

Stellar distance and velocity (III)

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Abstract

The use of parallax angles is one of the standard methods for determining stellar distance. The problem that arises in using this method is how to measure that angle. In order for the measurement to be correct, it is necessary for the object we are observing to be stationary in relation to the sun. This is generally not true. One way to overcome this problem is to observe the object from two different places at the same time. This would be technically possible but will probably never be realized. Another way to determine the distance is given in [1]. With certain assumptions, this is a mathematically completely correct method. After the publication of the third Gaia's catalog [2], we are now able to test the proposed method using real data. Unfortunately, for the majority of stars it is not possible to obtain the distance directly, but with the help of some additional measurements we would be able to indirectly determine the distance of such stars.

Keywords: *stellar distance, stellar velocity, The Gaia Catalogue's*

1. Determining the stellar distance and velocity

Suppose that the observed star Z is moving with a uniform, rectilinear space motion regarding the sun Fig. 1. Let us denote by τ_1 the time when the signal was sent from the point noted by Z_1 and by t_1 the time when the signal is registered at point noted by A . We assume that τ_1 and t_1 are expressed in the same units of time. The unit vector of the direction AZ_1 is denoted by $\hat{\mathbf{a}}$. In an analogous way, we will define triples $(\tau_2, t_2, \hat{\mathbf{b}})$ and $(\tau_3, t_3, \hat{\mathbf{c}})$ for pairs of points (B, Z_2) and (C, Z_3) , respectively.

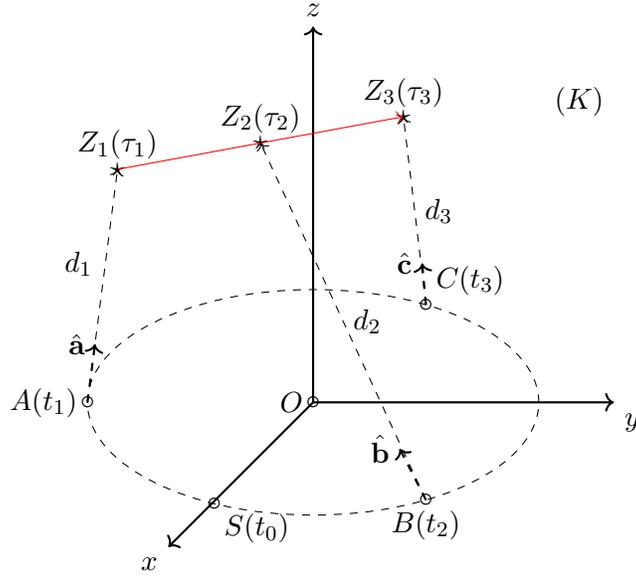


Figure 1: Star Z moves uniformly regarding the sun. There is one nontrivial solution.

Coordinate system (K) is heliocentric ecliptic coordinate system. Coordinate axes are determined in accordance with the *ICRS* standard.

This is a list of constants that will be used in the calculations:

$$\Pi = 3.14159265358979 \quad (1)$$

$$R = AU = 149597870.7 \text{ [km]} \quad (2)$$

$$c = 299792.458 \text{ [km/sec]} \quad (3)$$

$$\text{yearday} = 365.25 \text{ (number of days in one year)} \quad (4)$$

$$\text{daysec} = 24 * 3600 \text{ [sec]} \text{ (number of seconds in one day)} \quad (5)$$

$$\text{yearsec} = \text{yearday} * \text{daysec} \text{ [sec]} \quad (6)$$

The reference epoch for Gaia DR1 is J2015

J2015 = 2015/01/01 12:00(?) (GMT) or 0.5 days from the beginning of the year 2015.

The reference epoch for Gaia DR2 is J2015.5

J2015.5 = 2015 July 2, 21:00:00 (GMT) or $365.25 * 0.5$ days from the beginning of the year 2015.

The reference epoch for Gaia DR3 is J2016

J2016 = J2015 + yearday = 2016/01/01 18:00(?) (GMT)

Vernal equinox in 2015 happened on March Mar 20, 22:45 (GMT) or 78.94791667 days from the beginning of the year 2015.

