u = 0$: This gives us the flat Minkowski metric of Special Relativity.
2. $\frac{u}{r} < 1$: This describes the metric for an *eternally* spherically-symmetric vacuum centered in space. This metric is also used to describe the vacuum outside a spherically symmetric object occupying a finite amount of space (like a star or planet).
3. $\frac{u}{r} \geq 1$: This describes the metric for a spherically symmetric vacuum centered on a point in *time*. Analogous to the second case, this metric should also describe a vacuum *of time* outside a spherically-symmetric object spanning *infinite space*. The center of the metric is *everywhere in space, but at a single point in time* (just like one could say that the vacuum described in the second case is centered at all times on a single point in space).

An important observation is that the internal metric describes a vacuum solution to the field equations. But the Universe is clearly filled with energy, so how can this solution apply? In order to satisfy the requirements of the metric, the Universe must be "*a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time*". For this metric to be a cosmological description, it must be that Universe only truly exists in the present and in a very real sense moves into the future. The surrounding vacuum is the future, and the Universe is freefalling through time toward the temporal center of the metric.

Time being the radial dimension of the metric combined with the fact that the solution is a vacuum solution gives a mathematical justification for our intuitive notions of past, present, and future. The anisotropy along the radial direction gives us an arrow of time that distinguishes the 'past' and 'future' analogous to the way the external solution gives us an absolute distinction between 'up' and 'down'. And the vacuum as described above gives us a boundary between them, that boundary being the 'present' time.

Observation has shown that the Universe is:

- Spherically symmetric.
- Homogenous in space
- Inhomogeneous across time.

We will also make one further assumption in this paper:

- The Universe only ever occupies a single instant of Cosmic time[1] and moves from one moment of cosmic time to the next where the time measured by observers between cosmic times depends on their respective motions. In

[1] In the classical approximation. Quantum uncertainty would blur that instant.

other words, the 3D spatial distribution of energy in the Universe is physically moving through the time dimension from the past into the future, and energy only exists in the present. So if one were to view the Universe on a spacetime diagram, they would only see the Universe at one value of time with the rest of the diagram empty.

This further assumption implies that the spherically symmetric Universe is 'surrounded' by vacuum in the time dimension, analogously to how the aforementioned 2D shell was surrounded by a vacuum of space. Since the only spherically symmetric vacuum solution in General Relativity is the Schwarzschild metric, this assumption implies that the global metric of the Universe is the internal Schwarzschild metric. Relativity of simultaneity does not prohibit the idea of the energy existing at a specific Cosmological time because of the nature of the metric. In Cosmology, we can determine absolute motion and absolute simultaneity because we have the Cosmic Microwave Background. For example, consider two events that are causally disconnected. If observers at each event see the CMB temperature to be uniform in all directions (the observers are comoving), then if both observers measure the CMB to have the same temperature at both events, then we know the events are absolutely simultaneous, even if an observer in motion sees them as non-simultaneous. Any observer in motion through space, inertial or otherwise, will see a dipole on the CMB, and that dipole will provide all the info about the state of motion of the observer. Therefore, because of the structure of the metric, we can define past, present, future, and motion in an absolute sense. To put it another way, the fact that cosmological time is finite into both the past and future allows us to specify the distance of any event from either the beginning or end of time absolutely.

Consider the celestial spheres around an observer in the Universe. When we look out to distant events, we can use the redshift from these events to determine their distance from us. Events with the same distance from us can be thought of as residing on a celestial sphere, such that all these events are separated from us by the same magnitude of space and time. We can classify these spheres into three types:

1. <u>Dynamic Spheres</u> – These are the spheres that galaxies reside on. Objects on these spheres maintain a constant coordinate distance from us and move forward in time. We are able to move toward or away from objects on these spheres by moving through space. If we fix our sights on a particular galaxy, the light we see from that galaxy is being emitted later in time as we ourselves move through time.

2. <u>Static Spheres</u> – These are spheres fixed in time. The Cosmic Microwave Background is the most obvious example of these spheres. Light from the CMB sphere is always emitted from the same cosmological time, but as we ourselves move through time, we see light from that time emitted from farther and farther away from us in space, giving the impression that the CMB sphere is growing. We cannot move toward or away any objects on this sphere because it is frozen in time. Both metrics are able to capture this behaviour, but they do so in different ways.

