A Proof For 3X+1 Guess

Baoyuan Duan

duanby@163.com, Yanliang, Xian, People's Republic of China

Abstract

Build a special odd tree model according with $(\times 3+1)\div 2^{k}$ algorithm, to depart odd numbers in different groups.Carry out research into the tree,found that counts of elements in the tree reduce and converge downward one by one layer,values of elements converge downward to 1,and all odd numbers can appear in the tree only one time.Then prove the "3x+1"guess indirectly.At last,find a logic error of deduction of the tree,and prove 3x+1 guess in another ways,and give a solution for any odd converge to 1 equation.

Key Words:3x+1 odd tree

Ⅰ Prove Some Lemmas

Lemma 1: All odd numbers can be written in one of three forms:6a+1,6a+3,6a+5(a>=0).

Lemma 2: Transform odd 6a+1: $\left((6a+1)\times 2^{2k}-1\right)\div 3$,(a>=1,k>=1),generate an infinite data sequence,which each element is odd,and any element in the sequence is 4 times of the previous element(if exists) plus 1;The No. k odd make $(\times 3+1)\div 2^{2k}$,(k>=1)transformation get a result 6a+1.

Prove: k=1: $\left((6a+1)\times 2^{2}-1\right)\div 3=(24a+3)\div 3=8a+1$, is odd

k=2: $\left((6a+1)\times 2^{4}-1\right)\div 3=(96a+15)\div 3=32a+5$, is odd

Use method of induction,suppose $\left((6a+1)\times 2^{2k}-1\right)\div 3=2\text{m}+1$ m>0, k>=1

Then $\left((6a+1)\times 2^{2(k+1)}-1\right)\div 3=((6a+1)\times 2^{2k}-1)\div 3+((6a+1)\times 2^{2k}\times 3\div 3$

$=2m+1+((6a+1)\times 2^{2k}$ is odd

Hence each element in the sequence is odd.

$\left(\left(\left(\left((6a+1)\times 2^{2k}-1\right)\div 3\right)\right)\times 4+1=\left((6a+1)\times 2^{2(k+1)}-4\right)\div 3+1=\left((6a+1)\times 2^{2(k+1)}-1\right)\div 3$

Hence No. k+1 element is 4 times No. k element Plus 1.

With $\left((6a+1)\times 2^{2k}-1\right)\div 3$ make transformation of $(\times 3+1)\div 2^{2k}$ obviously get 6a+1.

Lemma 3: Transform odd 6a+5: $\left((6a+5)\times 2^{2k-1}-1\right)\div 3$,(a>=0,k>=1),generate an infinite data sequence,which each element is odd,and any element in the sequence is 4 times of the previous element(if exists) plus 1;The No. k odd make $(\times 3+1)\div 2^{2k-1}$,(k>=1)transformation get a result 6a+5.

Prove: k=1: $\left((6a+5)\times 2\ \ -1\right)\div 3=(12a+9)\div 3=4a+3$ is odd k=2: $\left((6a+5)\times 2^{3}-1\right)\div 3=(48a+39)\div 3=16a+13$ is odd

Use method of induction,suppose $\left((6a+5)\times 2^{2k-1}-1\right)\div 3=2\mathrm{m}+1$ m>0,k>=1

Then $\left((6a+5)\times 2^{2k+1}-1\right)\div 3=((6a+5)\times 2^{2k-1}-1)\div 3+((6a+5)\times 2^{2k-1}\times 3\div 3$

$$=2m+1+((6a+5)\times 2^{2k-1}$$ is odd

Hence each element in the sequence is odd.

$$\left(\left(\left((6a+5)\times 2^{2k-1}-1\right)\div 3\right)\right)\times 4+1=\left((6a+5)\times 2^{2(k+1)-1}-4\right)\div 3+1=\left((6a+5)\times 2^{2(k+1)-1}-1\right)\div 3$$

Hence No. k+1 element is 4 times No. k element Plus 1.

With $\left((6a+5)\times 2^{2k-1}-1\right)\div 3$ make transformation of $(\times 3+1)\div 2^{2k-1}$ obviously get 6a+5.

Lemma 4: Transform odd 6a+3: $(6a+3)\times 2^{k}-1$,(a>=0,k>=1),data generated can not be divided exactly by 3.

