The fundamental role of proper time in general relativity and in quantum mechanics

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Special relativity provides time with a precise physical concept: In a first step, time is generated by rest energy in the form of proper time, and in a second step, an observer may measure the corresponding coordinate time.

1. Time equations in physics

Preliminary, we want to recall the physical basics of the notion of time. Time is very frequently used in physics, but there are only very few fundamental equations on time. Nevertheless, the fundamental equations exist, and two of these equations will permit the derivation of the nature of time.

We may distinguish three different types of equations:

1. General time measurement, e.g. v = ds/dt, P = dW/dt etc.

2. Specific time-dependent evolutions:
- The time-dependent **expansion of the universe** - the scale factor **a** provides the universe with a sort of clock:

$$d(t) = a(t)\, d_0$$

- The time-dependent evolution of the **kinetic and the potential energy** of quantum systems - the time evolution operator (Hamiltonean):

$$i\hbar \frac{\partial}{\partial t}\psi(t) = \hat{H}\,\psi(t)$$

- The time evolution of the **rest energy** mc^2 of particles - the action of a point particle:

$$S = mc^2 \int d\tau$$

3. The conversion between coordinate time and proper time
Two equivalent equations may be distinguished:
- The spacetime interval

$$ds^2 = d\tau^2 = g_{\mu\nu} x^\alpha dx_\mu dx_\nu$$

- The time dilation equation

$$dt = \gamma\ d\tau$$

Two of these equations are sufficient for providing a precise concept of what the nature of time is:

The action of a point particle

$$S = mc^2 \int d\tau$$

and the time dilation equation

$$dt = \gamma \; d\tau$$

2. Proper time is not "just one more reference frame"

The equation

$$dt = \gamma \; d\tau$$

has been introduced by special relativity. This twofold time concept is in sharp contrast to Newton's absolute time concept. The Newtonian time concept was very simple: $t_1 = t_2$, there was one unique absolute time. Very simple, however, not compatible with the fact of constancy of speed of light.

This problem was resolved by special relativity where the principle of time dilation replaced the absolute time concept of Newtonian spacetime and put an end to the research for an ether.

Time dilation is the key notion of special relativity, and the whole special relativity could be reduced to this equation of time dilation because it makes harmonize the two postulates, the principle of relativity and the constancy of light. Newton's unique absolute time is split into two notions of time: The proper time of the observed object is time before time dilation, and the coordinate time of the observer is the measured time after time dilation. Both are linked by the time dilation factor, the Lorentz factor. Of course, this principle is not limited to special relativity, it applies also to gravitational time dilation of general relativity.

So we could say that special relativity is a time concept, it is splitting absolute time into two different time concepts by the means of time dilation.

However, this fact is not always clearly recognized. Currently, special relativity is dominated by Lorentz transformation, which is permitting to switch from one reference frame to another, and at first sight it might seem that the time dilation equation is included within Lorentz transformation, such that proper time may be considered as "just one more reference frame". It seems that we can always put ourselves in the position of a mass particle in order to adopt its frame with its respective proper time, by Lorentz transformation.

However, this is an error. Lorentz transformation puts the observer and the observed particle on an equal footing, it is symmetric. Time dilation is not symmetric, the roles of observer and observed particle are well-defined, and accordingly, there is no such thing as "time contraction".

Moreover, there is another difference: Lorentz transformation is limited to timelike reference frames, but there are lightlike metrics without clocks which do not have any reference frame. This is no problem for Lorentz transformation because the transformation between reference frames remains possible without restriction. But special relativity consists not only of timelike but also of lightlike worldlines, and lightlike phenomena are not negligible, the zero interval of lightlike phenomena is an important part of special relativity.

To be precise: even the time dilation equation does not apply to lightlike phenomena because there is a problem of division by zero, instead we have to resort to the inverse, the proper time equation:

$$d\tau = \frac{1}{\gamma} dt = \sqrt{1 - \frac{v^2}{c^2}}\, dt$$

We see that Lorentz transformation and time dilation are two different things. Lorentz transformation covers "nearly all" cases of time dilation, but not "all" cases, and for those who do not make this difference, the importance of proper time and of the twofold time concept of relativity gets completely lost. Unfortunately, this is what seems to be happening because very little attention is granted to proper time although many authors are highlighting the fundamental importance of proper time.

That means that Lorentz transformation is a good concept for the application of special relativity, but it is the wrong concept for the understanding of special relativity. It is hiding the fact that special relativity is a time concept and not a transformation concept.

3. Proper time as the fundamental time concept

We saw that special relativity has split up the Newtonian time into two complementary time concepts, the proper time **dτ** before time dilation and the coordinate time **dt** after time dilation.