We will now define the times when the measurements were made, and the time t_0 that will be considered as the initial time.

$$t_0 = 78.94791667 * \text{daysec} \text{ [sec]} \text{ - Vernal equinox in 2015}$$

$$t_1 = 0.5 * \text{daysec} \text{ [sec]} \text{ - the time of the first measurement (corresponds to J2015)}$$

$$t_2 = 0.5 * \text{yearsec} \text{ [sec]} \text{ - the time of the second measurement (corresponds to J2015.5)}$$

$$t_3 = \text{yearsec} \text{ [sec]} \text{ - the time of the third measurement (corresponds to J2016)}$$

The spherical coordinates (lon_{2015}, lat_{2015}) , $(lon_{2015.5}, lat_{2015.5})$ and (lon_{2016}, lat_{2016}) are given in the Gaia's catalogs:

- $lon2015$ - Ecliptic longitude of the source in ICRS at the reference epoch J2015.0
- $lat2015$ - Ecliptic latitude of the source in ICRS at the reference epoch J2015.0
- $lon2015.5$ - Ecliptic longitude of the source in ICRS at the reference epoch J2015.5
- $lat2015.5$ - Ecliptic latitude of the source in ICRS at the reference epoch J2015.5
- $lon2016$ - Ecliptic longitude of the source in ICRS at the reference epoch J2016.0
- $lat2016$ - Ecliptic latitude of the source in ICRS at the reference epoch J2016.0

We will now transform the spherical coordinates into Cartesian coordinates.

$$a_x = \cos(lon2015) * \cos(lat2015) \quad (7)$$

$$a_y = \cos(lat2015) * \sin(lon2015) \quad (8)$$

$$a_z = \sin(lat2015) \quad (9)$$

$$\hat{\mathbf{a}} = [a_x, a_y, a_z] \quad (10)$$

$$b_x = \cos(lon2015.5) * \cos(lat2015.5) \quad (11)$$

$$b_y = \cos(lat2015.5) * \sin(lon2015.5) \quad (12)$$

$$b_z = \sin(lat2015.5) \quad (13)$$

$$\hat{\mathbf{b}} = [b_x, b_y, b_z] \quad (14)$$

$$c_x = \cos(lon2016) * \cos(lat2016) \quad (15)$$

$$c_y = \cos(lat2016) * \sin(lon2016) \quad (16)$$

$$c_z = \sin(lat2016) \quad (17)$$

$$\hat{\mathbf{c}} = [c_x, c_y, c_z] \quad (18)$$

The origin of the (K) coordinate system is at the barycenter of the solar system, therefore the velocity \mathbf{v} at which the solar system moves relative to the (K) is equal to zero [1].

$$\mathbf{v} = [v_x, v_y, v_z] \quad (19)$$

$$v_x = 0 \quad (20)$$

$$v_y = 0 \quad (21)$$

$$v_z = 0 \quad (22)$$

Let the angles α , β and γ be defined as follows:

$$\alpha = \angle(S, O, A) = \frac{2 * \Pi * (t_1 - t_0)}{yearsec} \quad (23)$$

$$\beta = \angle(S, O, B) = \frac{2 * \Pi * (t_2 - t_0)}{yearsec} \quad (24)$$

$$\gamma = \angle(S, O, C) = \frac{2 * \Pi * (t_3 - t_0)}{yearsec} \quad (25)$$

We will now determine the coordinates of points $A(t_1)$, $B(t_2)$, and $C(t_3)$, which indicate the positions of the observer at the time the measurements were made Fig. 1.

$$A(t_1) = (A_x, A_y, A_z) \quad (26)$$

$$A_x = t_1 * v_x + R * \cos(\alpha) = R * \cos(\alpha) \quad (27)$$

$$A_y = t_1 * v_y + R * \sin(\alpha) = R * \sin(\alpha) \quad (28)$$

$$A_z = t_1 * v_z = 0 \quad (29)$$

$$B(t_2) = (B_x, B_y, B_z) \quad (30)$$

$$B_x = t_2 * v_x + R * \cos(\beta) = R * \cos(\beta) \quad (31)$$

$$B_y = t_2 * v_y + R * \sin(\beta) = R * \sin(\beta) \quad (32)$$

$$B_z = t_2 * v_z = 0 \quad (33)$$

$$C(t_3) = (C_x, C_y, C_z) \quad (34)$$

$$C_x = t_3 * v_x + R * \cos(\gamma) = R * \cos(\gamma) \quad (35)$$

$$C_y = t_3 * v_y + R * \sin(\gamma) = R * \sin(\gamma) \quad (36)$$

$$C_z = t_3 * v_z = 0 \quad (37)$$

Let define the time intervals Δt_1 and Δt_2 as follows:

$$\Delta t_1 = t_2 - t_1 \quad (38)$$

$$\Delta t_2 = t_3 - t_1 \quad (39)$$

One can define the matrices M and N :

$$M = \begin{bmatrix} \Delta t_2 * c & 1 & 1 & 1 \\ C_x - A_x & a_x & b_x & c_x \\ C_y - A_y & a_y & b_y & c_y \\ C_z - A_z & a_z & b_z & c_z \end{bmatrix} \quad (40)$$

$$N = \begin{bmatrix} \Delta t_1 * c & 1 & 1 & 1 \\ B_x - A_x & a_x & b_x & c_x \\ B_y - A_y & a_y & b_y & c_y \\ B_z - A_z & a_z & b_z & c_z \end{bmatrix} \quad (41)$$

With τ_1 , τ_2 and τ_3 we denote the times when the signals have been sent Fig. (1). Of course, these times mean nothing to us, but we can define the times $\Delta\tau_1$, $\Delta\tau_2$ as it follows:

$$\Delta\tau_1 = \tau_2 - \tau_1 \quad (42)$$

$$\Delta\tau_2 = \tau_3 - \tau_1 \quad (43)$$

The time on the star Z may be slower or faster than the time on Earth, but we assume that the stellar time $\Delta\tau_i$ is converted to terrestrial time.

Coefficient k is defined in the following way.

$$k = \frac{\overline{Z_0 Z_2}}{\overline{Z_0 Z_1}} = \frac{\Delta\tau_2}{\Delta\tau_1} \quad (44)$$

It has been proved in [1] that :

$$k = \frac{\det(M)}{\det(N)} \quad (45)$$

Let us define matrices D, D_1, D_2, D_3 in following way:

$$D = \begin{bmatrix} (k-1) * a_x & -k * b_x & c_x \\ (k-1) * a_y & -k * b_y & c_y \\ (k-1) * a_z & -k * b_z & c_z \end{bmatrix} \quad (46)$$

$$D_1 = \begin{bmatrix} (1-k) * A_x + k * B_x - C_x & -k * b_x & c_x \\ (1-k) * A_y + k * B_y - C_y & -k * b_y & c_y \\ (1-k) * A_z + k * B_z - C_z & -k * b_z & c_z \end{bmatrix} \quad (47)$$

$$D_2 = \begin{bmatrix} (k-1) * a_x & (1-k) * A_x + k * B_x - C_x & c_x \\ (k-1) * a_y & (1-k) * A_y + k * B_y - C_y & c_y \\ (k-1) * a_z & (1-k) * A_z + k * B_z - C_z & c_z \end{bmatrix} \quad (48)$$

$$D_3 = \begin{bmatrix} (k-1) * a_x & -k * b_x & (1-k) * A_x + k * B_x - C_x \\ (k-1) * a_y & -k * b_y & (1-k) * A_y + k * B_y - C_y \\ (k-1) * a_z & -k * b_z & (1-k) * A_z + k * B_z - C_z \end{bmatrix} \quad (49)$$

Assuming that $\det(D) \neq 0$, we found [1] that the distances d_1, d_2, d_3 are given by the following equations :

$$d_1 = \frac{\det(D_1)}{\det(D)} \quad (50)$$

$$d_2 = \frac{\det(D_2)}{\det(D)} \quad (51)$$

$$d_3 = \frac{\det(D_3)}{\det(D)} \quad (52)$$

The triple (d_1, d_2, d_3) represents a unique solution. Therefore, the collinear points Z_1, Z_2 and Z_3 are uniquely determined. If there were some other three collinear points Z'_1, Z'_2 and Z'_3 which would correspond to the three positions of the observed object, then we would have two different solutions, which is contrary to the fact that there is only one triple as a solution. In this way we proved that:

- d_1 - denotes distance between the Earth (satellite) and a star at the time (t_1)
- d_2 - denotes distance between the Earth (satellite) and a star at the time (t_2)
- d_3 - denotes distance between the Earth (satellite) and a star at the time (t_3)

For brevity in writing this method will be denoted by $(3P)$.

Let us define the arithmetic mean noted by d , standard deviation noted by σ and the coefficient of variation (CV) noted by c_v as follows:

$$d = \frac{d_1 + d_2 + d_3}{3} \quad (53)$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^3 (d - d_i)^2}{3}} \quad (54)$$

$$c_v = \frac{\sigma}{d} \quad (55)$$

The distances d_1 , d_2 and d_3 are defined in different ways but we will assume that the differences between them are relatively very "small" compared to d .

