New theoretic fractal framework for material science and engineering

Izotov A.D.
Member-correspondent of RAS, head of laboratory,
Institute of General and Inorganic Chemistry RAS;
Izotova V.O.
Head technologist Institute of General and Inorganic Chemistry RAS;
Mavrikidi F.I.
Senior researcher, PhD, Oil and Gas Research Institute, RAS.

Abstract. In present paper we accept the dynamic nature of fractal geometry as the equipoise of two universal formative processes, which are generated by two classes of forces in nature - forces of attraction, convergence and repulsion, divergence. This allows to introduce the divisibility of matter as a new degree of freedom in creating materials with desired bulk properties. It is shown, that this step connects previously disparate mathematical branches, as counterparts of fractals, into synthetic theory of matter. In the last section, a hypothesis about continuation of model to mathematical extent of Periodic Law, restricted by interpretation of nature of quantum numbers, is described. All mathematical structures are considered in parallel to basic notions of materials science.

Introduction. Materials science is one of the pillars of modern civilization, providing a solution of various problems of mankind. Technological progress, the creation of new medicines, artificial human organs, the development of communication and information technologies, the development of and access to the practical needs of space technology - all this is unthinkable without advanced materials science. Modern development of materials, its access to the frontiers of nanoscale science has put it at the center of environmental, humanitarian and political problems [1, 2]. Their novelty and complexity provide the science of matter with inter-disciplinary nature, dictate the need for logical consistency of the data between the various branches of science, and the synthesis of various theories within each of the disciplines.

This trend contradicts with general specialization of science, when each discipline has its own language and methods. Necessary to mention the mathematics – the science, professing the strongest abstraction, radical separation from the matter of other sciences. This fact hinders development of theoretical and computational methods in materials technology.

The science of matter is not a highly technical discipline, its roots go back in antiquity, pre-Socratic philosophy and Plato. In the idea of synthesis it is associated with many deep results in mathematical physics and theoretical computer science, which remained in the shadow of the standard physical paradigm.

With the advent of the theory of fractals situation took a new perspective. It was shown by vast volume of investigations the typicality of the fractal structure of material objects and the phase spaces of mathematical theories. This fact provides a new logical and mathematical basis for synthesis the disparate branches of science into coherent theory

The purpose of this article - to describe the new opportunities of approach to mathematical modeling in material science more adequate for modern problems and directed to work closely with the technology of creation and transformation of materials. The main result of our consideration looks as follows: transferring mathematical technique to fractal space-time, we get a continuous chain of formal results from set theory, number system, logic, functional analysis, geometry to matrix theory and theoretical computer science. This line can be considered as a candidate for system theory of matter, more adequate to materials science problems and technology. This chain is described in parallel with basing of materials science theory.

Description of the problem. Peculiarity problems of modern science is the transition from the use of finished materials to the creation of materials with desired bulk properties, i.e. to design and programming matter and its properties. What, in turn, implies the existence of some "theory of properties", allowing one to see their genesis and connection with material support.

Today, mathematical modeling in materials held with the intermediary role of mathematical physics. Categories of properties, their transformations and interconnections are not included in the standard of physical and mathematical theories and, therefore, are not the subject of science, while remaining fully in the experiment, the weight of which is difficult to overestimate. His principal contrast to the theory is in making possibility to obtain the desired characteristics as a function of the cumulative effects of different qualitative and formally irreducible to each other factors - heat, mechanical, field-like, geometrical and of shape. Experiment aims to reduce shortcomings mathematical theory, it complements fragmentary description of its isolation abstract methods. The experiment is a laboratory prototype technological process and actual operating conditions. Direct application of isolated theories is simply impossible without a "visa" of experiments. However, as necessary, he nevertheless did not form connected patterns of behavior related substances.

In view of financial and social responsibility of science the task of constructing a synthetic model of matter - formal analogue of experiment and process, is urgent and meets modern requirements.

Antinomy of matter, Zeno's paradoxes, and symmetry breaking. In materials technology one has two main objectives. The first problem is a way to build on a *"bottom-up"* – how bulk properties of the whole sample depends on its structure and composition. The second, *"top-down"* – how the internal structure depends on the behavior of the whole in its environment. From the general scientific viewpoint, this pair is a variant problem *"One-and-Many"*, formulated by Plato and known in modern science as a problem *"part and whole."* Its wording – *one requires formally describe and model interdependence of properties of the whole and its parts. How integral characteristics of the system arise from the local, and, conversely, how the properties of the parts are determined by integral sample.*

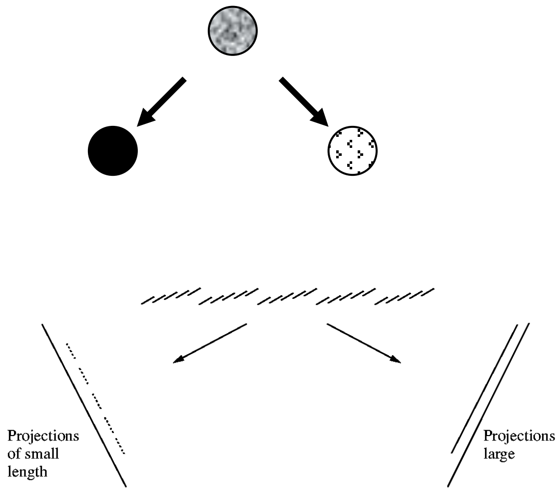
In materials science it becomes the problem of formalization of relations and properties of matter with the peculiarities of its atomic and molecular structure. In the second half of the twentieth century during the development of cybernetics and straightforward mathematization of sciences when was made the attempt to reduce all scientific knowledge, including chemistry, to physics, the problem *"part and whole"* has appeared in the spotlight. The overall experience has shown that the solution to this problem by methods of mathematical physics has not reached the goal - a formal mathematical theory of this problem does not exist. Today, there is its "chemically homogeneous" approximation - Multiscale Modeling [3, Ch. IV].

