Noname manuscript No.
(will be inserted by the editor)

Margenau's reduction of the wave packet

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Received: date / Accepted: date

Abstract Margenau wanted to see reduction of the wave packet in terms of the Schrödinger equation. Here we will look at it in the context of nonlocality.

Keywords Schrödinger equation; reduction of the wave packet; nonlocality

1 Introduction

Margenau [1] was opposed to the additional projection theorem in the reduction of the wave packet. If we only allow the Schrödinger equation $H\psi = i\hbar(\partial\psi/\partial t)$, the the time progression of ψ is after Δt seconds is: $\psi + (\partial\psi/\partial t)\Delta t$. In this sense we can define a Margenau operator as

$$M = 1 - \frac{i}{\hbar}\Delta t H \tag{1}$$

The question in this paper is as follows. Is the M operator capable of obtaining the same reduced form of the wave function as in reduction of the wave packet. And if so, is the claimed equivalence of a Greenberger wave function [2] giving the same result under a Margenau operator as in (1). Let us define the following Hamiltonian $H = H_0 + H_M$ and apply that to entangled spin states. This is done in the next sections.

2 Application to entangled spin

The topic here is an equivalent of the entagled wave function: i.e. $|\psi\rangle_{12} = \psi(AB)$ in the sense of Einstein. With $\begin{pmatrix} 1 \\ 0 \end{pmatrix}_j$ the up spin state is denoted and

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equivalently with $\begin{pmatrix} 0 \\ 1 \end{pmatrix}_j$ the down spin state. Furthermore, with $j = 1$ we denote the spin travelling towards Alice and with $j = 2$ the spin travelling towards Bob. Of course, it is a particle, e.g. an electron, with a spin that is doing the travelling. For abbreviation we call it spin moving towards Alice or towards Bob.

The e^{-if} equivalent form of an entangled wave packet $|\psi\rangle_{12}$ is defined by

$$|\psi\rangle_{12} = \frac{e^{-if}}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right\} \tag{2}$$

Here the phase factor e^{-if} is different from the usual description where $1/\sqrt{2}$ is employed as normalization factor instead of $e^{-f}/\sqrt{2}$. But because $e^{if}e^{-if} = 1$ we are allowed to introduce the phase with a phase variable f and obtain ${}_{12}\langle\psi\,|\psi\rangle_{12} = 1$. Do also note that *either* $\begin{pmatrix} 1 \\ 0 \end{pmatrix}_1$ at Alice *and* $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ at Bob; *or* $\begin{pmatrix} 0 \\ 1 \end{pmatrix}_1$ at Alice *and* $\begin{pmatrix} 1 \\ 0 \end{pmatrix}_2$ at Bob is found. This is in accordance with the quantum theoretically required discreteness of the spin so that *no* linear combination of the basic spin states exist in either separate wing of the experiment. It is assumed that this agrees with Einstein (10) below.

In addition let us here define the Hamiltonian H that plays a crucial part in (1). We can have

$$H_0 = \frac{\hbar}{\Delta t}\frac{\partial}{\partial\phi} \tag{3}$$

$$H_M = \frac{\hbar}{\Delta t}\begin{pmatrix} 0 & 0 \\ 0 & \frac{\partial}{\partial f} \end{pmatrix} = \frac{\hbar}{\Delta t}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\frac{\partial}{\partial f}$$

Note that the Hamiltonian $H = H_0 + H_M$ is Hermitian.

Given an observer defined frame of reference, the ϕ in the H_0 of the Hamiltonian is the azimuthal angle at measurement that the spin makes with the orientation vector $\hat{\mathbf{n}}$ of the measurement instrument of the observer.The angle ϕ exists because of observation. Because M_1 is an expression of observational operation, an operation with ϕ can be present in the M_1.

In the wave function (2) we don't have a ϕ dependence. The differentiation to f, in the H_M part of the Hamiltonian, refers to the f in the equivalent wave function (2). The ϕ belongs to the measurement instrument. The f belongs to the description of the entangled particle spins.

The next step is to restrict the activity of H to the Alice side. Let us assume an experiment where Bob waits a time unit before measuring the spin that is heading towards him. The first reduction is at the side of Alice. This reduction of the wave packet at the side of Alice is replaced by a Margenau operator M_1.