Transverse velocity(speed) v_t is defined as usual:

$$v_t[km/sec] = \frac{d * \tan(PM)}{\Delta t_2} = \frac{d * \tan(PM)}{yearsec} \quad (56)$$

In fact Equation (56) should be written in the following form:

$$v_t[km/sec] = \frac{d * \tan(PM)}{\Delta \tau_2} \quad (57)$$

Where total proper motion is noted by PM while Δt_2 and $\Delta \tau_2$ are defined by equations (39) and (43). Time interval $\Delta \tau_2$ can be measured using Doppler effect [4]. The difference between the times $\Delta \tau_i$ and Δt_i is due for two reasons, first the speed of light is finite and second the distance d is constantly changing. For example if the observed object moves away then $\Delta t_2 > \Delta \tau_2$.

2. Analysis of the obtained results

Depending on whether it is possible to determine the distance d and with what precision, all observed cosmic objects can be divided into three groups:

1° $(d_1 > 0) \wedge (d_2 > 0) \wedge (d_3 > 0) \wedge (c_v < \varepsilon)$ where ε represents some small number and $d_1 = AZ_1$, $d_2 = BZ_2$ and $d_3 = CZ_3$

In this case we will say that the star Z moves uniformly at a distance d from the Earth Fig. (1). The accuracy in determining the distance d is significantly greater when points A , B and C form a triangle instead of lying on one line. This means that the intervals between the two measurements should be four (sixteen, twenty-eight..) instead of six months. This case is presented in Table 1.

Source_id	Ecl.lon(2016)	Ecl.lat(2016)	d ₁ [km]	d ₂ [km]	d ₃ [km]	c _v	Dist[km]
3961616055021330688	179.7821975005790	29.6831549248165	2.142 E16	2.134 E16	2.126 E16	0.0038	2.135 E16
3961742979894505856	180.1228034635390	29.4999358548088	1.850 E17	1.8472 E17	1.8438 E17	0.0018	1.8472 E17
2342878993244280448	359.6681908	-29.52927592	1.33132 E17	1.33173 E17	1.33213 E17	0.0003	1.33173 E17

Table 1: Distance d is calculated on the basis of the data obtained from the Gaia's catalogs, where coefficient of variation $c_v < 0.004$

2° $(d_1 > 0) \wedge (d_2 > 0) \wedge (d_3 > 0) \wedge (c_v \geq \varepsilon)$

where ε represents some small number and $d_1 = AS_1$, $d_2 = BS_2$ and $d_3 = CS_3$

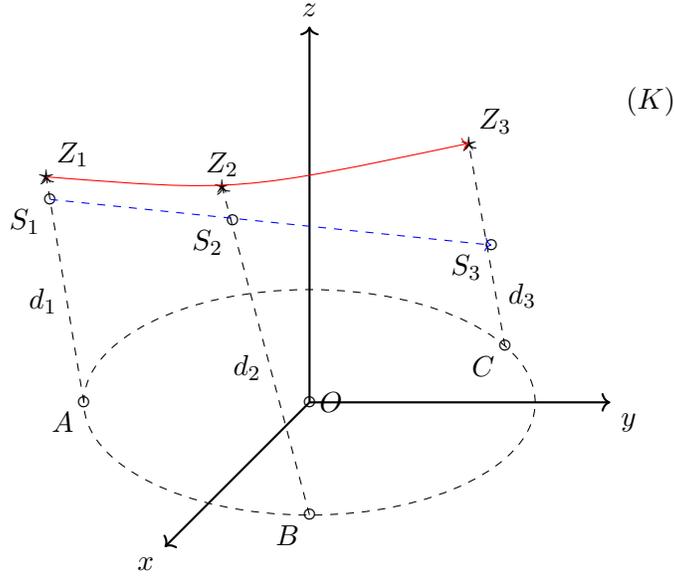


Figure 2: The star moves along a curve that is close to a straight line. All distances d_i are positive, but we are still unable to determine the distance d

In this case we will say the measurements are not accurate enough or that the star does not move uniformly with respect to the sun Fig. (2). The time difference between the two measurements is six months. If we were to reduce this time to 2-4 months, then in some cases the trajectory along which the observing star moves would be closer to a straight line. This means that the accuracy in determining the distance d would eventually increase. This case is presented in Table 2.