Problem *"part and whole"* is not only a problem of materials, but also biology, engineering and the humanities. Very many scientific problems can easily be reformulated as a problem of materials science. For example, in the oil and gas industry there is a problem of increase oil recovery factor, which is also a synthetic physicochemical problem [4]. Here the role of atomic structure of matter plays complex heterogeneous reservoir rock geometry, and sample properties have the form of the cumulative oil recovery. The same structure arise for the planning activities, design and decision-making in the management of complex technical, organizational, and ecological-economic systems.

Thus placing the problem of designing the material in the context of the problem of *«part and whole,"* one can move beyond fragmented theories to the crossroad of interdisciplinary and intertheoretical ties that corresponds substantially to the role of materials science.

The problem of reconciling formal logic and human experience is known in the history of science as the paradoxes or antinomies of Zeno of Elea. Etymology of the term "*antinomy*" is "*contradiction between the two laws*." One of them ties parts into the whole, creating a *One*, second - divide the whole into parts, producing a *Many*. This means, that materials technology works with antinomic matter.

The antinomy of matter as a duality of its nature, can be illustrated as follows. Any volume of material (Figure 1 below) is on the one hand a continuous indivisible, whole, *One* (left arrow), and on the other – is *Many*, a compound objects (right arrow). In physics connected objects are called continuous media, compounded ones - an ensemble of particles, clusters or atomic-molecular structures.



From K. Falconer Fractal Geometry of Nature. Mathematical Foundations and Application. Wiley, 2003, P.96, Fig. 6.3

Fig. 1 Antinomy of matter and antinomy of fractals

Iteration relationship *"whole-part-whole-part - ..."* leads deep into the matter - its "elementary" particles, consisting of "more elementary", which in turn consist of even more "fundamental", etc. It is evidently, that there is nothing abstract in this antinomy, this is what one deals with in the practice of materials. The only problem is to formally distinguish integrity from divisibility and then link them into a single scheme. The difficulty is that, in practice of mathematical physics both divisibility and connectedness are formally described identically, and on a sheet of paper are indistinguishable. Formally *One* is identical *to Many*. For example, mentioned multiscale modeling, which obviously should bind phenomena at different "*depths of divisibility*" of matter, is formulated in the same variables as the mechanical motion of the indivisible solid.

Consider the process of infinite division of any volume of the substance. It is not a simple mechanical operation. Let us explain this. We take a sample of a substance with a given set of properties - strength, thermal conductivity, electrical conductivity. We run the county district on the process of infinite division and get to the elementary particles of matter. Where and when will disappeared bulk properties of matter? Obviously, the reverse movement, "bottom-up" raises the question - "where and when new properties of matter do appear?" This version of the paradoxes of Zeno "Dichotomy", it illustrates one of the main problems of materials – the phenomenon of size effects in the structure and properties of materials. In science, this is known as the broken symmetry in the process of formalization. It happened to be impossible after the reduction of sample of matter to elementary parts formally deduce from them its properties as a holistic object. In other words, at different scales the size of a substance exhibits different properties [5].

These logically contradictory ways "bottom-up" and "top-down" are typical of all the natural sciences. At one extreme materiality prevails - molecular biology, physical and chemical methods in geology, geography, physical methods, quantitative methods in economics, etc. Opposite pole of sizes formed transparent *unity of variety* of qualities - evolutionary theories and historical character in the sciences. Chemistry in this pair looks like *"quantum mechanics - Mendeleev Periodic Table."*

Fractals, physical chemistry, functional asymmetry of Nature. Fractal translated from Latin means "crushed", "fragmented." From the starting position model - fractal set is regarded as a formal counterpart of the divisible material object. Parts of which, in turn, are divisible, and so on to infinity. This ensures that the basic requirement of a mathematical model - the model structure is to be similar to structure of original object, which underlies, for example, all design schemes engineering.

The divisibility or "macroscopic quantization" of the matter as a special property took attention only recently, when a number of works of general scientific significance has appeared [6,7].

Divisibility, severability, fragmentation, dispersion, "zooming-into-details" – all this terms are fractal-nuanced descriptions of a native property of matter, independent of the bio-physical and chemical nature of the objects. Its universality goes on also into mathematical entities - numbers, functions, phase spaces. *"Finely divided matter"* has the same geometry as granular - *"coarse - grained"* mathematical phase spaces. Therefore, a chain of physical sizes of the "aggregate - cluster - molecule - atom - elementary particles - ..." series of parallel with fractal (sub)spaces of decreasing diameter.

Therefore fractals are binary, matter-symbol character of substance, common to physics and mathematics. Metaphor - *"fractals are fused continuum"* sufficiently accurately conveys this relationship. Relations to biology expresses formula - *"fractals are anatomy of matter"*. Fractals can be likened to the next step in the mathematical description - *"sharpening"* of the eucledean image, when its internal structure becomes more apparent and emerges *"the back side of the matter"*, invisible superficially. This is known as scaling or *"deepening in the details"*, or *"zooming - in"* in the theory of fractals (Fig.2).

Figure 2. Sharpening, and the appearance of parts of the structure.

Specificity of the problem of formalizing the fractality, adequate for technology issues arises while perception / measurement / studies / simulation in two modes observability - as a whole object and its structure directly on the set of scales (in the limit on all scales). Respectively, each characteristic will have a double meaning - as a macroscopic variable and as a unit of microscopic characteristics. So, for example, the second sense of the mass density in continuum mechanics is a probability density function. Therefore *"dual observability"* matches dynamics and thermodynamics [10, Part III].