Let us therefore look at $M_1\,|\psi\rangle_{12}$. The M_1 contains $H_1 = H_{01} + H_{M1}$. The second index in H_{01} is $j = 1$ and therefore refers to Alice. Because $|\psi\rangle_{12}$ does not contain ϕ information, we immediately can conclude: $H_{01}\,|\psi\rangle_{12} = 0$.

Let us then turn our attention to H_{M1}. Hence

$$H_{M1}\,|\psi\rangle_{12} = \frac{\hbar}{\Delta t}\begin{pmatrix} 0 & 0 \\ 0 & \frac{\partial}{\partial f}\end{pmatrix}_1 \frac{e^{-if}}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}_1 \otimes \begin{pmatrix}0\\1\end{pmatrix}_2 \qquad (4)$$
$$-\frac{\hbar}{\Delta t}\begin{pmatrix} 0 & 0 \\ 0 & \frac{\partial}{\partial f}\end{pmatrix}_1 \frac{e^{-if}}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}_1 \otimes \begin{pmatrix}1\\0\end{pmatrix}_2$$

Because of the matrix form of H_{M1} onlly the second term is non vanishing. And, aknowledging that

$$\frac{\hbar}{\Delta t}\begin{pmatrix} 0 & 0 \\ 0 & \frac{\partial}{\partial f}\end{pmatrix}_1 e^{-if} = -ie^{-if}\frac{\hbar}{\Delta t}\begin{pmatrix} 0\,0 \\ 0\,1\end{pmatrix}_1 \qquad (5)$$

and because

$$\frac{\hbar}{\Delta t}\begin{pmatrix} 0\,0 \\ 0\,1\end{pmatrix}_1 \begin{pmatrix}1\\0\end{pmatrix}_1 = \frac{\hbar}{\Delta t}\begin{pmatrix}0\\0\end{pmatrix}_1 \qquad (6)$$
$$\frac{\hbar}{\Delta t}\begin{pmatrix} 0\,0 \\ 0\,1\end{pmatrix}_1 \begin{pmatrix}0\\1\end{pmatrix}_1 = \frac{\hbar}{\Delta t}\begin{pmatrix}0\\1\end{pmatrix}_1$$

it follows that

$$-\frac{i\Delta t}{\hbar}H_{M1}\,|\psi\rangle_{12} = \frac{e^{-if}}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}_1 \otimes \begin{pmatrix}1\\0\end{pmatrix}_2 \qquad (7)$$

Therefore, the Margenau form $M\varphi = \psi$, viz. [1] is here equal to

$$M_1\,|\psi\rangle_{12} = \frac{e^{-if}}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}_1 \otimes \begin{pmatrix}0\\1\end{pmatrix}_2 \qquad (8)$$

This implies that an M_1 operator is possible where a similar form arises as with reduction of the wave packet when Alice measures $\begin{pmatrix}1\\0\end{pmatrix}_1$. Measurement here is entirely Hamiltonian / Schrödinger equation based without reduction of the wave packet.

3 Greenberger wave function

In [2] use is made of linear combinations of basic up and down states viz. their appendix A.

$$|\,\hat{\mathbf{n}},+\rangle = (\cos\theta/2)e^{-i\phi/2}\begin{pmatrix}1\\0\end{pmatrix} + (\sin\theta/2)e^{i\phi/2}\begin{pmatrix}0\\1\end{pmatrix}, \qquad (9)$$
$$|\,\hat{\mathbf{n}},-\rangle = (-\sin\theta/2)e^{-i\phi/2}\begin{pmatrix}1\\0\end{pmatrix} + (\cos\theta/2)e^{i\phi/2}\begin{pmatrix}0\\1\end{pmatrix}$$

The $\hat{\mathbf{n}}$ represents a unit normal vector in a frame of reference. The ϕ is the azimuthal angle and θ is the polar angle. The length of $\hat{\mathbf{n}}$ is unity. The states

