SELF-CONSISTENT EM FIELD

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Abstract. This article try to unified the four basic forces by Maxwell equations, the only experimental theory. Self-consistent Maxwell equations with the e-current coming from matter current is proposed, and is solved to electrons and the structures of particles and atomic nucleus. The static properties and decay are reasoned, all meet experimental data. The equation of general relativity sheerly with electromagnetic field is discussed as the base of this theory. In the end the conformation elementarily between this theory and QED and weak theory is discussed.

Contents

1. Bound Dimensions

A rebuilding of units and physical dimensions is needed. Time s is fundamental. We can define:

The unit of time: s (second)

The unit of length: cs (c is the velocity of light)

The unit of energy: $\hbar/s$ (h is Plank constant)

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The unit dielectric constant ϵ is

$$[\epsilon] = \frac{[Q]^2}{[E][L]} = \frac{[Q]^2}{\hbar c}$$

The unit of magnetic permeability μ is

$$[\mu] = \frac{[E][T]^2}{[Q]^2[L]} = \frac{\hbar}{c[Q]^2}$$

The unit of Q (charge) is defined as

$$c[\epsilon] = c[\mu] = 1$$

then

$$[Q] = \sqrt{\hbar}$$

$$\sqrt{\hbar} = (1.0546 \times 10^{-34})^{1/2} C$$

C is charge's SI unit Coulomb.

For convenience, new base units by unit-free constants are defined,

$$c = 1, \hbar = 1, [Q] = \sqrt{\hbar} = [1]$$

then the units are reduced.

Define

$$UnitiveElectricalCharge : \sigma = \sqrt{\hbar}$$

$$\sigma = 1.03 \times 10^{-17} C \approx 64e$$

$$e_{/\sigma} = e/\sigma = 1.57 \times 10^{-2} \approx 1/64$$

The system is redefined and rebuilt as:

$$s \to C's : 1 = m/e =: \beta, \quad m := |k_e| \approx m_e$$

$s \to Cs$ means the value of the redefined second as becomes C seconds. Then all units are power σ^n. This unit system is called *bound dimension* or *bound unit*. We always take the definition latter in this article

$$\beta = 1$$

With physicals

$$[V] := \sigma^n, [W] := \sigma^m$$

such is defined with measure of a physical of energy k:

$$V = M, \quad k = 1$$

it means

$$\frac{V}{k^n} = \frac{W}{k^m} =: [W]_{k=1}$$

We always use the measure $\sigma = 1$ when the measure are default, hence some way we think unit disappears in this case.

2. Inner Field of Electron

Try the self-consistent Maxwell equation for the inner electromagnetic (EM) field of electrons

$$\partial^l \partial_l A_\nu = i A^*_\mu \partial_\nu A^\mu /2 + cc. = J_\nu, \quad \sigma = 1 \tag{2.1}$$

$$\partial^\nu \cdot A_\nu = 0$$

with definition

$$(A^i) := (V, \mathbf{A}), (A_i) := (V, -\mathbf{A})$$
$$(J^i) = (\rho, \mathbf{J}), (J_i) = (\rho, -\mathbf{J})$$
$$\partial := (\partial_i) := (\partial_t, \partial_{x_1}, \partial_{x_2}, \partial_{x_3})$$
$$\partial' := (\partial^i) := (\partial_t, -\partial_{x_1}, -\partial_{x_2}, -\partial_{x_3})$$
$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The equation 2.1 has symmetry:

$$cc.PT$$

3. General Electromagnetic Field

We find

$$(x', t') := (x, t - r)$$
$$\partial_x^2 - \partial_t^2 = \partial_{x'}^2 =: \nabla'^2$$

The following is the energy of a piece of field A:

$$\varepsilon := \frac{1}{2}(< E, D > + < H, B >) \tag{3.1}$$

The time-variant part is neglected, as a convention for energy calculation. If the field has Fourier transformation then the *field energy* becomes

$$\varepsilon = \frac{1}{2} < A_\nu | \partial_t^2 - \nabla^2 | A^\nu > \tag{3.2}$$

4. Solution of Electron

The solution by *recursive re-substitution* (RRS) for the two sides of the equation is proposed. For the equation

$$\hat{P}' B = \hat{P} B$$

Its algorithm is that (It's approximate, the exact solution needs a rate on the start state in the re-substitution for the normalization condition)

$$\hat{P}'(\sum_{k \le n} B_k + B_{n+1}) = \hat{P} \sum_{k \le n} B_k$$

A function is initially set and is corrected by RRS of the equation 2.1. Here is the start state

$$A_i = A_r e^{-ikt}, \partial_\mu \partial^\mu A_i = 0$$

The fields' correction A_n with n degrees of A_i is called the n degrees correction.