Fractal geometry has largely indicated as the science of the internal geometry of the material. Today it is often presented via "fractional" versions of "smooth" physical - mathematical theories [11 - 18]. Relatively new is an extension of apparatus by ultrametric hierarchical spaces [19 - 22]. However, analysis shows that all variations of such approach are aimed to improve known physical theories without giving significant increase knowledge. Therefore there is a widespread sceptical opinion of physicists [23], and its weight is unreasonably small in Chemistry - "fractional" versions do not affect the "chemistry of fractals", i.e. dependence of the properties of matter on the scale. That is why, we set aside the popular theories of "mathematical" and "physical" fractals: the exact self-similarity - the triangle and the Sierpinski carpet, Menger continuum, nonlinear dynamics, fractional Brownian motion version etc. All of them are chemically indifferent and do not match main problem of materials science, as indicated above.

Chemical specifity appears whith the coherent development of ideas by I.V. Tananaev to consider dispersion or *particle size*, i.e. divisibility of substance throughout the range of values as a separate parameter, characterizing the properties of the substance. Then physicochemical analysis becomes *"composition-structure-dispersion-property*" theory. From this perspective, nano- and ultrafine system can be regarded as physical (pre)fractals, enlarged by chemical degree of freedom. Analogue of this idea in the theory of fractals has the form of material properties depending on the number of features and / or peculiarities of the reacting surfaces on the level of observation / measurement scale [24]:

$$\text{“number of properties”} \propto \text{“scale of measurement”}^{-D}$$

where D - fractal dimension of the sample geometry. So on, the single-scaled measurement and averaging technique adopted in physics are meaningless. At each level of the hierarchy there is definite language and description parameters, so the *multiscale description* requires universal protolanguage as a background for languages of all scales. Here we present triggered universatility of mathematical fractals in their number-theoretic, *2-adic* interpretation (see below), which have the form of binary strings of 0’s and 1’s. Universatility of binary encoding, along with feedback, i.e. nonlinearity, is, perhaps the most significant legacy of cybernetics.

There is a fundamental - a dynamic source of fractals, that generates antinomy of matter. It has been observed that fractals are traces of nonequilibrium processes of evolution and interaction of various media [25, 6]. It turns that two basic ways of creating materials - *"bottom up"*, i.e. aggregation, condensation, and *"top down"*, i.e. dispersion, scattering, are exact counterparts of two natural causes - form-created processes of material objects, which are known by different names: the convergence and divergence, entropy and negentropy, dispersion and coagulation, diffusion and aggregation, condensation and rarefaction, homogenization and differentiation. Convergence creates dense objects, this is the way *the "bottom-up"* approach. Divergent processes - crushed, scatter them - this way is *" top down"* one.

This pair of processes geometrically looks like a convergent - divergent arrows to the nodes of network. As a result, we come to the famous image of fractal attractors - they are created by a sequence of contraction maps, i.e. converged processes, and generate Lyapunov exponential divergence, i.e. divergence of trajectories. As a result, each cluster substances in a fractals space-time is obtained as composition of convergent-divergent resultant couples:

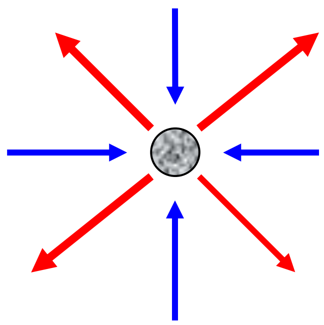


Fig.3 Convergent - Divergent formation of matter.

This situation is known as morphological symmetry - they are identical in geometry, and functional asymmetry - they are opposite in action. Each of these processes is the condition of existence of opposite one. Therefore, the *"double observability"* of substance gives a picture of alternation of continuity and discontinuity, aggregation and dispersion. Invariant of this situation – is the bodies, surrounded by fields and containing fields inside. For example, the atom - is the nucleus, surrounded by the electron shell, the living cell - the nucleus immersed in protoplasm. Thus, in a fractal space technology of materials and technology of nature are similar.

The idea of fractal synthesis. Word *complexity*, which characterizes the current state of the science of matter [8], means "woven together." Substance is plexus, or even a *"topological* *solution*" of spaces of different dimensions, each of which runs definite physical-chemical process. Technology of creating substances is the preparing this "*solution*". In it irreducible field, thermal, elastic-plastic, electromagnetic, etc. physical theories interact instantly with discrete, geometric, continuous, algebraic and analytic, deterministic and stochastic mathematical models. A separate problem is geometry - atomic-molecular structure of matter acts as an independent factor.

So, the idea of synthesis is to join by the general "*fractal denominator* " the entire spectrum of processes of different nature. Interaction of these processes is organized by overall fractal sample volume instantly on all scales.

In modern science, synthetic theories have no mathematical incarnation, despite that this fact attracts a lot of attention. With the help of non-wellfounded sets by D. Mirimanoff (1917-1921) and p -adic numbers Z_p in the physical interpretation, which gave them S.Ulam in the first half of the XX century it is possible to formalize the synthetic potency of fractal geometry. Mirimanoff introduced iteration of relationship " *part-whole* " as the process of generating a set of elements of the initial set, formed by dividing (dispersion), in addition to the standard Cantor notion of set, which is formed by collecting (aggregation). Ulam has noticed the identity topologies of matter, formed by the processes of infinite division, classical fractal "*Cantor perfect set C"* and p -adic numbers. p -Adic numbers are invariants of infinite divisibility of matter, numerical expression, digitalization of hierarchical coordinate of matter. [26, p.56-70].

The result is the following model of the fractal structure of matter. Fractals are equipoise of two universal formative processes, which are generated by two classes of forces in nature - forces of attraction, convergence and repulsion, divergence. These forces are two causes – i.e. "*double-cause*", which generate fractal matter. Together with the forces of mechanical displacement, they form the physico-chemical space - physical, extended with hierarchy. Hierarchy is a subspace in which appear and disappear the properties and qualities of the substance. So, fractals are extensive-intensive, binary objects. Logic and semantics of fractals is two-dimensional. The solution of problem "*part and whole* " appears naturally as a result of two additional causes of substance. Multiscale analysis - bonding phenomena on micro- and macro-levels, i.e. physico-chemical analysis, is included in the causal chain. All numerical characteristics of the substance, including spatial and temporal break into extensive, eucledean and intensive, hierarchical ones, which are interconnected by the Legendre transform [26, P.88]. Intermediary between extensive and intensive degrees of freedom is fractal geometry [27].