3. The Dark Sphere – The Dark Sphere is the Big Bang and lies beyond the CMB. It is in principle unobservable for two reasons. First, the CMB is opaque so that any light from the Big Bang cannot penetrate it. Second, even if the CMB was not blocking our view, any light from that sphere would be infinitely redshifted in the frame of all future observers since the scale factor on that sphere is zero. But though we cannot see the Dark Sphere, it must be there if the model of the Universe is consistent.

These spheres are shown in terms of the internal Schwarzschild metric in Figure 1. Figure 1 shows the Schwarzschild coordinates of the internal metric plotted on the Kruskal-Szekeres coordinate plane. In these coordinates, space is the 't' coordinate emanating from the center of the diagram (Big Bang) and time is the 'r' coordinate depicted as hyperbolas (time is flowing forward as r goes toward zero). The upper right quadrant of this diagram represents a single fixed direction ($\theta = const, \phi = const$). So each bold line representing a sphere would be a point on each sphere over time. Note that light on this diagram travels on 45-degree lines.

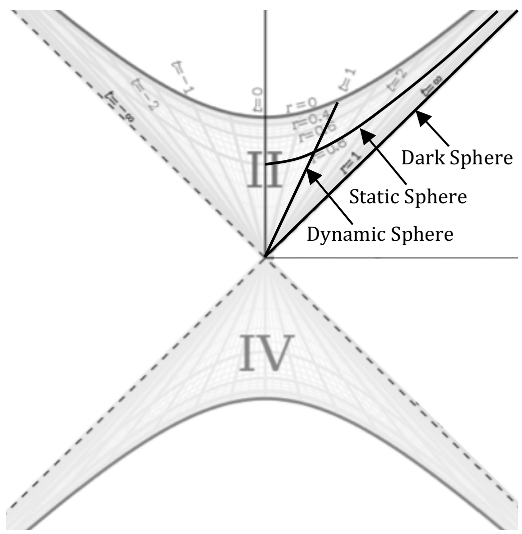

Figure 1 – Celestial Sphere Types on Kruskal-Szekeres Coordinate Chart[2]

The Antimatter Universe and Regeneration

Figure 2 shows the full Schwarzschild metric in Kruskal-Sezekeres coordinates. The diagram can be split in two along the diagonal where in the top right half, forward time points up while in the bottom right half, forward in time points down. Left and right are also swapped when looking at the upper and lower halves.

We can therefore conjecture that the diagram is describing both a matter Universe flowing up from the center and an antimatter Universe flowing down from the center both toward

[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

the singularity. The reason we expect an antimatter Universe is both because the directions of time and space are reversed relative to each other and because we expect that equal amounts of matter and antimatter are created at the beginning of the Universe and therefore, we expect the particles of the second Universe to have opposite charges relative to the first. Thus, the pair of Universes satisfy CPT symmetry.

At $r = 1$, we can see that the Universe must be massless because, as will be shown, the scale factor there is zero. This means that infinite space can be traversed in zero time there, which can only happen with massless particles. Thus, we can say that the Universe must be pure radiation at $r = 1$. After $r = 1$, this radiation would decay into matter/antimatter pairs that seed the two different Universes as each Universe begins falling in different directions in the diagram. Each matter particle in one Universe will also be entangled with its sister antimatter particle in the other Universe. Therefore, the Universes are maximally entangled at this point, giving the combined Universes a low entropy initial state at the beginning of expansion. Decoherence between the Universes over time will lead to an increase in entropy as time moves forward in each Universe. If the Universes then meet at the singularity, the matter and antimatter in the two Universes would annihilate, leaving only a Universe filled with high energy radiation. This regenerates the $r = 1$ state, cycling the entire process over and over again. The metric is singular at $r = 0$ because the annihilation would end the worldlines of the matter particles.