Firstly

$$\nabla^2 \phi = -k^2 \phi$$

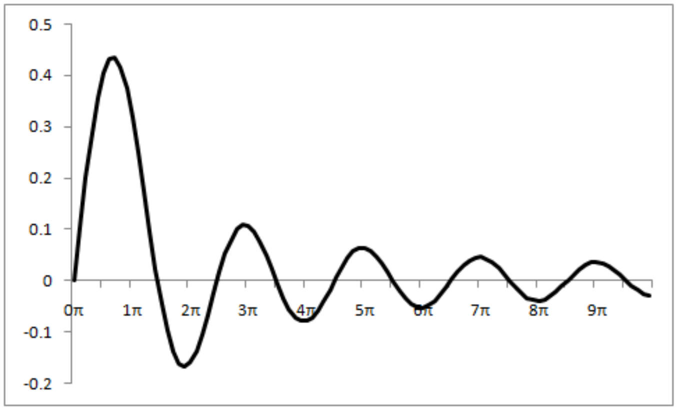

FIGURE 1. The function of j_1

is solved. Exactly, it's solved in spherical coordinate

$$-k^2 = \nabla^2 = \frac{1}{r^2}\partial_r(r^2\partial_r) + \frac{1}{r^2\sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{r^2\sin^2\theta}(\partial_\varphi)^2$$

Its solution is

$$\Omega(x) := \phi = j_l(r)Y_{lm}(\theta,\varphi)$$
$$\Omega_k(x) := j_l(kr)Y_{lm}(\theta,\varphi)$$
$$j_1(r) = \frac{\sin(r)}{r^2} - \frac{\cos(r)}{r}$$

As integrated or derived, its high-order singularity is eliminated by measure $k=1$. In fact

$$\nabla j_1(kr) = kj_1'(kr)\widehat{\mathbf{r}}$$

has problem at O in the next derivative.

In fact, it has to be re-defined in truncated form (T-form) and in process of limit

$$\Omega^\beta(x) = \Omega(x)(h(r) - h(r-\beta^{-1})), \quad \beta \to 0$$

We use the following definition

$$\omega_k := N\Omega_k e^{-ikt}, \quad k > 0$$
$$= \sum_{\widehat{\mathbf{k}}} \mathtt{F}(\widehat{\mathbf{k}})e^{i\mathbf{k}x - ikt}, \quad \widehat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|}$$
$$< k\Omega_k(x)|k\Omega_k(x) >= 1$$
$$< k\omega_k(x)|k\omega_k(x) >\approx e^2\delta^3(0)$$

There are calculations:

$$(\partial_t^2 - \nabla^2)u = -\nabla'^2 u = \delta(x')\delta(t') = \delta(x)\delta(t),$$
$$u := \frac{\delta(t-r)}{4\pi r} = \frac{\delta(t')}{4\pi r'},$$
$$fg * u\cdot\delta(x,t) = \mathcal{F}(f)(-) * \mathcal{F}(g)(-) * \mathcal{F}(u)(w)|_{w=0} = f(-)\cdot u * g\cdot\delta(x,t)$$
$$f * g\cdot\delta(t-t') = \delta(t-t')\int_I d\tau\cdot f(t/2-\tau)g(t/2+\tau)$$

$$\int_I dx(\Omega(x)^*\Omega(x))^n = (\int_I dx\Omega(x)^*\Omega(x))^n$$

In the frequencies of $\Omega(x)\cdot\Omega(x)$ the zero frequency is with the highest degrees of infinity.