Empirical binarity of fractals is an exact match in mathematics as a coherent (see below). results of duality. This factor suggests the possibility to talk about new approach of formalization of the technology.

Number asymmetry. Today mathematics of fractals is created on fractal set as a irregular part of a smooth Euclidean space, often as an attractor of a dynamical system. The above considerations allow us to transmit the modeling into physico-chemical fractal space. Then the causality of phenomena is complemented with opposite one, and two-dimensional *extensive-intensive* logic model arises as a formal counterpart of technological practice.

In the arguments Ulam the divisibility of matter has a modern look of multiplicative processes, which under the guise of *iterated function system, IFS* in short, is the most advanced and popular techniques for generating fractal images [28]. The key moment of synthesis is the simultaneous use of all of interpretations of *IFS'* geometric preimage - lexicographic binary tree. The following are basic - a basic data type in theoretical computer science *TCS*, hierarchical structure of matter, 2-adic numbers Z_2, space of double causality, and, finally, Cantor perfect set *C* - a classic example of a fractal, it often pops up in many areas of mathematics. Intuitively, this idea is as follows.

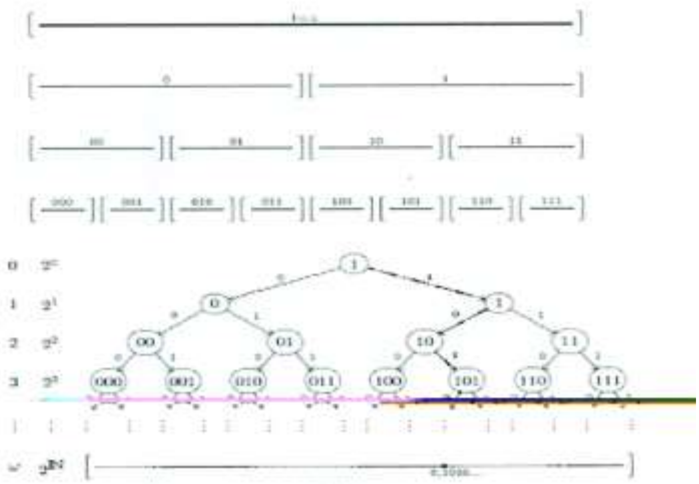


Fig. 4 Binary lexicographical tree

(from Verelst K. *Zeno 's Paradoxes* / / arXiv: 0606639 v 1 [math. HO] 2006)

Figure 4. shows divisibility / dispersing of whole (cut up) into fragments of reduced size across scales (powers of 2 on the left) to the Cantor set *C* (dotted line below). In the tree, sequences of nodes show the formation of binary strings - basic data type in computer science. Driving down the branches, acting as causality "*top-down*", is a multiplicative cascade of contractive, converged processes, directed by divergence branches. Backscattered, dispersed set of branches is directed by their convergence, acts as causality "*bottom-up*".

From this figure becomes evident role of the famous Stone duality [29]. It establishes the identity, i.e. isomorphism, of two main images of this tree – fractal structure of matter and Boolean algebra. Thereby this volume of material has a dual representation - as part of the matter in the physical space and a formal language. Languages and Boolean algebras are tools for expression of properties of objects in the form of equations, functions, variables and dimensions etc. Therefore Stone duality provides a theoretical basis (see *the legacy of cybernetics*), for the technological design and programming of the properties of matter.

Tree in Figure 4 is a tree of *2* -adic numbers Z_2 , a special case of *p* -adic numbers Z_P . Here p = 2,3,5, ... - is prime number, the base of the number system. Currently, the physical interpretation of prime numbers is an open question. Probably in a dynamic picture of fractals number other than 2 is a relative concept, depending on the choice of units. [6]. We restrict ourselves to $p = 2$, because of its universality. As a physical meaning of 2 we fix a pair of "*continuity - discontinuity* ", by which everything always is divided. Then the binary structure of the model lies in the accurately on a tree - causality coincides with logic, transformation of material (physics) interfaced with transformations of formal languages (computer science). Here it remains open pairing mathematics to chemistry, its periodic law (see, however, our hypothesis below).

Consider the thought experiment. One person counts *1,2,3, ..., N* , ... At this time, another one puts bricks in the wall, and the third person has a hammer on a single brick. Both follow the count, i.e. the sequence of natural numbers, but the results account directly opposite. In the first case, increasing the mass of matter in the second matter disappears into the dust. Therefore, any number, for example $p = 2$ has a double meaning. Formula $y = 2 \cdot x$ could mean doubling the length of the segment, and the doubling of the number of nodes in the binary tree. The first option is the embodiment of a standard series of extensive natural generalizations *R*, the second - an intense and generalizations Z_2 , which remained in the shadows. This is the essence *number asymmetry* – semantic of each variable, function, equation is defined over two number systems at once, and, therefore carries two opposite meanings.

It is well known the double number-field isomorphism of Cantor set *C:*

$$R \cong C \cong Z_2 \qquad (*)$$

where two extreme members of which represent the semantic duality of fractals. By the number asymmetry we shall mean the union of real *R* and 2-adic numbers Z_2 in the model of functional asymmetry [26 PP.56-84;27]. More precisely: the real and *p* -adic numbers have identical syntax and many properties in common. However, physically they are opposites: describe opposed topologies of matter, their magnitude correspond inversely, reals are observable, *p* -adic - invisible. Let us explain this.