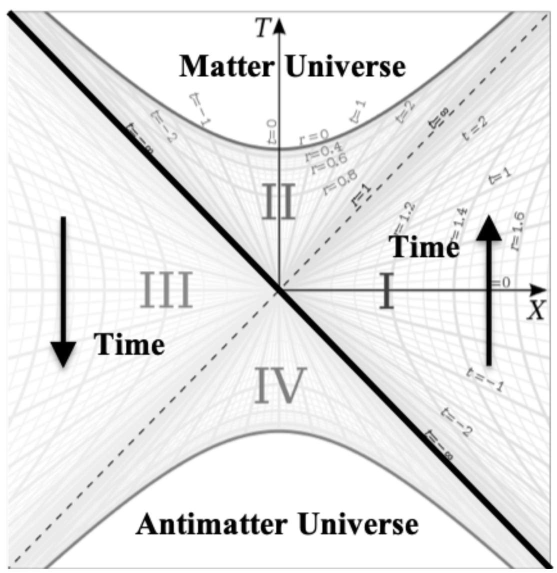

Figure 2 – Matter and Antimatter Universes on Kruskal-Szekeres Coordinate Chart[3]

[3] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

Freefall Through Time

Let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ). There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $\frac{u}{r} \geq 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 showed the $\frac{u}{r} \geq 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant *r* represent spacelike slices of constant cosmological time and the rays of *t* represent spatial distances. We will not be considering differences in angles until a later section in the paper, so we only need to consider the two halves of Figure 1. We will focus on the upper half where the half represents an observer pointed in a particular direction and the positive *t*'s represent the coordinate distance from the observer in that particular direction while the negative *t*'s represent coordinate distance in the opposite direction.

We must first determine the paths of inertial observers in the spacetime. For this we need the geodesic equations for the internal Schwarzschild metric [1] given in Equation 1. In these equations *u* represents a time constant that in the external metric would be the Schwarzschild radius (in Figure 1, the value of *u* is 1). The following equations are the geodesic equations for *t* and *r* $(r \leq u)$:

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)} \frac{dr}{d\tau} \frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2} \left[\frac{u-r}{r} \left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r} \left(\frac{dr}{d\tau}\right)^2 \right] - (u-r) \left(\frac{d\Omega}{d\tau}\right)^2 \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2}\left[1 + \left(\frac{d\Omega}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{4}$$

For $\frac{d\Omega}{d\tau} = 0$, notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the Schwarzschild constant was 2*GM*, as it would be in the external solution, Equation 4 would be Newton's gravity). Also, anyone moving inertially starting with non-zero $\frac{dt}{d\tau}$ will experience the same acceleration through time as someone with zero $\frac{dt}{d\tau}$ since dt does not appear in Equation 4.

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant *t* (and any such observer can choose $t = 0$ as their coordinate). The 'Big Bang' will have occurred in Figure 1 along the line $r = 1$. We know this because the above analysis showed that space expands if $\frac{dr}{d\tau}$ is negative, so for our current cosmological time, our worldlines must be moving toward $r = 0$.

The Scale Factor

Expressions for the proper time interval along lines of constant *t* and Ω and the proper distance interval along hyperbolas of constant *r* and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{5}$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{6}$$

Where *a* is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the *t* coordinate. This is good because the *t* coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time *r*.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$. When $r = u$, Equations 5 and 6 are both 0. At this point (the Big Bang), it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident (the observer can reach every point in space without moving through time, all paths are light-like). So this why the scale factor goes to zero there and why the lines of *t* in Figure 1 converge at that point; it is an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from positive to negative (we can see that if we draw a worldline through the center point,

$\frac{dr}{d\tau}$ will change signs as it passes the $r = u$ point). In fact, for any choice of time coordinate, that point will be a stationary point in those coordinates.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed of light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite. A plot of the scale factor vs. *r* (with $u = 1$) is given in Figure 3 below:

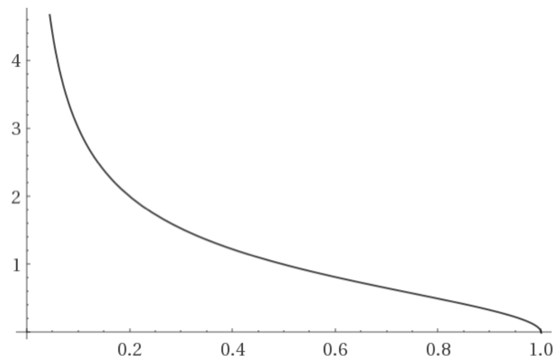

Figure 3 – Scale Factor vs. *r* for $u = 1$

Cosmological Parameters

In order to compare this model to cosmological data, we must solve for *u* and find our current position in time (r_0) in the model. Reference [3] gives us a 95% confidence interval for the measured transition redshift at $z_t = 0.426^{+0.27}_{-0.089}$. We can use the fact that $\sqrt{\frac{u-r}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1] (note that this Equation was derived from the FRW metric in the reference, but the internal metric, when setting $d\Omega = 0$, can be put in the same form as the FRW metric with a coordinate change, so the equation below is still valid for the internal metric):

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{u-r}{r}} - 1 \tag{7}$$

We can see in Figure 2 that there is an inflection point that corresponds to the transition redshift in the model. To find this inflection point, we need to derive the Hubble parameter and deceleration parameter equations using the scale factor. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \tag{8}$$

And the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 \tag{9}$$

The transition redshift occurs when $q = 0$, giving us $\left(\frac{r}{u}\right)_t = 0.75$. With this and Equation 7, we can find $\left(\frac{u}{r}\right)_0$:

$$z_t = 0.426^{+0.27}_{-0.089} = \sqrt{\frac{1}{\frac{1}{0.75}-1}}\sqrt{\left(\frac{u}{r}\right)_0 - 1} - 1 \rightarrow \left(\frac{u}{r}\right)_0 = \left(\frac{1.426^{+0.27}_{-0.089}}{\sqrt{3}}\right)^2 + 1 \tag{10}$$

Giving:

$$\left(\frac{u}{r}\right)_0 = 1.678^{+0.281}_{-0.082} \tag{11}$$

The current Hubble constant, as measured by the Planck mission was found to be $H_0 = 67.8 \pm 0.9\ (\mathrm{km/s})/\mathrm{Mpc}$ and from the Hubble Space telescope $H_0 = 73.48 \pm 1.66\ (\mathrm{km/s})/\mathrm{Mpc}$. With these and Equation 11, we can solve for limiting values of u and r_0 (after converting the units of H_0 so that u is measured in Gly):

$$H_0 = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2u\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \rightarrow u = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2H_0\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \tag{12}$$

Note that in Equation 12, H_0 is in units of $(Gy)^{-1}$. Before presenting the results, let us derive the expression for t vs. r along a null geodesic where the geodesic ends at the current time r_0. We can do this by setting $d\tau = rd\Omega = 0$ in Equation 1 and integrating:

$$t = \int_{r_0}^{r} \frac{r}{u-r}dr = u\ln\left(\frac{u-r_0}{u-r}\right) + (r_0 - r) \tag{13}$$

Table 1 below gives the values of u, r_0, a_0, q_0, r_t (coordinate time at transition redshift), H_t (Hubble constant at the transition redshift), and t_t (coordinate distance of transition redshift) given the measured bounds of z_t and H_0. All times are in Gy, distances are in Gly, and H are in $(\mathrm{km/s})/\mathrm{Mpc}$.

z_t	H_0	u	r_0	$u - r_0$	a_0	H_t	r_t	t_t	$r_t - r_0$	q_0
0.337	68.7	30.4	19.0	11.4	0.77	85.8	22.8	8.5	3.8	-0.5
0.337	66.9	31.2	19.5	11.7	0.77	83.6	23.4	8.8	3.9	-0.5
0.337	75.14	27.8	17.4	10.4	0.77	94.3	20.9	7.9	3.5	-0.5
0.337	71.82	29.1	18.2	10.9	0.77	89.5	21.8	8.1	3.6	-0.5
0.696	68.7	28.5	14.3	14.2	1.00	91.8	21.4	12.7	7.1	-1.0
0.696	66.9	29.3	14.7	14.6	1.00	89.3	22.0	13.0	7.3	-1.0
0.696	75.14	26.0	13.0	13.0	1.00	100.4	19.5	11.5	6.5	-1.0
0.696	71.82	27.3	13.7	13.6	1.00	95.8	20.5	12.1	6.8	-1.0