Ⅱ Build A Tree Model

Below we build digit tree using Lemma 1--4.

Use 1 as root place in layer 0,data sequence generated by transformation of $(1\times 2^{2k}-1)\div 3$,(k>1) place in layer 1,as up node of 1 in layer 0.Here because case k=1, $(1\times 4-1)\div 3=1$,is duplication of 1 in layer 0,remove it from layer 1.

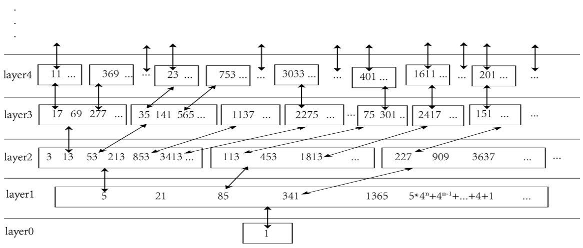
Build layer 2 using layer 1,for every element in layer 1,make transformation in order as follow:

Case element is 6a+1: do $\left((6a+1)\times 2^{2k}-1\right)\div 3$,(a>0,k>=1),generate an odd sequence place behind of the sequence last generated in layer 2,as up node of 6a+1 in layer 1.

Case 6a+5: do $\left((6a+5)\times 2^{2k-1}-1\right)\div 3$,(a>=0,k>=1) ,generate an odd sequence place behind of the sequence last generated in layer 2,as up node of 6a+5 in layer 1.

Case 6a+3:do nothing.

Use same method do transformation in order for every element of every sequence in one layer ,and generate data and node of next layer.Finally we can build a determined unique digit tree,as shown in graph 1.



Graph 1: 3x+1 odd tree

Ⅲ Prove Some Properties Of The Tree

Below prove some mathematical properties of the tree.

Property 1: All elements in the tree are odd.

According to Lemma 1--4 can get it.And all elements downward calculation accord to $(\times 3+1)\div 2^{2k}$,(k>=1)algorithm.I call it "3X+1 odd tree".

Property 2: If a data is an element of the tree,use the data as root build a tree using same method,then all elements in the new tree are elements in the tree.

Property 3: Elemens in random one sequence are arrayed by the order…6a+1,6b+5,6c+3…,first element is one of three forms.

Prove:We can easily know random one form of 3 forms can appear in the first position of a sequence as regulation of graph 1.To random layer i sequence j,if no. k element in the sequence is 6a+1,then no. k+1 element is $(6a+1)\times 4+1=24a+5=6b+5$,is form of 6b+5;No. k+2 element is $((6a+1)\times 4+1)\times 4+1=96a+21=6c+3$, is form of 6c+3.

Similarly we can prove case of first element is 6b+5 or 6c+3.

We can also prove that the first element of the sequence generated by odd 6b+5 upward calculation is 2 times of the first element generated by odd 6a+1(if exists) plus 1.

Property 4: Any element(except 1)in the tree do downward calculation $(\times 3+1)\div 2^k$,(k>=1),get an unique data,drop and only drop one layer.

Property 5: Numbers of elements in the tree diverge upward,reduce and converge downward one by one layer.

Prove:According to Lemma 2--4 and Property 3,each element(except 6a+3) in one layer build an infinite sequence in the up layer,on the contrary,one sequence in a layer can only build one element in the down layer.Hence,numbers of elements in the tree reduce downward one by one layer.The lowest layer have only one element 1,then numbers of elements converge downward.Because all elements are generated from 1 upward calculation one by one layer,all elements do downward calculation converge to 1.This is to say,although each element does downward calculation,data generated per time is sometimes big,sometimes small varying,the numbers of data it can build reduce per time,data value finally converge to 1.

Property 6: If the position of a data is determined in the tree,then the downward calculation route of the data is unique.

Property 7: The first element of random one sequence(except sequences in layer 0,1) can not be built from an odd $(\times 4+1)$ calculation.

Prove:With random layer i sequence j(i>1,j>=1),if no. k element is 6a+1,the first element of the corresponding sequence in layer i+1 is $((6a+1)\times 2^2-1)\div 3=(24a+3)\div 3=8a+1$, $(8a+1-1)\div 4=2a$,is an even,hence first element can not be gotten from odd;If no. k element is 6a+5,the first element of the corresponding sequence in layer i+1 is $((6a+5)\times 2\ \ -1)\div 3=(12a+9)\div 3=4a+3$, $(4a+3-1)=4a+2$,can not be divided exactly by 4.Sequences in layer i+1 can only be built from 6a+1 or 6a+5,hence we prove it.This is to say,the first element can not be written in binary form x01…01(odd x>1).