For the objected function $\Omega(2kx) * \Omega(2x)$:

$$\nabla^2 = -\sum_{\mathbf{k}}(k^2 e^{-i\mathbf{k}x} * + e^{-i\widehat{k}x} *)$$

In order to avoid calculating singularity, calculate

$$\Omega^*(-x) * \nabla^2\Omega(x)$$

by

$$= \nabla\Omega^*(-x) * \nabla\Omega(x)$$

Because with the calculation of the part out of the high degree singularity,

$$[\Omega^*_{-k}(x)\Omega_{-k}(x)]_{k=-1} = -[\Omega^*_k(x)\Omega_k(x)]_{k=1}, \quad k > 0$$

it's valid that

$$i\nabla_\nu\Omega^*_k(-x) * i\nabla_\nu\Omega_k(x) = \widehat{k}\sigma k^2\frac{\delta^3(kx)}{\sigma^3\delta^3(0)}$$

5. Electrons

It's the start electron function for the RRS of the equation 2.1:

$$A_i^\nu := \pm i\lambda\partial^\nu\omega_k(x,t)/\sqrt{2}, \quad k > 0, \lambda \approx 1$$

which meet the covariant condition

$$\partial_\nu A_i^\nu = 0 \tag{5.1}$$

Some states are defined as the core of the electron, which's the start function $A_i(x,t)$ for the RRS of the equation 2.1 to get the whole electron function of field A: e or e_c:

$$e_r^+ : \omega_m(x,t), \quad e_l^- : \omega^*_{-m}(x,-t)$$
$$e_l^+ : \omega^*_m(x,-t), \quad e_r^- : \omega_{-m}(x,t)$$
$$e_r^+ \to e_l^+ : (x,y,z) \to (x,-y,-z)$$

r,l are the direction of the Magnetic Dipole Moment (MDM) of the electron. r direction is right-hand rotation.

Using the equation 2.1, the electron function is normalized with charge as

$$Q_e = m < A^\mu(-x)| * i\partial_t|A_\mu > /2 + cc. = e$$
$$|k| \approx m$$

The MDM of electron is calculated as the second degree proximation

$$\mu_z = m < A_{i\nu}(-x)| * -i\partial_\varphi|A_i^\nu > \cdot\hat{z}/4 + cc.$$
$$= \frac{Q_e}{2m}$$

The spin is

$$S_z = \mu_z k_e/e = 1/2$$

The correction in RRS of the equation 2.1 is calculated as

$$A - A_i = \frac{(A_i^* \cdot i\partial A_i/2 + cc.) *_4 u}{1 - \cdot i\partial(A_i - A_i^*)/2 *_4 u}$$

$$A = A \cdot h(t)$$

We find the Lorentz gauge:

$$\partial_\nu e^\nu = 0$$

It's valid that the *interactive potential between electrons*:

$$\varepsilon =< e'(-x)| * \partial^\nu \partial_\nu /2, *im\partial_t, * - m^2 | e >= C_{e'e}(1,1,1) \tag{5.2}$$

The function of e_r^+ is decoupled with e_l^+

$$< (e_r^+ + e_l^+)^\nu(-x)| * -m^2 | (e_r^+ + e_l^+)_\nu > - < (e_r^+)^\nu(-x)| * -m^2 | (e_l^+)_\nu >$$
$$- < (e_l^+)^\nu(-x)| * -m^2 | (e_r^+)_\nu >= 0$$

The following is the increment of the energy ε on the coupling of e_r^+, e_r^-, mainly between A_{2-n} and A_{2+n}

$$\varepsilon_e =< (e_r^+ + e_r^-)^\nu(-x)| * -m^2 | (e_r^+ + e_r^-)_\nu > - < (e_r^+)^\nu(-x)| * -m^2 | (e_r^+)_\nu >$$
$$- < (e_r^-)^\nu(-x)| * -m^2 | (e_r^-)_\nu >$$
$$\approx -2e^4\beta = -\frac{1}{1.66 \times 10^{-16}s}$$

The calculation is simply unit-dimension analysis.

The following is the increment of the energy ε on the coupling of e_r^+, e_l^-, mainly between A_{4-n} and A_{4+n}.

$$\varepsilon_x =< (e_r^+ + e_l^-)^\nu(-x)| * -m^2 | (e_r^+ + e_l^-)_\nu > - < (e_r^+)^\nu(-x)| * -m^2 | (e_l^+)_\nu >$$
$$- < (e_l^-)^\nu(-x)| * -m^2 | (e_r^-)_\nu >$$
$$\approx -\frac{1}{2}e^8\beta = -\frac{1}{1.08 \times 10^{-8}s}$$

These calculations are applied:

$$\Omega = -Y_{11}\frac{\cos(r)}{r}, \quad \nabla\Omega = Y_{11}\frac{\sin(r)}{r}\widehat{r}$$