Table 1: Limiting Cosmological Parameter Values Based on z_t and H_0 Measurement

Note that these values cannot be calculated for the CMB because of lack of precision in z_t and H_0 measurements (The CMB is too close to $r = u$ to get meaningful values given the imprecise measurements). Table 2 has the proper times from the Big Bang to the transition redshift and current time for stationary, inertial observers ($dt = rd\Omega = 0$) by integrating Equation 1 (there is not enough precision in the measurements to calculate this for the CMB). The column τ_{tot} gives the time from $r = u$ to $r = 0$. The expression for τ_{tot} turns out to be quite simple[4]:

$$\tau_{tot} = \frac{\pi}{2}u \tag{14}$$

The column τ_{remain} gives the time between $r = r_0$ and $r = 0$.

z_t	H_0	τ_0	τ_t	τ_{tot}	τ_{remain}
0.337	68.7	34.6	29.1	47.8	13.2
0.337	66.9	35.7	30.1	49.2	13.5
0.337	75.14	31.7	26.5	43.7	12.0
0.337	71.82	33.3	27.9	45.7	12.4
0.696	68.7	36.3	27.2	44.8	8.5
0.696	66.9	37.4	28.0	46.0	8.6
0.696	75.14	33.3	25.1	41.0	7.7
0.696	71.82	34.8	26.2	42.9	8.1

Table 2: Limiting Proper Times Based on z_t and H_0 Measurements (Time is in Gy)

Note that while the coordinate times for the current age of the Universe ($u - r_0$) are close to current estimates (for high z_t), the proper time τ_0 is actually much larger. This is because in the early Universe, observers are moving slower through the time dimension and therefore they accrue more proper time per unit coordinate time early on. But the speed through the time dimension increases over time such that even though we are presently only about halfway through the "coordinate life" of the Universe (according to Table 1), the amount of proper time remaining is actually much less than the amount of proper time that has already passed (according to Table 2).

Next we would like to use the u and r_0 values found to create an envelope on a Hubble diagram to compare to measured supernova data. First we need to find r as a function of redshift. We can do this by solving for r_{emit} in Equation 7 where $a_0 \equiv \sqrt{\frac{u-r}{r}}$, the present value of the scale factor:

$$r = u\frac{z^2+2z+1}{a_0^2+z^2+2z+1} \tag{15}$$

4 Thinking of τ_{tot} as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2}u$ for time and space. Equation 14 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2}u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

Next we substitute Equation 15 into Equation 13 to get coordinate distance in terms of redshift:

$$t = u\left[\ln\left(\frac{r_0(a_0^2+z^2+2z+1)}{u}\right) - \frac{z^2+2z+1}{a_0^2+z^2+2z+1}\right] + r_0 \tag{16}$$

Finally, we convert Equation 16 to the distance modulus, μ, which is defined as:

$$\mu = 5\log_{10}\left(\frac{t}{10}\right) \tag{17}$$

Where t in Equation 17 is in units of parsecs. A plot of distance modulus vs. redshift is shown in Figure 4 below plotted over data obtained from the Supernova Cosmology Project [6]. Curves calculated from all combinations of u and r_0 in Table 1 are plotted, giving an envelope for the model's prediction of the true Hubble diagram.

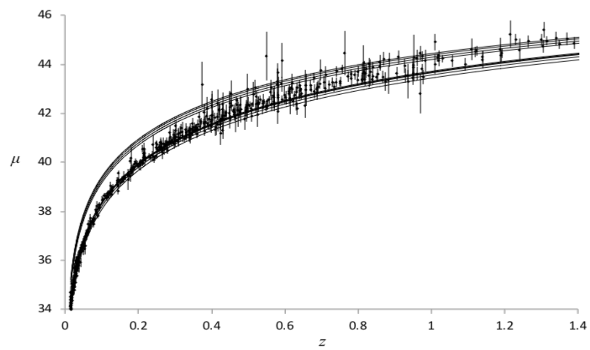

Figure 4 – Distance Modulus vs. Redshift Plotted with Supernova Measurements

Note that the lower curves correspond to the $z_t = 0.696$ data, suggesting that, if this model is correct, the true transition redshift is closer to 0.696 than 0.337.