Property 8: If two elements in two sequences are equal,then the first elements in the two sequences are equal,the two sequences are complete equivalence.

Prove:According to Lemma 2--3 and Property 7,if the first element of layer i sequence j A_{ij} is a_{ij1},the sequence can be written in binary form a_{ij1}, a_{ij1}01…01, layer l sequence m can be written in binary form a_{lm1}, a_{lm1}01…01, a_{ij1} and a_{lm1} itself can not be written in binary form x01…01(odd x>1).If a_{ij1} is equal to a_{lm1},then two sequences are complete equivalence.If a certain a_{ij1}01…01 is equal to a certain a_{lm1}01…01,do $(-1)\div 4 = \text{odd}$ calculation separately to them,datas gotten must be equal.Continue do until can not perform this calculation,should get data a_{ij1} and a_{lm1}, a_{ij1}= a_{lm1}, two sequences are complete equivalence.

Property 9: Elements in same layer are not equal to each other.

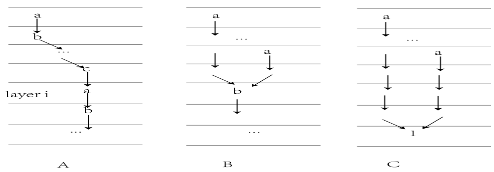
Prove:Obviously,elements are not equal to each other in layer 0 and layer 1.To prove elements in other layers are not equal to each other in same layer,we only need to prove first elements of sequences in same layer are not equal to each other.With random 6a+1 and 6b+5,do $((6a+1)\times 2^2 - 1)\div 3 = (24a+3)\div 3 = 8a+1$,

$((6b+5)\times 2 \ -1)\div 3 = (12b+9)\div 3 = 4b+3$,if 8a+1=4b+3,then 4a-1=2b,odd=even,contradict.Hence,first elements of random sequences built from 6a+1 and 6b+5 are not equal to each other.If elements are not equal to each other in layer i,with all 6a+1 and 6b+5 elements in layer i,do calculations above,get sequences in layer i+1,which first elements are not equal to each other.

Property 10: Elements in different layers are not equal to each other.

Prove:If data a exists in two different layers,there are 3 cases,as shown in graph 2.

Case A:downward calculation route of a include a.From layer i downward,each layer should have a sequence which is complete equivalence to one sequence in some one up layer,but obviously sequence in layer 1 is not complete equivalence to any sequence in up layers,because sequence in layer 1 has the binary form 101,101…01, sequences in up layers have the form x,x01…01(odd x>1),x itself can not be written in binary form y01…01(odd y>1).Hence,case A is not established.In fact,in case A we even can not use data a to build a tree(upward,downward,horizontal expand) which counts of elements reduce downdward.



Graph 2: Same Data In Different Layers

Case B and C: downward calculation routes of a in two different layers intersect in some one down layer or layer 0.Since each downward calculation get a unique result,to let two different routes intersect, one of the two routes must do cross-layer calculation,do not accord to Property 4.

Attention,in the proof of Property 9 and Property 10,we use special characters of layer 0 and layer 1,is just to make proof easier,during the procedure of tree model building,we only use Lemma 1--4,this is to say, use any odd number to build a tree,it should also accord to Property 1--10.

Ⅳ Prove 3X+1 Guess

Below prove all odds do $(\times 3+1)\div 2^k$ calculation must converge to 1.

Prove:If the data is in “3X+1 odd tree” built above,it must converge to 1.If exists an odd a do not appear in the tree,use a build a tree with same methods above,then all elements in the new tree do not appear in “3X+1 odd tree”,but the new tree also accord to Property 1--10.

Since $(2x+1)\times 3+1 = 6x+4$,(x>0),to random x>0,k>0, $(6x+4)\div 2^k \neq 2x+1$,hence a do not converge to itself.Suppose with a do downward calculation converge to odd b,and b>1,this case b can continue do $(\times 3+1)\div 2^k$ calculation,and layer number reduce for every calculation,possible datas can be generated are fewer and fewer,until build 1,can not continue do calculation.Hence a must converge to 1,and must exist in “3X+1 odd tree”.