The question may arise: <u>From an axiomatic point of view, which time concept is more fundamental, coordinate time dt or proper time dτ?</u>

The answer is surprisingly simple and clear, it is provided by the definition of proper time:

"The time measured by a clock following a given object"[1].

This definition does not even refer to spacetime or to coordinates, it refers only to the object, more concretely: to the mass particle. That means: Proper time is an intrinsic characteristic of mass particles. And particle worldlines are completely independent of spacetime when they are parameterized by their respective proper time.

How can one imagine a worldline without spacetime? Spacetime seems to be always there, so proper time must depend in some way of spacetime. But this is an illusion. Observer-dependent spacetime is mere observation. The particle does not need any spacetime for its existence (the underlying manifold is threedimensional space instead of spacetime). It is only the observer who needs spacetime for retrieving the proper time of the particle, as shown in **fig. 1**:

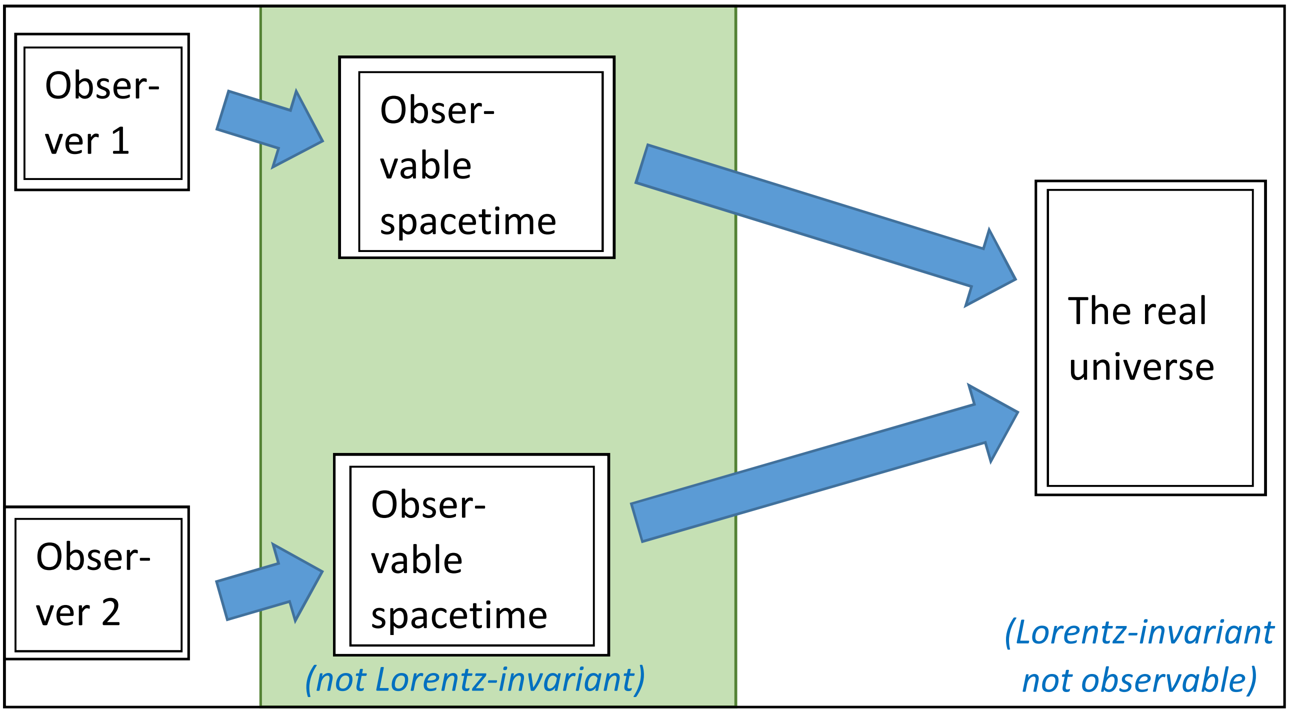

Fig. 1: Observable spacetime is just a tool (a sort of interface) of the observer, the real universe is located beyond observable spacetime.

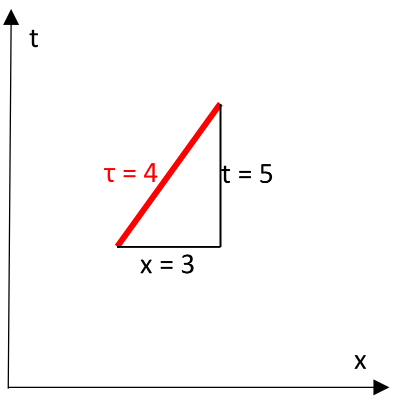

Fig. 2: Spacetime diagram

The observer uses the spacetime diagram of **fig. 2** where the proper time parameter is replaced with the coordinate time parameter of the observer. In this diagram dx = 3, dt = 5, and logically proper time is dτ = 4, but proper time is not visible in the spacetime diagram, it is hidden. We can measure the length of the red line in the diagram, it is 5.83, but this length is meaningless and does not correspond to any real metric, but every observer may calculate the underlying proper time.