All measurements in science are expressed by rational numbers, which are written as a string of finite length:

$$x = a_k 2^k + a_{k-1} 2^{k-1} + \ldots + a_0 + a_1 2^{-1} + a_2 2^{-2} + \ldots + a_n 2^{-n} \qquad (1)$$

or, in the form of a sequence of digits $x = a_k a_{k-1} \ldots a_0\, a_{-1}\, a_{-2} \ldots a_{-n}$. In arithmetic operations except division, the first digit is leading, and in the division - the last one. Therefore, the real and *p* -adic numbers are related in (1) by reading the line backwards, or a reflection to relation of a decimal point:

$$x^* = a_k 2^{-n} + a_{-n+1} 2^{-n+1} + \ldots + a_0 + a_1 2^1 + a_2 2^2 + \ldots + a_k 2^{-k} \qquad (1*)$$

so a string of digits is defined instantly over the two number systems, so it has a double numerical and topological meaning :

$$x \in R \leftarrow (\ a_k a_{k-1} \ldots a_0\, a_{-1}\, a_{-2} \ldots a_{-n}) \rightarrow x^* \in Z_2 \qquad (2)$$

Operation connecting (1) and (1*) is called an involution. Involution relates syntactically identical, but physically opposite two basic numerical system of mathematics *R* and Z_2. The phenomenon of their connection is known in science as universality of power laws, i.e. dependences of a form $y \propto x^{\pm D}$.

About it in more detail. Quantity or metric *2* - adic (*p* -adic number) is determined by the first non-zero digit of the expansion (1 *) and is called non-Archimedean or ultrametric value:

$$\left| x^* \right|_2^{\alpha} = 2^{-(-n)} = 2^n \qquad (3)$$

It is easy to check that this value reverse-proportional to the usual archimedean value $\left| x \right|_\infty$ of number in (1). That is, if one introduces two kind of *order* on rational numbers (1): $\wedge$ - up and $\vee$ - down the tree, related via involution, then

$$\left| x \right|_2^{\alpha} \approx const \cdot \left| x \right|_\infty^{-1} \qquad (4)$$

This technique will be needed in the future, when the theory of order and lattices in connection of atomic-molecular lattice, will be involved. All the basic facts about fractals, are combinations of pairs of topologies and metrics (3) and (4) in number asymmetry.

Measurements and values. Connection of formal mathematical methods to the subject area goes through measurement methods. Without them any theory hangs in the air, and the area remains inaccessible for the theory it was built. In the general theory of measurement is also possible to see the duality of nature of values, which connects the binary nature of fractals and matter. It goes back to the Hölder and Ostrowski theorems: there are two ways to measure - extensive (additive) and intensive (multiplicative). According to this duality, measured values fall into two classes, which are referred to in different contexts in different ways: extensive and intensive, covariant and contravariant, longitudinal and transverse [30, Ch. 2.3, 3.2].

In the model of number asymmetry it is natural to compare this division, generated via pair of metrics (3)-(4), which are the basic functions for developing methods of physico-chemical measurements, with extensive-intensive pairs of thermodynamic variables. The basis of this analogy is that the Archimedean metric $|x|_\infty$ is additive

$$|a+b|_\infty = |a|_\infty + |b|_\infty = |a| + |b| \le 2 \cdot \max\{|a|_\infty, |b|_\infty\}$$

while the nonarchimedean ultrametrics $|x|_2$ - subadditive:

$$|a+b|_p \le \max\{|a|_p, |b|_p\}$$

In thermodynamics, the first type consist of natural variables: ***S*** - entropy, ***V*** - volume, ***M*** - mass, ***N*** - number of particles. The second is: ***T*** -temperature, ***P*** - pressure, ***M (J)*** - the chemical potential. A detailed theory of twin measurement in this case is the embodiment of a separate paper. In our model, R corresponds with extensive and Z_2 - intensive quantities, exclusiveness which is known in algebra courses as the exclusivity of number systems R and Z_p, and theorems by Hölder and Ostrovsky. It can be shown that the pair of functions metrics corresponds to the general principle of duality between two base spaces and, thus, provides the Legendre transform [26, pp. 85-89]. Thereby expanding the mathematical theory of measurement, bringing new connections is in accordance with technological experience.

Formalization of a binarity . Its basis is the number asymmetry

$$R = inv\, Z_2 \;,\; R = inv\, Z_2$$

as a formal counterpart of the binary nature of the substance.

In the context of the laws of nature, it has the form of a formal identity of basic equations for topologically distinct physical processes. In the p -adic physics [22] and mathematics is the principle of invariance of the laws of nature under changing the number field; in the theory of Boolean algebras - double meaning of continuity [31, C. 342, 357-358, 411] allows to transfer formulas and equations from one number system to another. In other words, one equation on a fractal structure defines two opposite processes that relate to each other as deterministic and random processes. We describe the main ingredients of the binary scheme, omitting technical details.

1. Numerical characteristics are formed from binary string $\xi \in Z_2$, using two measurement functions - archimedean $|\xi|_\infty$,and non-archimedean $|\xi|_2^D$ metrics that are linked by a power law:

$$|\xi|_\infty \cdot |\xi|_2^D = Const \qquad (5)$$

2. Any selected volume V is described in double coordinate system

$$(V(x,t) \subset R^3) \leftarrow V \rightarrow (V^*(\xi,\tau) \subset Z_2) \qquad (6)$$

here x, ξ, t, τ - spatial and temporal coordinates R and Z_2, respectively. That is, on the one hand it is an element mechanics continuous media, on the other - some ensemble of particles or fragment of a lattice.

3. All observed and measured values are described by rational numbers Q , which are a combination of real R and p -adic Z_2. They have the form

$$x \in \langle R \oplus \{Q\rangle \oplus Z_2\} \qquad (7)$$

In this entry intersecting pair of different brackets denote the dual nature of the measured values. Accordingly, each variable $x \in Q$ has two meanings - as a parameter continuous volume, e.g. density ρ, corresponding to angle brackets, and the density distribution function of uncertainty φ - to figure brackets:

$$\rho \leftarrow x \rightarrow \varphi \qquad (8)$$

4. In the proposed approach as a model uncertainty is taken possibility theory [32], which corresponds to a non-archimedean metric in (1). It describes multiple sources of uncertainty, both physical and informational. Its main advance is invariance under scaling transformation, i.e. divisibility, unlike traditional probability measure, which dispersed under dispersion/ divisibility of matter.