Come here,we have proved “3X+1”guess.In fact, “3X+1 odd tree”include all odd numbers,and each odd number appear only one time.

Ⅴ Some Strong Proof

Proof above I think is enough,although is weak.Below make it stronger.

Property 11: Use any odd a as root build a “3x+1” odd tree(if a is with form 6b+3,use 4a+1 as root),all elements in the tree are different.

Prove:Through Lemma 1--4 and Property 1--10 ,we know if two elements in the tree are equal,their data sequences are equivalence,the first element of each sequence must be equal.and we know all first elements of all dada sequences are built from x or y through (2x-1)/3 or (4y-1)/3(a can also be built from odd in its next layer,except 1),if they are equal,should have (2x-1)/3=(4y-1)/3,or(2x-1)/3=(2y-1)/3,or(4x-1)/3=(4y-1)/3,the last two formulas have no meaning. If (2x-1)/3=(4y-1)/3, then x=2y,x is even,contradiction.This means an element in the tree can only be built by only one element through only one method,there exist only one route to produce it,it is not possible to exist two same data in different routes.

On the other hand,in a normal a root tree,layer 0 has one or two elements,layer 1 has n($n=\infty$) elements,layer 2 has about n^2 elements,layer 3 has about n^3 elements…,and elements in up layer can produce all elements in down layer through one step.If there is a loop a->b->c->d->a->b->c->d->a through $(\times 3+1)\div 2^k$ calculation.use a to build a tree,each layer should has about n^4 elements,count of elements in each layer are same,then n^4 elements produce n^4 elements through one step $(\times 3+1)\div 2^k$ calculation,this is not possible,because a can produce b,a01,a0101 can also.

Hence,all elements in the tree are different.

We can also find, if a normal tree do not expand downward and horizontal,the second layer from bottom has only one data sequence,although it still has infinite elements, its elements are equivalence to do downward calculation. 1 root tree also has this property.Hence any odd a>1 can do downward calculation,never repeat,layer number reduce for every calculation,possible datas can be generated are fewer and fewer,until build 1.

Ⅵ Some More Strong Proof

Above proof I have found a logic error.Although all elements of the tree model built using above way are different,we can not get conclusion that there is not exist a loop,because if elements are in the same layer,they could form a loop!here give an example.

if odd x do n times $(\times 3+2y+1)\div 2^k$ (y>1)calculation can build back to x,we get:

$$x=\frac{3^n x+(2y+1)\times(3^{n-1}+3^{n-2}\times2^{p1}+3^{n-3}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+..+p_{n-2}}+2^{p_1+p_2+..+p_{n-1}})}{2^{p_1+p_2+..+p_n}}$$

We can also use above way to build an odd tree,but this time,there exist many loops!

For example with 13 do $(\times3+11)\div2^k$ calculation,have:

$$13\xrightarrow{1}25\xrightarrow{1}43\xrightarrow{2}35\xrightarrow{2}29\xrightarrow{1}49\xrightarrow{1}79\xrightarrow{3}31\xrightarrow{3}13$$

These datas can form a loop,because they are in the same layer,do not satisfy up-down hierarchical structure,they produce themselves in the same layer.other datas which do not form a loop(including 1 layers,this is to say x->x) can according with propertys of the tree,and should be converged.

Continue to research this normal case, we suppose odd x do n times $(\times3+2y+1)\div2^k$ (y>1) calculation can build 2y+1,which is the nature root of $(\times3+2y+1)\div2^k$ calculation,because to any 2y+1, $((2y+1)\times3+2y+1)\div2^2=2y+1$.Then:

$$3^n x+(2y+1)\times(3^{n-1}+3^{n-2}\times2^{p1}+3^{n-3}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+..+p_{n-2}}+2^{p_1+p_2+..+p_{n-1}})=(2y+1)\times2^{p_1+p_2+..+p_n}$$

$3^n x$ should be divided by (2y+1),this is to say,only if x is multiple times of (2y+1),it is possible to converge to 2y+1,other case converged to other roots,or form a loop.

Below research $(\times3+1)\div2^k$.