6. System and TSS of Electrons

The movement of electron makes an EM field denoted by A, the unit of which is verified by interactions:

$$A := f *_3 \sum_i e_i = N \sum_X f(X,T)\delta(x - X, t - T) *_4 \sum_i e_i(x,t)|_{T=t}$$

with the particle number normalization:

$$< f|f >= 1$$

The following are naked stable particles:

particle	*electron*	*photon*	*neutino*
notation	e_r^+	γ_r	ν_r
structure	e_r^+	$(e_r^+ + e_r^-)$	$(e_r^+ + e_l^-)$

The following is the system of particle x with the initial state

$$A_0 := \sum_v e_x^v * E_v, \quad E_v := \sum_c n_{cv} e_c$$
$$e'_x := e_x^*(-t), \quad e_x^v := e_x, e'_x$$
$$e_x * -e := e_x(-x,-t) * e(x,t)$$

The *condition for the general EM field* is

$$\frac{1}{2}(\partial_t^2 - \nabla^2)A = im\partial_t A \tag{6.1}$$

The kinetic energy is the same of the EM energy. In fact $A, \partial_t A, J$ are all currents. The inner field of single electron is subjected to this equation in Fourier space.

$$\partial^\nu A_\nu = 0$$

$$p = m(\sum e_x * e|i\partial'|\sum e_x * e)/2 + cc.$$

$$\rho = m(\sum e_x * e|\sum e_x * i\partial_t e)/2 + cc.$$

$$\mathbf{J} = (\sum e_x * e| - i\nabla|\sum e_x * i\partial_t e)/2 + cc.$$

In fact for the initial A_0

$$(\partial_t^2 - \nabla^2)e_x = 0$$

Reference to the result 5.2 and its explanation. Then

$$e_x := k_x \Omega_{k_x}(x) e^{-ik_x t}$$

e_x is generally spherical Bessel function. With the charge conservation

$$Q_x =< e_x * E_v | e_x * i\partial_t E_v >$$

$$|k_x| \approx \frac{n}{|Q_x|} \quad \sigma = 1, \quad n := \sum_{vc} n_{vc}^2$$

In fact this state A_0 of electrons system is a (Transient) Steady State (TSS).

7. Muon

The initial of muon is

$$\mu_r^- : e_\mu * (e_r^- - e_r^- - e_l^+), \quad e_\mu = e_x(k_x = -m_\mu)$$

μ is approximately with mass $3m/e_{/\sigma} = 3 \times 64m$ [3.2][1] (The data in bracket is experimental by the referenced lab), spin S_e (electron spin), MDM $\mu_B m/k_\mu$.

The main channel of decay is

$$\mu_r^- \to e_r^- - \nu_l, \quad e_r^- \to -e_l^+ + \nu_l$$

$$e_\mu * e_r^- - e_\mu * \nu_l \to e_\mu * e_r^- - L(\delta^{1/2}(x)) * \nu_l$$

L is Lorentz transformation. The main life is

$$\varepsilon_\mu :=< e_\mu^* * e_\mu(-x)|e_r^{-*}(-x) * im\partial_t e_l^+ > + < e_\mu^* * e_\mu(-x)|e_l^{+*}(-x) * im\partial_t e_r^- >$$

$$= \frac{\varepsilon_x m}{k_\mu} = -\frac{1}{2.18 \times 10^{-6} s} \quad [2.1970 \times 10^{-6} s][1]$$

In fact the self-interactions of the two charges of neutrino are counteracted.

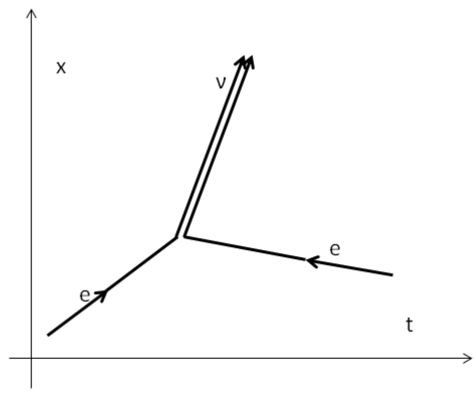

FIGURE 2. neutrino radiation

8. Pion

The initial of pion perhaps is

$$\pi_r^- : e'_\pi * (e_l^+ - e_l^+) + e_\pi * e_r^-$$

It's approximately with mass $3 \times 64m$ [4.2][1], spin S_e, MDM $\mu_B m/k_{\pi^-}$.