Source_id	Ecl.lon(2016)	Ecl.lat(2016)	$d_1[km]$	$d_2[km]$	$d_3[km]$	c_v	Dist[km]
2342610197010495616	0.353415475	-30.19461453	9.0678 E15	6.35516 E15	3.63505 E15	0.43	-
2342901155275643392	0.530649858	-29.58663349	2.94577 E16	2.33137 E16	1.71527 E16	0.26	-
3442647460064052224	85.923436780576	5.465370789112	1.50462 E15	2.27783 E15	3.05312 E15	0.3398	-

Table 2: Distance d cannot be precisely determined because coefficient of variation $c_v > 0.25$

$$3^\circ \quad (d_1 < 0) \vee (d_2 < 0) \vee (d_3 < 0)$$

where $d_1 = AS_1$, $d_2 = BS_2$ and $d_3 = CS_3$

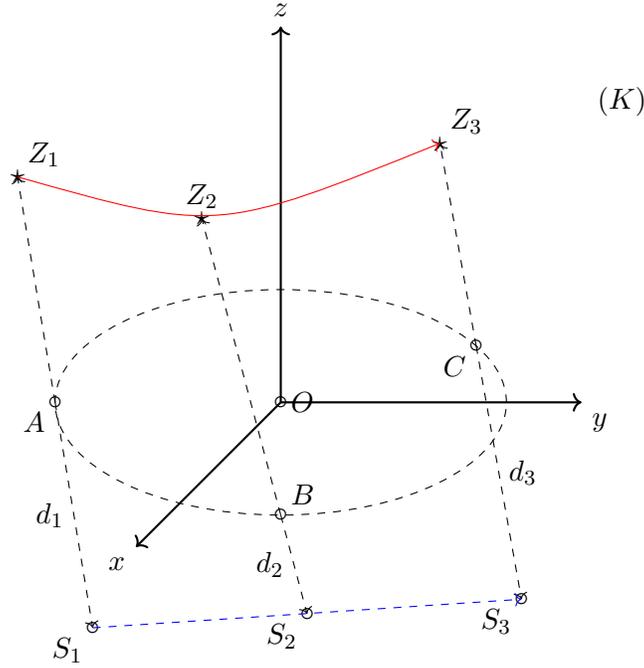


Figure 3: The star Z is moving along a curve. Some of distances d_i are negative

If at least one of d_i is less than zero it means that d does not represent the distance between the earth and the star Z . The points S_1, S_2 and S_3 lie on one line and since there is one solution it means that the points Z_1, Z_2 and Z_3 are not collinear but lie on some curve Fig. (3). As indicated in Table 3, the distance between the observer and the star is not possible to determine.

Source_id	Ecl.lon(2016)	Ecl.lat(2016)	d_1 [km]	d_2 [km]	d_3 [km]	c_v	Dist[km]
3961707520645185536	180.0038102526130	29.7935242455032	-4.651 E16	-4.474 E16	-4.297 E16	-0.0395	-
3961592522895189632	179.9320080075880	30.2431018805237	-8.873 E16	-9.828 E16	-1.8438 E17	-0.0973	-
3442624890011192320	86.08541291	5.146543438	-2.51603 E15	3.45772 E15	9.44761 E15	1.727	-

Table 3: Some of distances d_i are less than zero. The star moves along the curve and it is not possible to determine distant d

Therefore it could be concluded, in order to obtain optimal results in determining the distance d , the time interval between two measurements should not be fixed, but chosen according to which of the three groups the observed object belongs to.

3. Comparison between the two methods

After testing 320,000 randomly selected stars, assuming that the c_v was equal to 0.001, for only 321 stars we were able to determine the distance d . In percentage it is about 0.1%. We have considered only those cases where $ra_error < 1$ [mas] and $dec_error < 1$ [mas] for each of the three Epochs [2].