Suppose odd x do n times $(\times3+1)\div2^k$ calculation can build 1,then have:

$$3^n x+3^{n-1}+3^{n-2}\times2^{p_1}+3^{n-3}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+..+p_{n-2}}+2^{p_1+p_2+..+p_{n-1}}=2^{p_1+p_2+..+p_n}$$ Equation (1).

Below prove this equation always has solution.Do deformation to this equation:

$$3^n x+3^{n-1}+3^{n-2}\times2^{p1}+3^{n-3}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+...+p_{n-2}}=2^{p_1+p_2+...+p_n}-2^{p_1+p_2+...+p_{n-1}}$$
$$=2^{p_1+p_2+...+p_{n-1}}(2^{p_n}-1)$$

right side of equation should be divided exactly by 3,obviously,only if $p_n=2k$,or p_n is even.let $2^{p_n}-1=3b_n$,b_n is odd,then above equation become:

$$3^n x+3^{n-1}+3^{n-2}\times2^{p1}+3^{n-3}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+...+p_{n-2}}=2^{p_1+p_2+...+p_{n-1}}\times3b_n$$

Both side divided by 3:

$$3^{n-1}x+3^{n-2}+3^{n-3}\times2^{p1}+3^{n-4}\times2^{p_1+p_2}...+3\times2^{p_1+p_2+...+p_{n-3}}=2^{p_1+p_2+...+p_{n-2}}\times(2^{p_{n-1}}b_n-1)$$

Same way, let $2^{p_{n-1}}b_n-1=3b_{n-1}$, b_{n-1} is odd,this is easy to meet,because $3b_{n-1}$ +1 is even,then can be divided by $2^{p_{n-1}}b_n$ .if b_{n-1} is determined,p_{n-1} and b_n is also unique determined.if b_n is determined, b_{n-1} and p_{n-1} have infinite solutions.

Do this continue.finally we have:

$$3^2 \times x + 3 = 2^{p_1} \times (2^{p_2} b_3 - 1),$$

Let $2^{p_2} b_3 - 1 = 3b_2$,b_2 is odd,then have:

$3 \times x + 1 = 2^{p_1} b_2$, this is also easy to meet, because x is odd, $3 \times x + 1$ is even, then can be divided by $2^{p_1} b_2$,and p_1 and b_2 has unique soliution.Here should attention,x can be any odd,there is no any other limit.This is to say any odd is possible to converge to 1.

Through above,as long as we let

$3 \times x + 1 = 2^{p_1} b_2$, $2^{p_2} b_3 - 1 = 3b_2$ … $2^{p_{n-1}} b_n - 1 = 3b_{n-1}$, $2^{p_n} - 1 = 3b_n$,calculate back,we can get solution.In the last step $2^{p_n} - 1 = 3b_n$, b_n should be with binary form 101,101…01,if is not ,can continue do $(\times 3 + 1) \div 2^k$ calculation,until get solution,because here n,p_1,p_2…p_n are flexible if x is flexible,to one defined x,we can always find a group n,p_1,p_2…p_n to satisfy the Equation (1).And with each middle step, $2^{p_{n-1}} b_n - 1 = 3b_{n-1}$,there is no any limit to next step data b_n,it can be any odd in formula level.To one b_n,there exists infinite number of b_{n-1} and p_{n-1} in formula level,this enlarge the opportunity of finding solution.And $(\times 3 + 1) \div 2^k$ has not any other root,because if $(x \times 3 + 1) \div 2^k = x$,only get solution x=1.And if there exists a data x form a loop,there should not be any possible to produce 1,is discordant to above proof.Here we can not get a general formula solution of n,p_1,p_2…p_n,because with each x,the converge route is different.

Thus, we prove 3x+1 guess.

Ⅶ A Solution For Any Odd Converge To 1 Equation

We know 1 do random n steps $(\times 3 + 1) \div 2^2$ can converge to 1,have:

$$3^n + 3^{n-1} + 3^{n-2} \times 2^2 + 3^{n-3} \times 2^4 ... + 3 \times 2^{2n-4} + 2^{2n-2} - 2^{2n} = 0$$

Below we use it to prove and search for solution for any odd x converging to 1. Here we suppose n>4,because case n<=4 can be easily vertified directly.Suppose:

$$3^n x + 3^{n-1} + 3^{n-2} \times 2^{p_1} + 3^{n-3} \times 2^{p_1+p_2} ... + 3 \times 2^{p_1+p_2+..+p_{n-2}} + 2^{p_1+p_2+..+p_{n-1}} - 2^{p_1+p_2+..+p_n} = 0 \qquad \text{Formula (2)}$$