5. Transfer procedure formulas and equations from real field to the *2* -adic numbers consists

of the addition of true propositions of condensed matter by true ones for dispersed matter. The very form of the equations remains unchanged. Such equations are, for example, the continuity equation, the equation for the flow is as true for a continuous medium, the particle fluxes, molecules, and for heat, energy, entropy, electric current. This means that the fractal model and consists of at least of two differential equations, according to the double meaning of the variables (2) - (4). Fractal volume serves connecting link between them.

Illustration of the transfer principle is the expression for the derivative. In *R* is the rate of linear displacement in Z_2 - rate of entropy production:

$$v(x,t) \leftarrow f^{'}(x) \rightarrow s \qquad (9)$$

For example, the expressions for the velocity of the particles of matter are identical for a continuous medium, the Fourier law for the heat flux, Ohm's law for electricity, etc.

6. Analytical duality of fractal model for brevity can be written in a logical form.

For example, the equation for the velocity *v* elements of matter in the field of force gradient *F* and *continuity equation* - *CE* for mass density and possibilities (known as "*probability fluid*") (4) have the form of statements:

$$|= \quad v = k \cdot grad\ F$$

$$|= \quad CE(\rho, x, t) + CE(\varphi, \xi, \tau) + CE(V(x, \xi, t, \tau)) = 0 \qquad (10)$$

sign " | = " means truth in the physical sense. The third term is the equation for the volume of motion in two subspaces of the dynamics, which also has a view of the continuity equation. This technique - modeling transforming shape of matter, is used in different sections of materials science [33, Ch .5,15].

7. Initial output for binary-type equations (7) are the conservation laws in the weak integral form, as they have equal rights include two types of variables (3) and, therefore, becomes possible the above scheme (6) - (10). Analogue of (1) - (1*) for element of mass is

$$m_i = \int_{V_i} \rho \cdot \varphi \cdot dV_i$$

Therefore, the amount of substance in the volume *V* is equal to

$$m(V) = \sum_i m_i = \sum_i |\xi_i|_\infty \cdot |\xi_i|_p^D \cdot V_i$$

From here one can derive a general equation of continuity for the substance and "fluid probability (possibility)," i.e. possibility in this sense of (7), which are common to all variables

From consideration of continuity equation it becomes clear, that it is an interpretation of the basic relation of the model (1). His sense is of equality of the amount of matter transformed from convergent to divergent process. In its simplest form, to maintain the flow *j* of substance *m* through the surface *S* correspondence $Const \leftrightarrow m$, $j \leftrightarrow |\xi|_2^D$, $S \leftrightarrow |\xi|_\infty$ is a manifestation of the Legendre duality of fractal structures [26, PP.87-88].

Therefore, the continuity equation is invariant with respect to a shift along the coordinate divisibility and stays true to the substance of any nature. Therefore, it can serve as a starting point conjugate equations. His overall view is:

$$\frac{\partial \rho}{\partial t} + div_R(\rho v) \leftarrow \left(\frac{\partial V}{\partial t} + \frac{\partial V^*}{\partial \tau}\right) + div_R(\rho v) + div_{Z_2}(fu) \rightarrow \frac{\partial f}{\partial \tau} + div_{Z_2}(fu) \qquad (11)$$

should be supplemented by (2) - (6).

8. In the limit to the divisibility when "matter disappears" and only geometry remains, the selected volume appears as a surface consisting of surfaces, such as scale-free net of boundaries. This equation becomes the equation of conservation of level-set of the function $F_V(x,t;\xi,\tau)=0$, representing the selected volume as a set of hierarchical sections [33; 34, Ch. I, IV]

$$F^{'}_{t,\tau} + v_n \cdot F^{'}_x = 0 \qquad (12)$$

The meaning of this is the equation is motion of a given volume of matter represented by their fractal, i.e. net of boundaries of its parts, subvolumes.

9. This way leads to the computer technique of modeling of materials. In computers all objects are replaced by their fractal counterparts, i.e. by Stone prototypes, i.e. fractal images are filled with numerical and analytical sense. This is the way of giving computer 3D models deductive and analytical capabilities.

For example, relation (11) can be used as synthesis *universal* and *unique* in materials technology. *Universal item* refers to the physical laws, and the *unique* to the particles shape, topology defects, especially the atomic and molecular structure, etc. [3]. The left-hand side of (11) can be studied via standard technique, while the right-hand side is the dynamics of the Stone preimage of the substance and its exact coordinatization and digitization are available only with the help of computer modeling techniques, such as computerized tomography. The synthesis of these polarities occurs in a fractal volume - the computer's memory. Number asymmetry is embodied in the "*computational duality* "- a connection between two computers running additional modes.

Technology as a hologram . As it was historically shown, in the field of real numbers *R*, there can be no synthetic model approximates the theory to the needs of practice. Synthesis becomes real only in the *p*-adic numbers. The proposed approach assumes that the partial models of physical and chemical processes, i.e. correspondence of their theories to scales of hierarchy have already been developed. Synthesis is sound only via well-developed analysis. We give the logical-topological model of matter, i.e. picture connection between processes and objects, which is a necessary step in the development of computational schemes (for details see [26, P.100]).

Model of substance has expanded view of (*):

$$C_{matter} \cong \exp(C) \cong 2^C \cong Z_2 \cong [IFS \equiv \{0,1\}^N] \cong [Z_2 \rightarrow Z_2] \cong C(Z_2, Z_2) \cong C_{Bool} \qquad (13)$$

In this chain of isomorphisms is read from left to right as: the material fractal. its transformations, the truth set of Boolean algebra (propositional calculus), 2-adic numbers, iterative system functions as a formal language, the lattice of bonds, field of continuous functions, Boolean algebra. Movement from left to right takes the matter into its opposite, the symbol as its inverse or Stone preimage (*).

