On spin-charge separation

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Abstract

Recently, we have demonstrated that the Dirac equation can be cast into a form involving higher-order spinors. We have shown that the transformed Dirac equation splits into two equations, describing charged spin 0 and (massless) spin $\frac{1}{2}$ particles. We apply this result to the problem of spin-charge separation.

1 Introduction

It was found in a very recent experiment that in a solid-state, under extreme conditions, the electron behaves as if made of two particles – one spinless particle carrying a negative charge (known as a holon) and another having spin $\frac{1}{2}$ (a spinon) [1]. For a comment on this discovery, see [2]. Kivelson, Rokhsar, and Sethna proposed existence of such a spin-charge separation [3] in the context of quantum spin liquids (QSL), predicted by Anderson [4].

Recently, we have demonstrated that the Dirac equation can be cast into a transformed form involving higher-order spinors [5, 6]. Furthermore, we have demonstrated that such solutions can describe decaying, unstable particles – the transformed Dirac equation splits into two equations, describing spin 0 and (massless) spin $\frac{1}{2}$ particles.

We shall examine the possibility that this splitting of the Dirac equation can correspond to the spin-charge separation of the electron.

In the next Section, we split the Dirac equation in the interacting case, following approach described in [5, 6], obtaining three equations: two spin 0 equations, describing particles with charge q and $-q$, and one massless spin $\frac{1}{2}$ Weyl equation.

Finally, in Section 3, we apply our results to the problem of spin-charge separation.

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2 Splitting the Dirac equation

The Dirac equation:

$$\gamma_\mu \pi^\mu \Psi = m\Psi, \tag{1}$$

in spinor notation is [7]:

$$\left.\begin{array}{l} \pi^{A\dot{B}}\eta_{\dot{B}} = m\xi^A \\ \pi_{A\dot{B}}\xi^A = m\eta_{\dot{B}} \end{array}\right\}. \tag{2}$$

In what follows tensor and spinor indices are $\mu = 0,1,2,3$ and $A = 1,2$, $\dot{B} = \dot{1},\dot{2}$, respectively. Note that $\pi_{1\dot{1}} = \pi^{2\dot{2}}$, $\pi_{1\dot{2}} = -\pi^{2\dot{1}}$, $\pi_{2\dot{1}} = -\pi^{1\dot{2}}$, $\pi_{2\dot{2}} = \pi^{1\dot{1}}$. The Minkowski space-time metric tensor is $g^{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$ and we sum over repeated indices. Four-momentum operators are defined as $p^\mu = i\frac{\partial}{\partial x_\mu}$ where natural units are used: $c = 1$, $\hbar = 1$. The interaction is introduced via minimal coupling,

$$p^\mu \longrightarrow \pi^\mu = p^\mu - qA^\mu, \tag{3}$$

with a four-potential A^μ and a charge q.

We have demonstrated that for a class of longitudinal potentials [8] Eq. (2) can be written in a covariant form as [5,6]:

$$\left(\begin{array}{cccc} 0 & 0 & \pi_{1\dot{1}} & \pi_{2\dot{1}} \\ 0 & 0 & \pi_{1\dot{2}} & \pi_{2\dot{2}} \\ \pi^{1\dot{1}} & \pi^{1\dot{2}} & 0 & 0 \\ \pi^{2\dot{1}} & \pi^{2\dot{2}} & 0 & 0 \end{array}\right)\left(\begin{array}{cc} \psi^1_{1\dot{1}} & \psi^2_{2\dot{1}} \\ \psi^1_{1\dot{2}} & \psi^2_{2\dot{2}} \\ \xi^1 & 0 \\ 0 & \xi^2 \end{array}\right) = m\left(\begin{array}{cc} \psi^1_{1\dot{1}} & \psi^2_{2\dot{1}} \\ \psi^1_{1\dot{2}} & \psi^2_{2\dot{2}} \\ \xi^1 & 0 \\ 0 & \xi^2 \end{array}\right), \tag{4}$$

with higher-order spinors defined as:

$$\pi_{1\dot{1}}\xi^1 = m\psi^1_{1\dot{1}},\ \pi_{2\dot{1}}\xi^2 = m\psi^2_{2\dot{1}},\ \pi_{1\dot{2}}\xi^1 = m\psi^1_{1\dot{2}},\ \pi_{2\dot{2}}\xi^2 = m\psi^2_{2\dot{2}}, \tag{5}$$

however, some components of the spinor $\psi^C_{A\dot{B}}$ are missing.

The problem of missing components of spinor $\psi^A_{B\dot{C}}$ is quite severe because the theory is not fully covariant. Therefore, to solve the problem in the spirit of Ref. [9], we make the following assumptions:

$$\begin{array}{ll} \xi^1(x) = \alpha^1(x)\,\hat{\chi}(x), & \xi^2(x) = \alpha^2(x)\,\check{\chi}(7x), \\ \psi^1_{B\dot{C}}(x) = \alpha^1(x)\,\chi_{B\dot{C}}(x), & \psi^2_{C\dot{D}}(x) = \alpha^2(x)\,\chi_{C\dot{D}}(x), \end{array} \tag{6}$$

where

$$\chi_{A\dot{B}} = \frac{1}{m}\left(\begin{array}{cc} \pi_{1\dot{1}}\hat{\chi} & \pi_{2\dot{1}}\check{\chi} \\ \pi_{1\dot{2}}\hat{\chi} & \pi_{2\dot{2}}\check{\chi} \end{array}\right), \tag{7}$$

and $\alpha^A(x) = \hat{\alpha}^A e^{-ik\cdot x}$, $k^\mu k_\mu = 0$, is a two-component neutrino spinor, i.e. it fulfills the Weyl equation [7]:

$$p_{A\dot{B}}\alpha^A(x) = 0. \tag{8}$$

Substituting (6) into Eq. (4), with $\alpha^A(x)$ fulfilling (8), we get Klein-Gordon-type equations with rescaled four momentum $\tilde{\pi}_\mu = \pi_\mu + k_\mu$:

$$\left(\tilde{\pi}_\mu \tilde{\pi}^\mu + iqE\left(x^0, x^3\right) + qH\left(x^1, x^2\right)\right)\hat{\chi} = m^2\hat{\chi}, \tag{9a}$$

$$\left(\tilde{\pi}_\mu \tilde{\pi}^\mu - iqE\left(x^0, x^3\right) - qH\left(x^1, x^2\right)\right)\check{\chi} = m^2\check{\chi}, \tag{9b}$$

where $E = \partial_0 A_3 - \partial_3 A_0$, $H = \partial_2 A_1 - \partial_1 A_2$ and $\mathbf{E} = (0,\ 0,\ E)$, $\mathbf{H} = (0,\ 0,\ H)$.

3 Dual nature of the electron

Recently, quantum oscillations have been observed in the spin-liquid state of α-$RuCl_3$ at temperatures $T \lesssim 0.4$ K and in a magnetic field $H \in (7.3,\ 11)$ Tesla [1]. These observations suggest the existence of spinons in a QSL.

On the theoretical side, we have shown in Section 2 that the Dirac equation for the electron in longitudinal fields can be transformed into a spin 0 Klein-Gordon-type equations (9), describing particles with charge q and $-q$, and a spin $\frac{1}{2}$ Weyl equation (8), describing a neutrino. Therefore, we have achieved, within the formalism of the Dirac equation, a spin-charge separation into a holon and antiholon, described by Eqs. (9), plus a spinon, described by the massless Weyl equation (8).

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