With x do reform $x = a_m \times 3^m + a_{m-1} \times 3^{m-1} + ... + a_1 \times 3 + a_0$,a_m…a_0=0,1,0r 2.Then:

$$3^n x = 3^n \times (a_m \times 3^m + a_{m-1} \times 3^{m-1} + ... + a_1 \times 3 + a_0) = (2a + 1) \times 3^n$$

If a_m>1 or a_m=1 and $(a_{m-1}\times 3^{n+m-1}+...+a_1\times 3^{n+1}+a_0\times 3^n)>(3^{n+m-1}+3^{n+m-2}\times 2^2...+3^n\times 2^{2(m-1)})$,make

$x=3^{m+1}-3^m+a_{m-1}\times 3^{m-1}+...+a_1\times 3+a_0$ or :

$x=3^{m+1}-2\times 3^m+a_{m-1}\times 3^{m-1}+...+a_1\times 3+a_0$

Here suppose a_m=1 and $(a_{m-1}\times 3^{n+m-1}+...+a_1\times 3^{n+1}+a_0\times 3^n)<(3^{n+m-1}+3^{n+m-2}\times 2^2...+3^n\times 2^{2(m-1)})$.

Build equation:

$3^{n+m}+3^{n+m-1}+3^{n+m-2}\times 2^2+3^{n+m-3}\times 2^4...+3\times 2^{2(n+m)-4}+2^{2(n+m)-2}-2^{2(n+m)}=0$ Formula (3)

If x can converge to 1,Formula (2) and Formula (3) should be equivalence.Below we reform Formula(3) to form of Formula (2).

First let:

$t=(3^{n+m-1}+3^{n+m-2}\times 2^2...+3^n\times 2^{2(m-1)})-(a_{m-1}\times 3^{n+m-1}+...+a_1\times 3^{n+1}+a_0\times 3^n)=t_n\times 3^n$,

because x is odd,this is odd minus even,t_n should be odd.become t_n to binary form and let:

$t_n\times(2+1)\times 3^{n-1}+3^{n-1}\times 2^{2m}-3^{n-1}=t_{n-1}\times 3^{n-1}$,this is just with 3^n part multify (2+1) become 3^{n-1} part,and plus corresponding part in Formula (3),minus corresponding part in Formula (2),from now t_{n-1} become even.Continue:

$t_{n-1}\times(2+1)\times 3^{n-2}+3^{n-2}\times 2^{2m+2}-3^{n-2}\times 2^{p_1}=t_{n-2}\times 3^{n-2}$,and let 2^{p1} equal to minimum bit of even part.and here, to $t_{n-1}\times(2+1)$,odd part add 1 or 2 bits, if add 1, $+2^{2m+2}$ should be operate in MSB bit, if add 2, $+2^{2m+2}$ should be operate in MSB-1 bit. Continue:

$t_{n-2}\times(2+1)\times 3^{n-3}+3^{n-3}\times 2^{2m+4}-3^{n-3}\times 2^{p_1+p_2}=t_{n-3}\times 3^{n-3}$, and let 2^{p1+p2} equal to minimum bit of even part,because LSB bit no. of odd part of t_i increase continuouslly,this is easily to finish.

Watch t_i,during iteration,the count of succession 1 in high part should be unchanged or increased.Why?Because if t_{i-1} is with form 10…,obviously, count of succession 1 in high part is unchanged or increased. if t_{i-1} is with form 111…,after do $\times(2+1)$,should become 101…, $+2^{2m+2}$,become 111…, count of succession 1 in high part is also unchanged or increased.Other cases can be proved easily.Some cases can increase,for example, if t_{i-1} is with form 110110…, t_i become 1110…

Do this iteration continuously, count of succession 1 in high part of odd part of t_i is unchanged or increased,LSB no. is also increased,hence ,finally, t_i can become form of 11…,just $2^k\times(2^j-1)$ form.Stop here,do not do $\times(2+1)$ again,it is already converge to 1.do $-2^{2(n+m)}$,it should be operate in MSB+1 bit,because MSB of $+2^{2k}$ is forever equal to MSB+1 of t_{i-1},which is the previous item.Hence minus result can be equal to $-2^{p_1+p_2+..+p_n}$,thus prove 3x+1 guess and get solution of Formula (2).