Use reflexivity Z_2 (the fact of self-similarity of fractal sets):

$$l_1 \cdot Z_2 \cong l_2 \cdot Z_2 \qquad (14)$$

where l_1 and l_2 - arbitrary scales on the coordinate dimensions. By this logic, we have a topological model of the process, including the well-known macro-, meso-, and microphysics:

$$l_0 \cdot Z_2 \cong l_1 \cdot Z_2 \times l_2 \cdot Z_2 \times \ldots \times l_n \cdot Z_2 \qquad (15)$$

here each of the factors is associated with one of the models describing the process of scale l_i on axis of sizes $l_0 \geq l_i \geq l_n$, corresponding to one of the members (13). Left-hand side of (15) as fractal volume (6) can describe shape changes. As a Stone preimage it reflects the transformation properties, i.e. logic, causality, especially top-down one. Rewrite (15) in a form where the left-hand side is one of the processes, the properties of matter, moving the volume to the right:

$$l_i \cdot Z_2 \cong l_0 \cdot Z_2 \times l_1 \cdot Z_2 \times l_2 \cdot Z_2 \times \ldots \times l_n \cdot Z_2 \qquad (15^*)$$

Equations (15) and (15 *) have the form of the tensor product of spaces. They are a continuation of fractal universatility to Z_2 , which in this case it makes sense of the logical-semantic space of any dimension on the number of processes that make up the model.

Computing apparatus of this scheme is matrix theory, supplemented with the theory of Kronecker product, which generalizes the theory of iteration functions system. Evolution of a given volume in this case is described by an asymmetric matrix equation (for details see [26, p.108])

$$V(t,\tau) = M_R(t) V_0(t=0,\tau=0) \otimes M_Z(\tau) \qquad (16)$$

Here on the left stands for matrix realization, i.e. finite-difference scheme for the boundary value problem. To the right stand for Kronecker multiplication, that formalizes the internal dynamics of matter.

Thus, we come to yet another universal objects of mathematics – matrix and tensor algebra and analysis. For fractals, where smoothness and continuity have no definite sense, they are more adequate than smooth technique. Matrices are common to discrete (network, graphs, lattices) and continuous (finite difference schemes for solving differential equations) parts of mathematics, i.e. they are able to work in a two-dimensional fractal logic, that is completely absent in the standard of physical and mathematical theory.

Matrix mathematics is rich in computational and analytical capabilities, is well suited for computer mathematics and information technology, so it is a convenient device for further development of technical methods.

A glance towards Periodic table. Without connection of Periodic Table, the proposed model has a substantial flaw. In this section we sketch the reasons for continuation of above considered *number asymmetry* as candidate for a mathematical content of this main law of chemistry. Mathematical description of Periodic Table attracts permanent attention. Various branches of mathematics – from simple geometry to theory of information and quantum formalisms, are used to catch its main properties and interconnections [35,36,37,38,39].

As in all branches of natural science the main question is: does it Nature permanently solves differential or other type of equations, say Schrodinger one for chemistry, to produce various motions and properties of observed matter? Or, in contrast, all solutions are already present in the space-time? If so, it is not hard to consider them as *quality-free* substance, forming space-time, i.e. totality of matter embedded in various fields.

As a first step, we propose the interpretation of (inter)connections of quantum numbers. As it is known, the quantum numbers play an important role in the structure of the periodic system, but do not exhaust it.

Considerations that allow to introduce the Z_2 - the main ingredient of number asymmetry, as candidate for mathematical background of periodic table, are as follows:

1. Overall structure of chemical elements – nucleus, a dense core of matter, surrounded by an electron shell, is similar to the binary, convergent-divergent phenomenology approach.

2. Inverse relationship relative spread of elements by atomic number has the view of distorted hyperbolic law, typical of fractal structures (see Fig. 5) [40,41,42]. If we include into a picture the evolutionary processes of chemistry, i.e. that the atomic weights of the elements are not constant, and one is to expect their variations [40; 42 P.537 (theory by G.I. Pokrovskii)], this view coincides with the nature of fractals as traces of nonequilibrium processes [25] the accordance with fractal geometry should be sufficient for the next step.

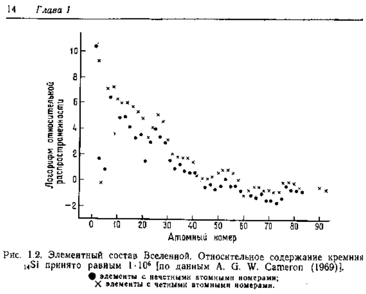


Fig.5 Distorted hyperbolic law of relative spread of chemical elements.

3. In this case, the power law can be seen as generated by two metrics on Z_2 , according to the theorem of the theory of *p-adic* numbers [43, PP.288-292]. Then, in accordance of the physical meaning of the base $p = 2$ of *p-adic* number system is a pair of substances, *"matter (nuckeus) - field (electronic shell)"*.

4. 2-adic numbers being ultrametric space, is itself computational model [44, 45,46]. In fractal space-time computational processes coincide with physical ones [47]. This explains the self-motion of Nature.

5. 2-adic numbers are Hilbert space [48, 49, 50]. Here we rewrite the hologram of Z_2, treated as Hilbert space *H*:

$$C_{matter} \cong \exp(C) \cong 2^C \cong Z_2 \cong [IFS \equiv \{0,1\}^N] \cong [Z_2 \to Z_2] \cong C(Z_2, Z_2) \cong H \cong BA$$

In notions of quantum theory: terms 1+7 represent "*particle – wave*" duality; composition of 2+3+5+7+9 gives a lattice of propositions of Hilbert space logic together with truth-functions in the sense of J. von Neumann and G. Birkhoff. The first and the last members represent "*matter-symbol*" duality, the central issue in the general systems theory. The fourth term gives to the Hilbert space and its ingredients coordinate-free, number sense dual to customary real-valued one.