We see that the study of the universe, of how the universe is, must go beyond observation: The spacetime interval dτ is not observable, but all observers agree on it, it may not be found by observation but only by calculation.

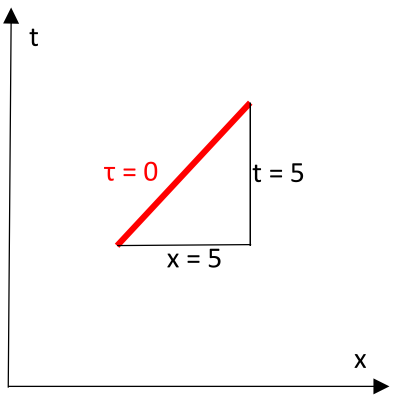

Fig. 3: Lightlike spacetime intervals

As we already saw, lightlike intervals are an extreme case: According to the second postulate, light is always observed as moving at speed of light c. That means that no observer is able to observe the underlying zero interval of photons.
By consequence, even the sum of all observations does not show the whole metric of the universe.

The twofold time concept of coordinate time and proper time becomes now clearer:

Experimental physics is starting off with a measurement of some coordinate time.
Based on the coordinate time, proper time may be retrieved.
Conversely, from the point of view of theoretical physics, proper time is object-related, and coordinate time is deriving from the proper time when it is measured by an observer.

<u>If proper time is fundamental, that means that coordinate time and spacetime in general is not.</u>
Indeed, this implies not only a fundamental time concept, but also a fundamental space concept which is simply threedimensional, flat space.

As coordinate time is not fundamental, it would be a mistake to try to resolve fundamental physical problems without taking into account proper time - **For fundamental physical questions on time we must refer to the proper time instead of coordinate time.**

Why is this rule so important?
At first sight there seems to be only a quantitative difference (for example, in **fig. 2** dt is 5 and dτ is 4), and it does not make a fundamental difference whether our approach refers to the coordinate time or to the proper time of a particle. But this rule is particularly important for lightlike worldlines whose spacetime interval is reduced to zero, that is not only a quantitative difference but a qualitative difference.

4. Proper time in the universe

For a closer understanding of the consequences of this rule of the priority of proper time, lets have a look at the universe and divide the universe into three main categories:

1. **Worldlines of mass particles**
2. **Worldlines of lightlike phenomena** (such as electromagnetic and gravity fields, photons in vacuum etc.)

3. The rest, that are **vacuum** points between worldlines

(For simplicity, we will not discuss the important category of interactions of lightlike phenomena with mass, such as photons traveling through media etc.)

Worldlines of mass particles: May be parameterized by proper time. They are producing proper time τ.
For **worldlines of lightlike phenomena:** Their proper time interval is reduced to zero, $\tau = 0$. The important consequence is that there is time symmetry of fields, because by definition, pointlike worldlines must be time symmetric.
Vacuum points between worldlines: For vacuum points, no proper time is defined, and by consequence, there is no coordinate time. Vacuum points are timeless. Only worldlines are provided with time.

This description of the universe by the proper time parameter is simplifying quantum mechanics a lot: There is the rule of timelessness, that means, there is only a time parameter where this is expressly defined, only quantum systems with mass are generating proper time, and lightlike fields are reduced to a simple point.

5. The generation of proper time by rest energy

Now we will present the process of the generation of proper time. The action of a point particle is perfectly describing this process:

$$\boldsymbol{S = mc^2 \int d\tau}$$

where the mass particle (or the quantum system with mass) is represented by it‘s rest energy mc^2.
But what is rest energy? Rest energy - The name seems to be a contradiction in itself, on the one hand "energy" which seems to mean something dynamic, and on the other hand "rest" which means doing nothing. The solution of this contradiction is that the action of rest energy goes only in time direction - in the same way as kinetic energy is pushing a particle through space, rest energy is pushing a particle through time, more precisely, the rest energy of a mass particle is providing the mass particle with aging. This is our interpretation of the action of a point particle.