Source_id	Ecl.lon(2016)	Ecl.lat(2016)	c_v	Dist[km]	Dist(Prx)[km]
3892032057203216512	179.464250611527	2.265023777597	4.93E-06	2.91551E+15	-
6708720503832637056	277.843888734382	-22.691401979412	3.58E-05	1.13911E+16	9.27202E+15
5960619316369918976	265.263620489822	-17.335282366231	3.56E-05	2.92601E+15	6.01184E+17
4283639513032943872	282.108514007983	27.488067363330	5.24E-06	6.13541E+16	1.33945E+17
4155142887628422656	280.073983972126	12.495318825158	1.42E-05	1.40158E+17	2.61482E+16
4052874077025202304	275.923229156324	-2.631458890691	8.30E-05	4.29259E+16	8.92399E+16
5627132972875759488	148.161330687293	-49.058942695502	9.98E-05	8.76936E+16	3.96268E+16
4073377151404982528	280.416811660799	-2.572231092517	3.65E-05	1.78858E+16	8.3734E+16
2774766954475533312	14.845191577444	6.420382868841	2.40E-04	5.53097E+15	1.30875E+16
166222071943317760	69.016283020408	9.305778028492	7.36E-04	2.18849E+16	8.07471E+15
5576498473844837120	110.866773513287	-60.378239028318	6.13E-04	1.14915E+17	6.98058E+15
214695656962915712	88.865916267096	27.505576601733	6.74E-04	6.33499E+16	5.51230E+16
3443394165898284544	86.843372879689	5.386570760321	2.24E-04	4.2984E+16	1.85977E+16

Table 4: Distances obtained by the (3P) method and the standard parallax method.

Table 4 shows only a few examples but also in all other cases there are significant differences between the two methods in determining the distance d .

4. Determining the distance d for stars with negative parallax

In the (3P) method, the parallax angle does not play any role, so it is completely irrelevant whether the parallax is positive or not.

Source_id	Ecl.lon(2016)	Ecl.lat(2016)	Parallax[mas]	c_v	Dist[km]	Dist(Prx)[km]
5870540554937313792	230.8123891030300	-44.4001685138039	-1.351731861	2.63E-04	1.68442E+16	-
5820665779779315968	252.6634092423410	-46.5826995849347	-1.073606785	7.96E-04	3.32807E+17	-
4316942586391274752	300.5130336201920	34.1575615028951	-4.338644109	4.94E-04	6.65775E+15	-
4026999166801713536	167.2093209988000	30.3870056006152	-1.052721103	1.81E-04	2.31501E+16	-
902781578495836544	117.2702243058950	13.7984472959831	-1.021739621	5.11E-04	7.73277E+15	-
4234495599583215872	301.187125786867	18.822706614695	-0.051135084	5.10E-05	1.72358E+17	-
4172623468949463936	273.405832104713	16.697263354807	-0.006873903	4.07E-06	5.8401E+16	-
4123917096702075264	265.296806899843	6.161622135268	-0.0001959	6.66E-05	6.62041E+16	-
4002033106025556992	173.793473382709	23.271354771437	-0.2758356	7.74E-04	1.27353E+18	-

Table 5: Distances for stars with negative parallax.

Using the proposed method, as shown in Table 5, it is easy to find the distance d for those stars for which this would not be possible if we used the standard parallax method.

5. The distance for stars whose parallax is greater than 10 [mas]

Just for comparison between the two methods, the distances of several stars, whose parallax is greater than 10, are shown in Table 6.

#	Source_id	Ecl.lon(2016)	Ecl.lat(2016)	Parallax[mas]	c_v	Dist[km]	Dist(Prx)[km]
1	3433155616700212736	93.087491261317	3.386323758837	22.08313702	1.58E-04	4.66583E+15	6.9865E+14
2	3444013946861337344	88.762042702292	6.976990094510	13.68358517	5.85E-04	5.6425E+15	1.12751E+15
3	6871699360068438784	295.411412404273	4.200803323270	20.70966019	2.89E-04	6.06583E+15	7.44985E+14
4	195463274447721728	84.208040079038	19.720229918935	13.08067231	5.43E-06	1.91901E+16	1.17948E+15
5	654704237414120192	118.569387968675	-5.049570286238	32.03269916	6.94E-06	2.50054E+16	4.81645E+14
7	5490774701892502016	138.499803200596	-74.508682013379	11.21404409	9.84E-06	3.45189E+16	1.37581E+15
8	5272621390101236608	199.803820553988	-76.083791197977	28.49767132	6.96E-06	1.01299E+17	5.41391E+14

Table 6: Distances for stars whose parallax is greater than 10 [mas]

The distance of the star marked with #8 is far greater than the distance of the star marked with #7, although the parallax angle of the star #8 is greater than the parallax angle of the star marked with #7.