Decay Channels:

$$\pi_r^- \to -e_l^+ + \nu_l, \quad e_l^+ \to -e_r^- + \nu_l$$

The mean life approximately is

$$-\varepsilon_x/2 = \frac{1}{2.2 \times 10^{-8}s} \quad [(2.603 \times 10^{-8}s][1]$$

The precise result is calculated with successive decays.

9. Pion Neutral

The initial of pion neutral is perhaps like an atom

$$\pi^0 : (e_r^+ + e_l^+, e_r^- + e_l^-)$$

It's the main decay mode as

$$\pi^0 \to \gamma_r + \gamma_l$$

The loss of interaction is

$$-2\varepsilon_e = \frac{1}{8.3 \times 10^{-17}s} \quad [8.4 \times 10^{-17}s][1]$$

10. Tauon

The initial of tauon maybe

$$\tau_r^- : e'_\tau * (ne_l^- - ne_l^-) + e_\tau * (e_r^- - e_r^+ - e_r^-)$$

Its mass approximately $53 \times 64m$ [54][1] ($n = 5$). It has decay mode with a couple of neutrinos counteracted

$$e_\tau * (e_r^- - e_r^+ - e_r^-) \to e_\tau * e_r^- - \gamma_r$$

The main life is

$$\frac{\varepsilon_e m}{k_\tau} = -\frac{1}{5.5\times 10^{-13}s} \quad [2.91\times 10^{-13}s; B.R. : 0.17][1]$$

Perhaps, it's a mixture with distinct coefficients n.

The stand-by part

$$e'_\tau * (ne_l^- - ne_l^-)$$

is invisible in reaction, although its static charge isn't zero alone.

11. Proton

The initial of proton may be like

$$p_r^+ : e_p * (4e_r^- - 3e_l^+ - 2e_l^+), \quad e_p = e_x(k_x = m_p)$$

The mass is $29\times 64m$ [29][1] that's very close to the real mass. The MDM is calculated as $3\mu_N$, spin is S_e. The proton thus designed is eternal.

12. Neutron

Neutron is the atom of a proton and a muon

$$n = (p_r^+, \mu^-)$$

The muon take the first track, with the decay process

$$\Phi * \mu^- = \Phi * e_\mu * (e_r^- - e_r^- - e_l^+) \to \Phi * e_\mu * e_r^- - e_\mu * \nu_l$$

Calculate the variation of the action of the open system that the energy of system subtracts the interactive, (of the fork) the interactive input is equal to the increment of energy of system,

$$i\partial_t\Phi + \frac{1}{2}\nabla^2\Phi = -\frac{\alpha'}{r}\Phi \quad m_\mu = 1 \tag{12.1}$$

two of the terms of EM energy are neglected. It's resolved to

$$\Phi = Ne^{-r/r_0}e^{-iE_1 t}$$

$$E_1 = -\frac{1}{2}c^2\alpha'^2 = -\frac{1}{2}c^2\alpha^2(\frac{\sigma^2}{k_\mu^2}) = -13.6eV\cdot 3^{-2}$$

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar c} \approx 1/137$$

It's approximately the decay life of muon in the track that

$$\varepsilon_n = \frac{-E_1}{m_\mu}e^3_{/\sigma}\varepsilon_x = -\frac{1}{936s}$$

13. Atomic Nucleus

We can find the equation for the sum field of Z' ones of protons: Φ, and the sum field of n ones of muons: ϕ

$$\frac{1}{2}\partial_t^2\Phi - ik_p\partial_t\Phi + \frac{1}{2}\nabla^2\Phi = (Z'+1)\frac{\alpha\sigma^4}{r} * \Phi - n\frac{\alpha\sigma^4}{r} * \phi$$

$$\frac{1}{2}\partial_t^2\phi - ik_\mu\partial_t\phi + \frac{1}{2}\nabla^2\phi = -Z'\frac{\alpha\sigma^4}{r} * \Phi + (n-1)\frac{\alpha\sigma^4}{r} * \phi$$

The more number on protons' interaction is from

$$< \Phi * e_p * \sum e|\Phi * (\partial^\nu\partial_\nu e_p) * \sum e > /2$$

The similar is for muon.