$|\,\hat{\mathbf{n}},-\rangle$ and $|\,\hat{\mathbf{n}},+\rangle$ are indeed orthonormal like the two basis vectors of a spin configuration space, $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. But because the Greenbreger functions contain linear combinations of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ they cannot in an Einsteinian sense, represent the spin state of a single particle. The spin state of a single particle is represented by either $\begin{pmatrix}1\\0\end{pmatrix}$ exclusive or $\begin{pmatrix}0\\1\end{pmatrix}$ and is essentially without any angular measurement instrument related direction *before* measurement. In quantum mechanics spin is a discrete variable. In a letter to Schrödinger of 19 june 1935 [4, page 179] Einstein writes (cite from Howard:)

> In the quantum theory, one describes a real state of a
> system through a normalized function, $\psi\dots$
> Now one would like to say the following:
> ψ is correlated 1-1 with the real state of the system. (10)

If (10) is possible, then Einstein calls the theory complete. If (10) is not possible Einstein would call that theory incomplete. Therefore, employing the functions in (9) to represent a spin of a particle, e.g. $|\,\hat{\mathbf{n}},+\rangle$ for "up" and noticing that the quantum theory requires discrete spins either $\begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$, gives rise to an incomplete theory in the sense of Einstein.

Based on the $|\,\hat{\mathbf{n}},-\rangle$ and $|\,\hat{\mathbf{n}},+\rangle$ from (9) the entangled state $|\psi(\,\hat{\mathbf{n}})\rangle$ of (9) below, is equvalent to (2).

$$|\psi(\,\hat{\mathbf{n}})\rangle_{12} = \frac{e^{-if}}{\sqrt{2}}\left\{|\,\hat{\mathbf{n}},+\rangle_1\,|\,\hat{\mathbf{n}},-\rangle_2 - |\,\hat{\mathbf{n}},-\rangle_1\,|\,\hat{\mathbf{n}},+\rangle_2\right\} \tag{11}$$

Regarding (10),the present author would call (11) overcomplete because of (9). However, because $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ given in (11) is demonstrated by Greenberger et al [2, their appendix A] equal to $|\psi\rangle_{12}$, in (2) one can try to argue that Einstein's completeness restriction does not apply here.

But in order to render Einstein's completeness criterion (10) irrelevant in this case, it seems likely that one must also demonstrate $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ is *equivalent* in all respects to $|\psi\rangle_{12}$ in (2).We check this equivalence to the direct application of our particular Margenau operator, with Hamiltonians given in (3), that gives the $M_1\,|\psi\rangle_{12}$ in (8).

Therefore, we may ask if the M_1 in $M_1\,|\psi(\,\hat{\mathbf{n}})\rangle_{12} = M_1\,|\psi\rangle_{12}$ via a direct computation of $M_1\,|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ first. The latter can explicitly be written down as

$$M_1\,|\psi(\,\hat{\mathbf{n}})\rangle_{12} = \tag{12}$$
$$\frac{1}{\sqrt{2}}\left\{\left(M_1 e^{-if}\,|\,\hat{\mathbf{n}},+\rangle_1\right)|\,\hat{\mathbf{n}},-\rangle_2 - \left(M_1 e^{-if}\,|\,\hat{\mathbf{n}},-\rangle_1\right)|\,\hat{\mathbf{n}},+\rangle_2\right\}$$

We will deal with each M_1 containing term on the right hand side of (12) separately.

3.1 The term $M_1 e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1$

Looking at the definition of M_1 in (1) and the Hamiltonian in (3) we can obtain

$$M_1 e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 = e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 - \frac{i\Delta t}{\hbar} H_1 e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 \tag{13}$$

And so, because $H_1 = H_{01} + H_{M1}$ from equation (3) and (9) it follows

$$H_{01} e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 = \tag{14}$$

$$\frac{\hbar e^{-if}}{\Delta t} \left\{ -\frac{i}{2} (\cos\theta/2) e^{-i\phi/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 + \frac{i}{2} (\sin\theta/2) e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \right\}$$

Concerning the H_{M1} in (3) and the equations (5) and (6)

$$H_{M1} e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 = -ie^{-if} \frac{\hbar}{\Delta t} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_1 \mid \hat{\mathbf{n}}, +\rangle_1 = \tag{15}$$

$$-ie^{-if} \frac{\hbar}{\Delta t} (\sin\theta/2) e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1$$

Combining the previous two equations, i.e. (14) and (15), gives

$$M_1 e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 = e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 - \frac{1}{2} e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 = \frac{1}{2} e^{-if} \mid \hat{\mathbf{n}}, +\rangle_1 \tag{16}$$