6. How big is the Milky Way galaxy

The Milky Way is the second-largest galaxy in the Local Group (after the Andromeda Galaxy), with its stellar disk approximately 170–200 [*kly*] and on average, approximately 1 [*kly*] thick. The Sun is 25–28 [*kly*] from the Galactic Center [3].

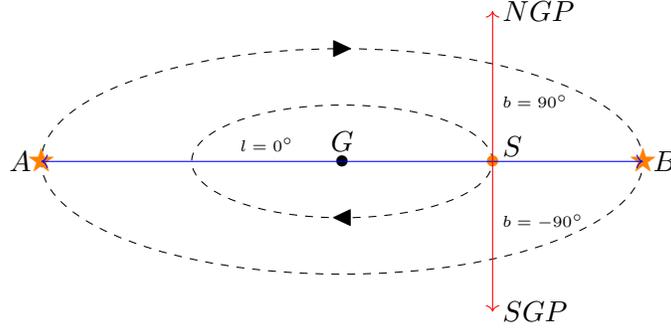


Figure 4: A schematic picture of the Sun's location in the Milky Way Galaxy

In referring to Fig 4. the following definitions apply:

- S - denotes position of the Sun
- G - denotes position of the Galactic center
- AB - denotes Galactic disc diameter
- NGP - denotes North Galactic Pole
- SGP - denotes South Galactic Pole
- l - denotes galactic longitude
- b - denotes galactic latitude

After we selected all the objects so that $abs(l - 180) < 1$ and among them we chose the three that have the greatest distances we got the final result shown in Table 7.

Source_id	Gal.lon(2016)	Gal.lat(2016)	PM[mas]	Prx[mas]	c_v	Dist[km]	Dist(Prx)[km]	v_t
3443331871693269888	180.537841235841	0.195830806156	13.633407	0.833999059	2.35E-05	2.61944E+17	1.84993E+16	548.63
3444241477048617472	179.145655747991	-0.154164937766	10.922541	1.513166702	5.27E-04	2.61773E+17	1.01961E+16	439.25
3443507209439201280	179.968690915909	0.715079161227	56.838364	6.075675427	6.42E-04	8.03899E+16	2.53937E+15	701.96

Table 7: The three farthest stars toward the galactic anticenter $abs(l - 180) < 1$

The same procedure was repeated assuming that $((l < 0.5) \vee (360 - l) < 0.5)$. The final result is shown in Table 8.

Source_id	Gal.lon(2016)	Gal.lat(2016)	PM[mas]	Prx[mas]	c_v	Dist[km]	Dist(Prx)	v_t
4057271883016119424	359.815611289133	-0.410278140213	2.231856	0.361633128	3.71E-04	1.560923E+18	4.266309E+16	535.20
4057204675365716736	359.709334091534	-0.834622324361	0.98699296	0.477899496	4.99E-04	1.215018E+18	3.228375E+16	184.23
4057431312201067008	0.992836333984	-0.819336706345	3.3731406	1.241408484	5.87E-04	6.324761E+17	1.242813E+16	327.75

Table 8: The three farthest stars toward the galactic center $(l < 0.5) \vee (360 - l) < 0.5$

From Tables 7 and 8 we have the following equations:

$$1 [ly] = 9.461E + 12 [km] \quad (58)$$

$$SB = 2.61944E + 17 [km] = 27,687 [ly] \quad (59)$$

$$SA = 1.560923E + 18 [km] = 164,985 [ly] \quad (60)$$

$$BA = (SA + SB) = 1.82287E + 18 [km] = 192,672 [ly] \quad (61)$$

If we assume that $GA = GB$ then it follows that:

$$SG = BA * 0.5 - SB = (SA + SB) * 0.5 - SA = (SB - SA) * 0.5 \quad (62)$$

$$SG = (1.560923E + 18 - 2.61944E + 17) * 0.5 [km] = 6.49490E + 17 = 68,649 [ly] \quad (63)$$

The galaxy does not have a perfectly symmetrical shape, so this result result should be taken with caution.

Tables 9 and 10 were defined in a similar way, taking into account those objects so that $(90 - b < 1)$ and $(b + 90 < 1)$, respectively.