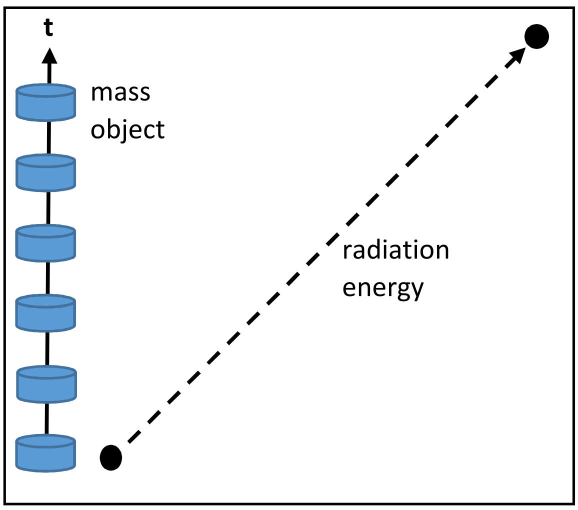

Fig. 4: Mass provides duration

For the closer understanding we compare the mass particle with a massless photon:
At first sight, radiation energy is also progressing in time direction: A light ray sent from the Sun may reach the Earth within 8 minutes after its emission. But we must remember here that the time interval of 8 minutes is mere observation: Fundamentally, the photon has not been pushed through time because its proper time is always zero.
And we notice that mass particles have a characteristic which is missing for massless particles: Whereas a mass object (the cylinder on the left) may be constantly observed and reobserved at any moment, the photon (on the right) may only be observed once - at its place of absorption.
That means, rest energy provides objects with durable existence, contrarily to radiation energy which is fugacious and which is existing only at the places of emission and absorption. Mass objects are distinguished by their continuous presence while the energy of radiation has not this property, it is fugacious and makes a sort of "leap".

While rest energy mc^2 progresses in time direction, the mass particle is generating its own aging, its proper time.

$$S = mc^2 \int d\tau$$

It is important to notice the independence of this equation:
The action is described exclusively by the independent characteristics of the particle itself:
m is observer-independent mass of the particle,
mc^2 is its rest energy
τ = the proper time (the clock) of the particle is also observer-independent.
There is no reference neither to

- spacetime (of general relativity)
- nor to position and momentum (of quantum mechanics) which are always zero from the point of view of the observed mass particle.

As a result, we may now use the two equations I mentioned at the beginning:

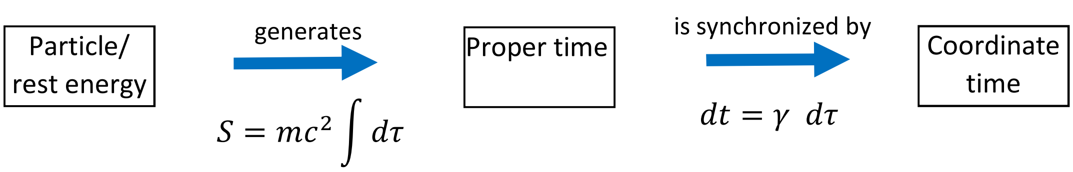

Fig. 5: The nature of time

In a first step, the rest energy of a particle creates proper time according to the rules of the action of a point particle, and in a second step, the observation is done according to the time dilation equation - this is what time is.

We get a global vision of the universe: In absolute space, worldlines parameterized by their respective proper time are transferring the whole causality of the universe, from one event to the next.

6. Proper time produces gravity

Finally, here is the most surprising point of the nature of time:

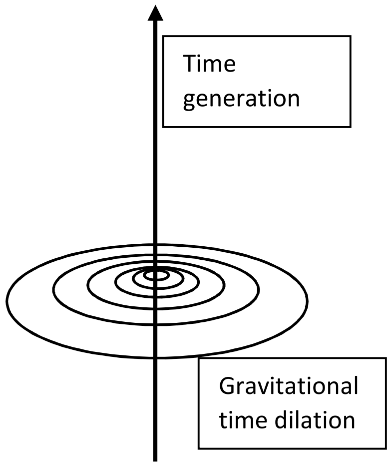

Fig. 6: Gravity as a field of time dilation around a time generation process

As explained above, mass particles are generating proper time. But as we all know, mass is also generating gravity and gravitational time dilation. That means that mass has two simultaneous effects, time and gravitational time dilation.

From these elements we get a surprising result - rest energy has a twofold time effect:

- On the one hand, rest energy produces its own proper time
- Simultaneously, the gravitational time dilation of rest energy slows down the proper time frequency of other particles. Gravitational time dilation reveals to be a side effect - a sort of surrounding trace of time generation, a field around the time generation process of the particle. We may speculate that there is a close physical interaction between proper time and gravitational time dilation.

7. Reference

[1] Landau/ Lifshitz, The Classical Theory of Fields, 1951, § 1.3. Proper time, p.8