NEUTRINO MASS

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cvavbc@gmail.com Neutrino interactions, particle properties

The neutrino mass eigen states are assumed to obtain their effective mass through their interaction with the Higgs field like their charged partners and the coupling constants are found as functions of the gauge constants of the electro-weak model and the electron, muon and Tau neutrino masses are estimated to be respectively 2.12098 eV, 2.1254 eV and 12.825 MeV.

The absolute value of the electron-neutrino mass comes from the Tritium beta decay [1] and appears to be about 2 eV. The muon neutrino also has mass. In the Standard Model [2] each lepton starts out with no intrinsic mass. The charged leptons obtain an effective mass through their interaction with the Higgs field. The neutrino is automatically massless because it is left-handed in the Standard model. Right-handed neutrinos have no interaction with other particles and so are not a functional part of the Standard Model. But neutrino oscillations confirm that Neutrinos have non-zero mass.

Does the neutrino obtain its mass through interaction with the same Higgs field like all other particles?

For this to take place we should start with a mass eigen state for the neutrino.

Let there be a mass eigenstate for the electron neutrino and its effective mass be generated through the Lagrangian,

$$L = -\mathrm{h}\,\bar{\nu}\nu\phi - h\bar{e}\mathrm{e}\phi - i\,a_1\bar{e}\;\gamma_5 e\phi\,, \qquad (1)$$

where, the Higgs field is ϕ with the VEV V_0 ,and h in Eq. (1), is very small and we are not assuming that the neutrino is left-handed. It is a full mass eigenstate. The electron is coupled to the Higgs field through a γ_5 coupling also. We recover the standard model scenario when h is set zero and i γ_5 is replaced by one. Given a Dirac field say,ψ, the Hermitian Scalar $\bar{\psi}\psi$ and i$\bar{\psi}\gamma_5\psi$ have opposite CP and T transformation properties (In this respect they are unlike the vector and axial vector.) The CP violation is now caused by the exchange of ϕ' fields. Since the coupling of Higgs Field is usually rather small, it is possible to arrange for the CP violation to be of roughly milliweak magnitude [Mohapatra 1980, Bilenkey, 3&4].

After spontaneous symmetry breaking from (1) we note that,

$$L = -hV_0\bar{\nu}\nu - \mathrm{h}\bar{\nu}\nu\phi' - hV_0\bar{e}e - h\bar{e}e\phi' - ia_1\bar{e}\gamma_5 eV_0 - ia_1\bar{e}\gamma_5 e\phi'\,.$$

And with, m $= hV_0$, which is now the electron-neutrino mass,

$$L = -m\bar{\nu}\nu - \mathrm{h}\bar{\nu}\nu\phi' - m\bar{e}e - h\bar{e}e\phi' - ia_1\bar{e}\gamma_5 eV_0 - ia_1\bar{e}\gamma_5 e\phi' \qquad (2)$$

In the above m, and also h are presumably very small because m is neutrino mass.

Let, $e = \exp(-\frac{1}{2} i\alpha_1\gamma_5)e'$, (3)

where α_1 is a real parameter. Vector and axial vector interactions are not affected by this transformation. We choose α_1 in such a way that the constant coefficient of $\bar{e}'\gamma_5 e'$ is zero. This gives,

$$-[mcos\alpha_1 + V_0 a_1 sin\alpha_1]\bar{e}'e' - [-imsin\alpha_1 + i\, a_1 V_0 cos\alpha_1]\bar{e}'\gamma_5 e'$$
$$-a_1\bar{e}'[sin\alpha_1 + i\gamma_5 cos\alpha_1]e'\phi', \quad (4)$$

and set the coefficient of the second term equal to zero to yield,

$$\tan\alpha_1 = \frac{a_1 V_0}{m}. \quad (5)$$

The mass of the electron is now given by,

$$m_e^2 = m^2 sec^2\alpha_1 = m^2\left[1 + \frac{V_0^2 a_1^2}{m^2}\right] = mV_0\left[\frac{V_0 a_1^2}{m} + \frac{m}{V_0}\right]. \quad (6)$$

The very last term within the bracket is independent of the VEV. It is the sum of $\left[\frac{a_1^2}{h} + h\right]$,which are the interaction constants of the Higgs field with the electron and its neutrino, [Eq.(2)]. Let,

$$q_0 = \left[\frac{a_1^2}{h} + h\right]. \quad (7)$$

A similar Lagrangian can be chosen to obtain the effective mass for the muon and its neutrino through their interaction with the same Higgs field

$$L = -h_1 \bar{\nu}_\mu \nu_\mu \phi - h_1 \bar{\mu}\mu\phi - i a_2 \bar{\mu}\gamma_5 \mu\phi \quad . \tag{8}$$