3.2 The term $M_1 e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1$

In this case we have

$$M_1 e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 - \frac{i\Delta t}{\hbar} H_1 e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 \tag{17}$$

And so similarly to the exercise in the previous paragraph

$$H_{01} e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = \tag{18}$$

$$\frac{\hbar e^{-if}}{\Delta t} \left\{ \frac{-i}{2} (-\sin\theta/2) e^{-i\phi/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 + \frac{i}{2} (\cos\theta/2) e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \right\}$$

For H_{M1} we find

$$H_{M1} e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = -ie^{-if} \frac{\hbar}{\Delta t} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_1 \mid \hat{\mathbf{n}}, -\rangle_1 = \tag{19}$$

$$-ie^{-if} \frac{\hbar}{\Delta t} (\cos\theta/2) e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1$$

And so, via $H_1 = H_{01} + H_{M1}$

$$H_1 e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = \tag{20}$$

$$\frac{i}{2} \frac{\hbar e^{-if}}{\Delta t} \left\{ (\sin\theta/2) e^{-i\phi/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 - (\cos\theta/2) e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \right\}$$

Therefore, we may conclude that

$$M_1 e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 - \frac{1}{2} e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 = \frac{1}{2} e^{-if} \mid \hat{\mathbf{n}}, -\rangle_1 \quad (21)$$

4 Result

If this result (21) and the one of the previous subsection in (16) is inserted in (12) it then quite easily follows that

$$M_1 \,|\psi(\,\hat{\mathbf{n}})\rangle_{12} = \frac{1}{2} \,|\psi\rangle_{12} \quad (22)$$

and, for completeness, the $|\psi\rangle_{12}$ on the right hand side of (22) is as defined in equation (2).

This demonstrates that direct computation, as in (12), of $M_1 \,|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ with $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ based on the Greenberger functions as in (11), does not give the same result as M_1 representing a direct measurement of an "up" state at Alice's as in (8). This shows that the Greenberger $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ of (11) is *not* mathematically equivalent in all the relevant aspects, to the basic entanglement function in equation (2).

5 Extension

It is noted that in this approach where only the Schrödinger equation is there in measurement, the complete or extended H_{M-+} Hamiltonian is

$$H_{M-+} = \frac{\hbar}{\Delta t} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_2 \frac{\partial}{\partial f} \quad (23)$$

The other operator, H_{M+-} is

$$H_{M+-} = \frac{\hbar}{\Delta t} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_2 \frac{\partial}{\partial f} \quad (24)$$

It can be verified that the Margenau operator with H_{M+-} from (24) gives

$$\left(1 - \frac{i\Delta t}{\hbar} H_{M+-}\right) |\psi\rangle_{12} = -\frac{e^{-if}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \quad (25)$$

With the use of a coin toss (e.g. $s = 1$ when Heads, $s = 0$ when Tails), the entangled pair is then selected with

$$H_M = s\, H_{M-+} + (1 - s)\, H_{M+-} \quad (26)$$

The $s = 1$ means: up is flying towards Alice and down towards Bob. Therefore, the down$_{Alice}$-up$_{Bob}$ combination in the entangled form is erased. The $s = 0$ means: down is flying towards Alice and up towards Bob and the up$_{Alice}$-down$_{Bob}$ combination is erased. For mathematical convenience only the Alice side of the measurement was inspected in this paper.

6 Conclusion & discussion

In this paper we looked at the early criticism of Margenau on the EPR paradox [3]. Margenau sought to save quantum theory by rejecting the projection or reduction of the wave packet, postulate. The reduction of the wave packet [3, in equation (7)] plays a crucial role in the EPR paradox. Einstein disagreed with Margenau [4, page 185] because, in my own words, irrespective of reduction of the wave packet, a joint state $\psi(AB)$ still would exist. Denying the reduction or projection postulate doesn't help much in denying the paradox. The $\psi(AB)$ existence would still give the inseparability of entangled particles. Nevertheless, the Margenau operator can serve to show a disparaty between a Greenberger entangled state $|\psi(\hat{\mathbf{n}})\rangle_{12}$ and an entangled state based on basic spin states $|\psi\rangle_{12}$.