Below give a specific example,x=7.

We know,to 7 do $(\times 3+1)\div 2^{k}$,have:

$$7\xrightarrow{1}11\xrightarrow{1}17\xrightarrow{2}13\xrightarrow{3}5\xrightarrow{4}1$$

Suppose:

$$3^{n}\times 7+3^{n-1}+3^{n-2}\times 2^{p_1}+3^{n-3}\times 2^{p_1+p_2}...+3\times 2^{p_1+p_2+..+p_{n-2}}\quad +2^{p_1+p_2+..+p_{n-1}}-2^{p_1+p_2+..+p_n}=0$$

$$3^{n}\times 7=3^{n}\times(2\times 3+1)=3^{n}\times(3^{2}-3+1)=3^{n+2}-3^{n+1}+3^{n}$$

Build:

$$3^{n+2}+3^{n+1}+3^{n}\times 2^{2}+3^{n-1}\times 2^{4}...+3\times 2^{2n}\ +2^{2n+2}-2^{2n+4}=0$$

$$3^{n+1}+3^{n}\times 2^{2}+3^{n+1}-3^{n}=(2^{3}+1)\times 3^{n}$$

*(2+1) and +2^4: $(2^{3}+1)\times(2+1)\times 3^{n-1}+2^{4}\times 3^{n-1}=(2^{5}+2^{3}+2\ +1)\times 3^{n-1}$

-3^{n-1}: $(2^{5}+2^{3}+2\ +1)\times 3^{n-1}-3^{n-1}=(2^{5}+2^{3}+2\ )\times 3^{n-1}$

*(2+1) and +2^6: $(2^{5}+2^{3}+2)\times(2+1)\times 3^{n-2}+2^{6}\times 3^{n-2}=(2^{7}+2^{5}+2^{4}+2^{3}+2^{2}+2)\times 3^{n-2}$,

Let p1=1,and delete item 2: $(2^{7}+2^{5}+2^{4}+2^{3}+2^{2}+2-2)\times 3^{n-2}=(2^{7}+2^{5}+2^{4}+2^{3}+2^{2})\times 3^{n-2}$

*(2+1) and +2^8: $(2^{7}+2^{5}+2^{4}+2^{3}+2^{2})\times(2+1)\times 3^{n-3}+2^{8}\times 3^{n-3}=(2^{9}+2^{8}+2^{5}+2^{4}+2^{2})\times 3^{n-3}$

Let p1+p2=2,and delete item 2^2: $(2^{9}+2^{8}+2^{5}+2^{4}+2^{2}-2^{2})\times 3^{n-3}=(2^{9}+2^{8}+2^{5}+2^{4})\times 3^{n-3}$

*(2+1) and +2^{10}: $(2^{9}+2^{8}+2^{5}+2^{4})\times(2+1)\times 3^{n-4}+2^{10}\times 3^{n-4}=(2^{11}+2^{10}+2^{8}+2^{7}+2^{4})\times 3^{n-4}$

Let p1+p2+p3=4,and delete item 2^4: $(2^{11}+2^{10}+2^{8}+2^{7}+2^{4}-2^{4})\times 3^{n-4}=(2^{11}+2^{10}+2^{8}+2^{7})\times 3^{n-4}$

*(2+1) and +2^{12}: $(2^{11}+2^{10}+2^{8}+2^{7})\times(2+1)\times 3^{n-5}+2^{12}\times 3^{n-5}=(2^{13}+2^{12}+2^{11}+2^{7})\times 3^{n-5}$

Let p1+p2+p3+p4=7,and delete item 2^7: $(2^{13}+2^{12}+2^{11}+2^{7}-2^{7})\times 3^{n-5}=(2^{13}+2^{12}+2^{11})\times 3^{n-5}$

Now become 111…,iteration finished,steps n=5.And
$2^{13}+2^{12}+2^{11}-2^{(2\times 5+4)}=-2^{11}=-2^{p_1+...+p_5}$.

Thus we have proved 3x+1 guess strictly.During the middle procedure of iteration,we may estimate the value of steps n through some estimating models.