#	Source_id	Gal.lon(2016)	Gal.lat(2016)	PM[mas]	Prx[mas]	c_v	Dist[km]	Dist(Prx)[km]	v_t
1	3961499231910404608	180.235046103830	28.928911422356	17.319605	0.604518241	4.82E-04	1.49221E+19	2.55218E+16	39,704
2	3961644607969332608	180.404725877696	29.854718309393	278.43823	9.61882474	1.37E-04	3.34243E+18	1.60398E+15	142,975
3	3961751157512399104	180.019001468704	30.490758148786	13.677024	0.061422163	9.42E-04	1.90751E+18	2.51186E+17	4008
4	3961624438797750656	180.582079994910	29.632830085481	52.60498	3.168342637	1.38E-05	1.0051E+18	4.86955E+15	8122
5	3961741399346682752	180.360953536279	30.589299839668	5.618753	0.840045033	7.84E-04	7.03273E+17	1.83661E+16	607

Table 9: The five farthest stars toward the north galactic pole $(90 - b < 1)$

#	Source_id	Gal.lon(2016)	Gal.lat(2016)	PM[mas]	Prx[mas]	c_v	Dist[km]	Dist(Prx)[km]	v_t
1	2342975612828207616	0.561739748958	-29.020121757189	15.172308	0.504548759	1.57E-04	2.24734E+20	2.55218E+16	523,832
2	2342630022579538176	0.541936108764	-30.264239493186	86.033516	1.537226261	8.10E-04	1.07195E+19	1.00365E+16	141,681
3	2342564876515950720	359.42887157572	-30.047353984642	15.577604	0.625130738	9.42E-04	2.92889E+18	2.46803E+16	7,009
4	2342704617571982208	0.298403519461	-29.704444217656	75.32534	1.13608906	3.25E-04	1.9785E+18	1.35803E+16	22,895
5	2342604080976989824	0.361966189375	-30.401246017438	9.464713	0.996803191	9.37E-04	1.24758E+18	1.54779E+16	1,814
6	2342906210451685760	0.528476982416	-29.454414577265	7.2573557	-0.128968379	6.99E-04	1.07286E+18	-	1,196

Table 10: The five farthest stars toward the north galactic pole $(b + 90 < 1)$

Obviously, for objects #2 in Table 9 and #1 and #2 in Table 10, the distances are extremely large, which is one of the reasons why the transverse velocities are extremely high. We can consider two possibilities. First, if they were Galactic objects then it is most likely a measurement error. If they were extragalactic objects then again there are two possibilities. Firstly, it could be a random measurement error. Another possibility is that due to the distortion of the space around the Galaxy, the line connecting the object and the observer is not a straight line but a curve. In this case the proposed algorithm cannot be applied.

7. Star constellations distance

For some stars from a stellar constellation it is not possible to determine the distance. Therefore, it is necessary to find those stars for which it is possible to find the distance. In this way, we can indirectly determine the distance of other stars if we are able to prove that these stars move in the orbit of some of the stars whose distance has already been determined. Table 11 shows one such simple example. This is just an example and it has not been proved that stars marked with #2, #3 and #4 move in the orbit of the star marked with #1.

#	Source_id	Ecl.lon(2015)	Ecl.lat(2015)	c_v	Dist[km]
1	4057492678697287680	266.824804064947	-5.4591431218883	4.96E-03	9.32042E+15
2	4057492678690136832	266.826252097473	-5.4494776145305	-1.27E-01	-
3	4057492678690657664	266.833184350181	-5.4564239180936	-8.65E-02	-
4	4057492678690661888	266.827298363692	-5.4488755851829	4.73E-01	-

Table 11: Star constellations distance is determined by the distance of the star marked with the #1

Things get much more complicated because generally each constellation is made of several hundred (thousand) stars. But the principle should remain the same. It is necessary to find those stars for which it is possible to determine the distance and then the others that move in their orbits. The star from the first group could be called a "mother star" while the star from the second group could be called a "daughter star".

We can say that it is easy to determine whether a star belongs to the mother-star group, but the question is

how to find its mother-star for a daughter-star. We will not deal with this problem in this paper, because it requires much more observation.

References

- [1] Čojanović M.
(2019) Stellar Distance and Velocity
Journal of Applied Mathematics and Physics , 7,181-209.<https://doi.org/10.4236/jamp.2019.71016>
- [2] (2018) Gaia Archive
<https://gea.esac.esa.int/archive/>
- [3] https://en.wikipedia.org/wiki/Milky_Way
- [4] Čojanović M. *(2020) Derivation of general Doppler effect equations*
Journal of Applied Mathematics and Physics, 18, 150–157. <https://doi.org/10.24297/jap.v18i.8913>