Suppose we are allowed to select a certain form of the Margenau operator $M = 1 - \frac{i\Delta t}{\hbar}H$. The H is Hermitian. In particular if the Schrödinger equation in the Margenau operator is construed so that one can derive a similar result as is obtained for reduction of the wave packet, then, it is possible to observe a difference between the basic entanglement of the two spins and the angular data containing Greenberger [2] spin wave functions. Margenau already discussed the point [1] that M is not unique. However, one can not off-hand discard the Margenau operator presented here. This is true because whether physical or not, with $M_1 |\psi\rangle_{12}$ the reduced form is obtained representing an Alice measurement.

It was found that $M_1 |\psi(\hat{\mathbf{n}})\rangle_{12} = \frac{1}{2} |\psi\rangle_{12}$ and this is *not* the wave packet that arose because of the measurement of Alice:

$$\frac{e^{-if}}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2.$$

Please do note that the f in e^{-if} is not necessarily equal to the azimuthal angle ϕ. The essential point is that entanglement is described, contrary to Greenberger, in a 1-1 relation to the basic spine wave functions; here represented by the states $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Therefore the description is Einstein complete (10).

The Greenberger wave functions resulting in $|\psi(\hat{\mathbf{n}})\rangle_{12}$ are employed to derive the quantum violation of the Bell inequality [2]. It is claimed by Greenberger [2, their appendix A] that $|\psi(\hat{\mathbf{n}})\rangle_{12}$ is equal to the entanglement of the basic spins, $|\psi\rangle_{12}$. If we however *first* employ the M_1 *before* employing mathematical equivalence between $|\psi(\hat{\mathbf{n}})\rangle_{12}$ and $|\psi\rangle_{12}$, then $M_1 |\psi\rangle_{12} \neq M_1 |\psi(\hat{\mathbf{n}})\rangle_{12}$.

This leads us to: the operation

$$\mathcal{E} = \text{"There is a, =, between } |\psi(\hat{\mathbf{n}})\rangle_{12} \text{ and } |\psi\rangle_{12}\text{"},$$

which does not commute with M_1. Or: $[\mathcal{E}, M_1] |\psi(\hat{\mathbf{n}})\rangle_{12} \neq 0$. Note that there is no reason to claim that $\mathcal{E}$ is tighter binding than M_1. If a reader objects to the $\partial/\partial\phi$ of H_{01} in (3), proper reasons *must* be given. The question is why a

Hermitian Hamiltonian referring to a measurement process may not contain operators that work on the angular position of the instrument in space. A similar question goes for the e^{-if} phase factor in (2) and (11). Considering ${}_{12}\langle\psi\,|\psi\rangle_{12} = 1$ the phase factor in $e^{-if}/\sqrt{2}$ doesn't make any difference from the usual $1/\sqrt{2}$. Then the question is: on what grounds is it forbidden to see a Hermitian Hamiltonian in a Margenau operator containing $\partial/\partial f$.

The present paper shows that there exists a difference between the entangled basic states $|\psi\rangle_{12}$ vs the angle information containing variant of Greenberger $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ viz. [2]. This is in terms of Margenau equivalence to reduction of the basic entangled spins wave packet. It is the $s = 1$ in terms of the extension of section - 5. As can be observed from (9) the $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$ contains angular information.

Or, to wrap it up. Given an attempt to give Margenau due credit and accept that only Schrödinger equation dynamics occurs in measurement, the Greenberger entangled wave packet is not "≡" to the basic entangled wave packet (2) in all relevant aspects. Wave packet reduction is hypothetical and following Einstein [4], irrelevant to the entanglement problem. One can argue Eintein incompleteness for theories based on $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$. But that can be ignored by Greenberger because of claimed equivalence. It was demonstrated in this paper: there is no equivalence. This was done with direct application of our version of M_1. There arise non-commuting operations due to azimuthal angle information in $|\psi(\,\hat{\mathbf{n}})\rangle_{12}$. The difference is first and foremost mathematically because, with direct M_1, it follows, $\mathcal{E}M_1\,|\psi\rangle_{12} \neq \mathcal{E}M_1\,|\psi(\,\hat{\mathbf{n}})\rangle_{12}$.

Declarations

The author has no conflict of interest.
No funding was received.
There is no data associated.

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