6. As, obviously, $Z_2 \cong R$, the number asymmetry $R = inv\, Z_p$ leads to isomorphism $U = R \times Z_2 \quad \Rightarrow U \cong C = R \times R$. So it is isomorphic to the field of complex numbers C - the main field of quantum mechanics (not to be confused with Cantor set, also denoted by C). Isomorphic mathematical spaces are not technically distinguishable. Therefore the conclusion of theorem by M. Sole about exclusivity *R, C* and the field of quaternions H for quantum theory, is not broken [51].

7. Superposition principle is obvious: Z_2 is a Banach algebra. Accordingly, in further considerations, the number asymmetry, that includes operation of involution, take a view as a *C** - algebra, one of the interpretations common to quantum mechanics and relativity in terms of function space [52]. In this form, *C** - algebra express the universality of fractals in both directions - towards the infinitely small and infinitely large, which accumulates in the literature the concept of fractal universe.

8. As a result of this brief quantum considerations, we can treat Z_2 as a space of wave functions. Probabilistic meaning of such wave functions follows from undefineability elements of Z_2 via technique of real analysis. Definition of ultrametrics coincides with the axioms of possibility measure, the complement to probability theory of uncertainty.

9. Whole hologram in pt.5 of this section provides characterization of Hilbert space by functional equation:

$$H = \exp(H)$$

As we left aside the limit of divisibility, i.e. Plank length scale, due to S. Ulam's considerations [53,54], and modern conclusion [55] - *"The existence of a minimum length in quantum mechanics and the speed of light are artifacts archimedean physics basis. Both of these assumptions are not supported by the non-archimedean physics*", the Z_2 - Hilbert space is zero-dimensional and exponentially complete. Therefore it can be reconsidered for functions, propositions as well for mass particles – both for material and informational entities.

10. The motion processes in Z_2 in their general sense are expressed via primitive symbolic equation of form

$$\frac{d\,\xi}{d\,\tau} = a \qquad (17)$$

where $\xi, \tau, a \in Z_2$. Here, in contrast with real-valued case, all differentials in left-hand side must be calculated in two metrics – both archimedean and non-archimedean, so as right-hand side. This equation determines 4 derivatives

$$e_1 = \frac{|d\,\xi|_\infty}{|d\,\tau|_\infty} \quad e_2 = \frac{|d\,\xi|_\infty}{|d\,\tau|_2} \quad e_3 = \frac{|d\,\xi|_2}{|d\,\tau|_\infty} \quad e_4 = \frac{|d\,\xi|_2}{|d\,\tau|_2} \qquad (18)$$

One can easily check that the product $e_1 \cdot e_2 \cdot e_3 \cdot e_4 = \tilde{n}_\xi^2 / \tilde{n}_\eta^2 \neq 0$, i.e. each of those derivatives is nonzero. Therefore Z_2 generates four its own copies via four kinds of motion. In field of real numbers they are undistinguishable due to indistinguishability of metrics. For not to entering in details the first derivative reflects smooth motion, three others – discrete ones. So we come to discreteness of quantum phenomena, underlying behavior of elements in periodic table, and, consequently to the division of all elements into four types.

10. Due to previous consideration one can transmit from the static field of Z_2 to four its copies, each filled with separate kind of motion. Therefore we have the following picture:

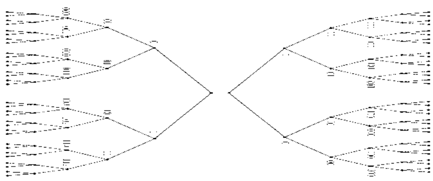


Fig. 6. 2-adic system of chemical elements

where Z_2 is represented by four branches with common root.

The hypothesis on *the 2-adic* relation between quantum numbers is as follows. On this tree trajectories, i.e. paths, connecting vertices, are compared with functions of Hilbert space Then, if we put the principal quantum number n equal to the number of the hierarchy division, the orbital quantum number $l = (0,1,2,\ldots n-1)$ coincides with the set of hierarchy levels. On Fig. $n=4$, $l = (0,1,2,3)$. These two numbers are common to all four branches. The magnetic quantum number m_l coincide with segment $(-l, l)$. When going to the root, path, i.e. function is deterministic, from the root – non-deterministic one. Left and right half of the figure matched the spin quantum number $m_s = (\frac{1}{2}, -\frac{1}{2}) \cong (\alpha, \beta)$.

Thus, the above trajectory $\xi = a_0 a_1 \ldots a_n \in Z_2$ as wave functions have double - deterministic and non-deterministic sense, and as elements of Z_2 number system naturally bind all four quantum numbers. One can say that Z_2 in the interpretation of the model is the space of solutions of the Schrödinger equation, which paves the way to a simple division of matter to bypass solving the differential equation. It looks like mathematical machinery of Nature.

The stated hypothesis about 2-adic background of quantum numbers, concerns the periodic system of free atoms, which, however, differs from the periodic table of chemical elements. Based on the foregoing, the closest interpretation of the periodic law appears as an infinite-dimensional function space by S.A.Schukarev [37, P.5]. In our considerations this thesis corresponds to reflexivity of Z_2 (14) - (16).

A brief summary . Adopting a dynamic model of fractal matter, we get at the disposal of logically related chain universal mathematical structures - number asymmetry, Boolean algebra, power laws, measurement theory, Legendre duality, formal languages , matrix mathematics and Kronecker product of matrices, in particular. This chain connects the technical measurements with numerical methods. Nature of bulk properties of substances recognized as a coordinate axis divisibility as changing the meaning of variables and equations.

The considered approach is consistent with the basic notions of materials science as synthetic physico-chemical discipline. However, still remains a problem mathematical representation of the periodic table. Our last speculations are to be considered as a hypothesis of continuation of universality of fractals and 2-adic numbers. It still requires more elaborate work toward checking and/or increasing adequateness.

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