In [7], the authors analyze a large sample of quasar data to obtain distance moduli at higher redshifts than is possible with supernova data. Although not definitive, the results of this analysis suggests that the "Dark Energy" density may be increasing with time, which does not fit with the ΛCDM model. However, the accelerated expansion predicted by the Schwarzschild solution *is* consistent with this type of expansion. Figure 5 shows the same predicted envelope from Figure 4 for the Hubble diagram plotted out to higher redshifts with the quasar data from [7] also shown with error bars. The black diamonds in the figure are the 18 high-luminosity XMM-Newton quasar points described in [7].

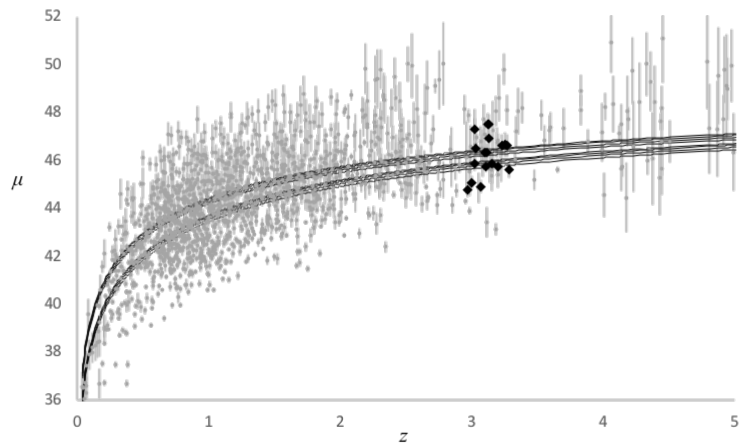

Figure 5 – Distance Modulus vs. Redshift Plotted with Quasar Measurements

Relationship to the External Schwarzschild Solution

Let us consider a meter stick at rest at the center of a collapsing spherically symmetric collapsing shell in space. The meter stick inside the shell stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat space). An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [5]:

$$\frac{dr}{d\tau} = -\sqrt{\frac{2GM}{r}} \tag{18}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 18. Since the meter stick is also at rest relative to observers at rest at any *r*, Equation 18 will also give the relative velocity between the freefall observer and the meter stick when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1 - V^2} = dt\sqrt{1 - \frac{2GM}{r}} \tag{19}$$

Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1 - \frac{2GM}{r}} \tag{20}$$

We see from Equation 20 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. So observers in freefall will see the space beyond $r = 2GM$ fully contracted as they approach $r = 2GM$. Furthermore, the clock of an observer at the center of the shell will be slowed as the shell collapses (the clock of an observer at the center ticks at a rate equal to an observer at rest at the location of the shell) such that if she exchanges light signals with the shell as it collapses, the time she measures for the light to return will shrink to zero as the shell reaches the Schwarzschild radius. Thus, she also effectively sees the space within the shell shrink to zero as the shell approaches the Schwarzschild radius.

But the freefalling observer of the external solution will never fall into a ‘black hole’. It would take an infinite amount of time in the frame of an observer at infinity for the freefalling observer to reach the event horizon. But the Universe itself will reach $r = 0$ in a finite amount of time in the frame of the infinite observer and therefore the freefalling observer will only reach the $r = 2GM$ location when the entire Universe has reached $r = 0$. Thus, she will never actually reach any event horizon, she will reach $r = 0$ when the entire Universe has reached $r = 0$.

Pair Production and the Charge and Spin Hypothesis

We can visualize the process of a pair of particles created at the Big Bang, moving through their respective Universes over time and then annihilating at the end of time in Figure 6 below.