Make

$$t = Ct'$$

to fit

$$\frac{1}{2}\partial_{t'}^2\phi - ik_p\partial_{t'}\phi + \frac{1}{2}\nabla^2\phi = -Z'\frac{\alpha\sigma^4}{r} * \Phi + (n-1)\frac{\alpha\sigma^4}{r} * \phi$$

Define

$$\Phi' e^{-iEt} = \Phi, \quad \phi' e^{-iEt} = \phi$$

$$\zeta = \Phi' + \eta\phi'$$

$$(Z'+1) - \eta Z' = -n/\eta + n - 1 =: N$$

$$\eta = \frac{(Z'-n+2) \pm \sqrt{(Z'-n+2)^2 + 4Z'n}}{2Z'}$$

then

$$-(E^2/2 + Ek_p)\nabla^2\zeta + \frac{1}{2}\nabla^4\zeta + 4\pi\alpha\sigma^4 N\zeta = 0$$

$$\nabla^2 = (E^2/2 + E) - \sqrt{(E^2/2+E)^2 - 8\pi\alpha\sigma^4 N} = -k^2 \quad k_p = 1$$

$$\zeta = j_l(kr)Y_{lm}(\theta,\varphi)$$

and

$$|k| = 1 \approx k_p$$

to delete the high-order singularity on ω of its derivatives. So that

$$E = -1 + \sqrt{-8\pi\alpha\sigma^2 N} \quad k_p = 1$$

$$N = \frac{1}{2}((Z'+n) - \sqrt{(Z'+n)^2 + 4(Z'-n) + 4})$$

$$\approx -\chi, \quad \chi := \frac{Z'-n}{Z'+n}$$

This state ζ is a TSS, so that $|k| = |E|$.

We find

$$\chi = 1/3: \quad E + 1 \approx 8.0MeV \quad k_p = 1$$

It's noticed that the gross interaction is least (zero) when

$$\eta \approx -1/2, 1$$

which means most stable nucleus is of the same protons (Z) and neutrons (n) approximately.

14. Basic Results for Interaction

For decay

$$W(t) = \Gamma e^{-\Gamma t}, \Gamma = 1 \tag{14.1}$$

$$\Gamma = \frac{1}{2} < A_\nu|\partial^\mu\partial_\mu|A^\nu > |_\infty^{t=0}$$

This result is deduced form the equation 6.1. It leads to the result between decay life and EM emission or the interactive potential.

The distribution shape of decay can be explain as

$$e^{-\Gamma t/2} e_x * \sum_i e_i \approx \Omega_x * \sum_i e_i \cdot e^{-\Gamma t/2 - ik_x t}, 0 < t < \Delta$$

It's the real wave of the particle x near the initial time and expanded in that time span

$$\approx \Omega_x * \sum_i e_i \cdot \int_{-\infty}^{\infty} dk \frac{Ce^{-ikt}}{k - k_x - i\Gamma/2}$$

15. Grand Unification

The General Theory of Relativity is

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij}/c^4 \tag{15.1}$$

Firstly the unit second is redefined as S to simplify the equation 15.1

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij}$$

Then

$$R_{ij} - \frac{1}{2} R g_{ij} = F^*_{i\mu} F_j{}^\mu - g_{ij} F^*_{\mu\nu} F^{\mu\nu}/4$$

We observe that the co-variant curvature is

$$R_{ij} = F^*_{i\mu} F_j{}^\mu + g_{ij} F^*_{\mu\nu} F^{\mu\nu}/8$$

16. Conclusion

Fortunately, this model explained all the effects in the known world: strong, weak and electromagnetic effects, and even subclassify them further if not being to add new ones. In this model the only field is electromagnetic field, and this stands for the philosophical that the unified world is from an unique source, all that depend on a simple fact: mechanical energy is the same of electromagnetic energy, and a hypothesis: in electron the movements of charge and mass are the same.

My description of particles is compatible with QED elementarily and depends on momentum quantification formula, and only contributes to it with theory of consonance state in fact. In some way, the electron function is a good promotion for the experimental models of proton and electron that went up very early.

Underlining my calculations a fact is that the electrons have the same phase (electrons consonance), which the BIG BANG theory would explain, all electrons are generated in the same time and place, the same source.

References

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