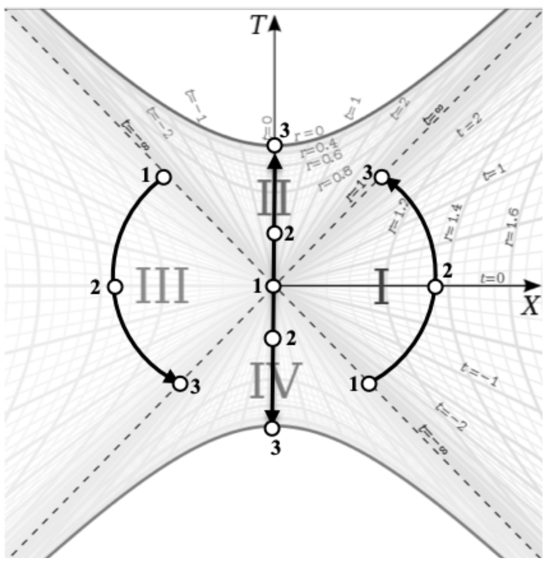

Figure 6 – Pair Production at the Big Bang and Annihilation at the End of Time[5]

In Figure 6, three stages are shown:

1. The line commonly thought of as a ‘White Hole Horizon (WHH)’ shows the matter and antimatter particles being emitted on opposite sides of the line into the external Schwarzschild metric. We also see the starting point in the internal Schwarzschild metric at the center of the diagram. All the events on

[5] Diagram modified from: “Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

the WHH are coincident because the particles enter the external metric at the same r and t coordinate.
2. The particles are travelling through their respective Universes in the external metric as their Universes fall in the internal metric
3. The line commonly thought of as the 'Black Hole Horizon (BHH)' shows the matter and antimatter particles coming back together and annihilating in the external metric since all the points on that line are also coincident for the same reason as the WHH. At this time in the internal metric, the Universes have reached the singularity.

Given that the matter and antimatter are moving in opposite directions in time, we can hypothesize that the electric charge of a particle is related to the particle's orientation in time. The sign of the charges of matter particles would indicate that these particles are oriented along the time radius in the same direction that the matter Universe is moving. The antimatter particles have opposite sign and so they are oriented in the direction of travel of the antimatter Universe. Chargeless particles such as photons would have no such orientation and have the same properties moving in both directions of time.

We can extend this hypothesis further by considering the spin of Fermions. Fermions can be measured to be spin up or spin down. We could interpret the spin to be a physical spin about the time dimension with, for instance, spin up indicating the spin vector is pointed in the direction of motion of the matter Universe along the time radius, and spin down indicating the spin vector is pointed in the direction of motion of the antimatter Universe along the time radius. This interpretation of Quantum spin also gives meaning to the angular term in the internal Schwarzschild metric in Equation 1. Fermion spins can be measured in any direction in three dimensions, so if the particles are spinning about the time radius, the angular term of Equation 1 gives us the space in which to interpret spin about the time dimension in multiple directions.

Equivalence of Time and Space

In Quantum Mechanics, the concept of spin was needed to achieve a relativistic description of quantum wave functions. We can see here that by modelling spin as rotation about the time radius, time gets treated on equal footing with space in that particles can rotate about either type of dimension. This equal treatment of time and space is the hallmark of relativistic theories.

The spacetime of the Universe can be thought of as being a spherically symmetric object having both a time and a space dimension along each of the directions of Ω. When we look out at the Universe in a particular direction we see both distant space and distant (past) time together. The origin of Ω can be placed anywhere in space at all times when observing the spatial structure of the Universe, but it has a fixed origin in time at all spatial locations when observing the temporal structure.

Equation 1 gives a clue as to how both space and time dimensions can be overlaid in all directions. Notice that the dr and $rd\Omega$ terms have opposite signs. As is the case in the external Schwarzschild and FRW metrics, we would expect the angular and pure radius terms to have the same sign. We can remedy this by changing Equation 1 to:

$$d\tau^2 = -\frac{u-r}{r}dt^2 + \frac{r}{u-r}dr^2 + (ir)^2 d\Omega^2 \qquad (21)$$

Making the radius in the angular term imaginary gives us the expected form of the metric. We can interpret this as every direction having a complex radius with a real spatial part and imaginary temporal part.

All observers have a minimum speed through time which depends on the present cosmological time, even if an observer is at rest spatially. And if an observer moves more rapidly through space in a given direction, they also effectively move more rapidly through cosmological time as a result of the time dilation of the observer's clock caused by the motion (i.e. a fast-moving observer will have aged less by the time they reach the singularity than an observer that remained at rest relative to the CMB during the expansion).

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