In Eq. (8), $h_1 and\ a_2$ are real positive numbers and after symmetry breaking, the muon neutrino obtains the following mass:

$$m_1 = h_1 V_0. \tag{9}$$

Following the same steps as in Eq. (2, 3, 4, and 5) for the Lagrangian (8) we readily observe that,

$$m_\mu^2 = m_1^2 sec^2\alpha_2 = m_1^2\left[1 + \frac{V_0^2 a_2^2}{m_1^2}\right] = m_1 V_0 \left[\frac{V_0 a_2^2}{m_1} + \frac{m_1}{V_0}\right]. \tag{10}$$

Again, we note that in Eq. (10) the parameter at the end in the brackets is independent of the VEV and is equal to $\left[\frac{a_2^2}{h_1} + h_1\right]$. Let,

$$q_1 = \left[\frac{a_2^2}{h_1} + h_1\right]. \tag{11}$$

After symmetry breaking, the muon neutrino obtains the mass m_1,

$$m_1 = h_1 V_0 \quad . \tag{12}$$

There is another massive charged lepton, the τ lepton. Its mass is,

$$m_\tau = 1.777\ GeV \quad . \tag{13}$$

This lepton also has a neutrino. Like the other leptons, they start out with no intrinsic mass and obtain effective mass with their interaction to the same Higgs field with the VEV = V_0. The Lagrangian in this case is

$$L = -h_2 \overline{\upsilon_\tau} \upsilon_\tau \phi - h_2 \bar{\tau} \tau \phi - i a_3 \bar{\tau} \gamma_5 \tau \phi \,. \tag{14}$$

Following by now the familiar steps, after spontaneous symmetry breaking, we have, [V_0 =246.22 GeV],

$$m_{\upsilon\tau} = m_2 = h_2 V_0 \quad , \tag{15}$$

where m_2 is the mass of the τ -neutrino and the mass of the charged τ lepton is now given by,

$$m_\tau^2 = m_2^2 sec^2 \alpha_3 = m_2^2 \left[1 + \frac{V_0^2 a_3^2}{m_2^2}\right] = m_2 V_0 \left[\frac{V_0 a_3^2}{m_2} + \frac{m_2}{V_0}\right] . \tag{16}$$

Again, the very last factor in the brackets is independent of the VEV.

And it is given by, q where,

$$q_2 = \left[\frac{a_3^2}{h_2} + h_2\right] \quad . \tag{17}$$

Even though we do not know what is the value of q, it is a real positive number. The masses of W and Z bosons are given in terms of the gauge constants g_L and g' of $SU(2)_L$ X$U(1)$ through Higgs Mechanism. The mass generating interaction constants of the charged leptons with the Higgs boson are put by hand in the Standard model as there is no theory which will require a particular choice for these constants. In Eq. (2) the electron is coupled to the ϕ [$\bar{e}e\phi$ and $\bar{e}\gamma_5 e\phi$] in two ways, and this sort of coupling is also there with the standard Z-boson. Using this clue, we tried to relate the masses of the electron and muon to the interaction constants g_V and g_A of the Z boson in Ref. [5].

First let us note how a simple choice of q_2 enables the prediction of Tau -neutrino mass.

If q_2= 1, then, $m_2 = 12.825\ MeV$, (18)

If q_2= 0.5 then $m_2 = 25.65\ MeV$, and (19)

If q_2= 21.33239× 10^3then, $m_2 = 601.19\ eV$, (20)

The choice of these values will be explained shortly. But it is clear that Neutrinos have mass. In the above, three possible values for the mass of the tau -neutrino are given. If h = h_1 = h_2 then all the three neutrinos will have the same mass. The electron neutrino mass can be computed from the following in which q_0 is chosen as a function of gauge constants [5].

$$m_e^2 = mV_0 q_0 = mV_0 \frac{(g_V/g_A)^4_{\nu e}}{(g_V/g_A)^4_{e\mu}} \left[1 - \left\{ 1 - \left({}^{g_V}/_{g_A} \right)^4_{e\mu} \right\}^{1/2} \right], \quad (21)$$

The factor $q_0 = \left(\frac{a^2}{h} + h\right)$ is given by the expression that is after mV_0 in Eq. (21), where g_V and g_A are the vector and axial vector coupling constants of the particles indicated by the subscripts with the Z-boson of the Standard model. The ratio g_V/g_A = 1 for all the neutrinos and it is introduced for future generalization. On the other hand,

$$(g_V/g_A)^2_{e\mu} = [-1 + 4sin^2\vartheta_W]^2 \quad . \tag{22}$$

Where $sin^2\vartheta_w$ is the Weinberg mixing parameter. Eq.(21) is approximately $\frac{1}{2}mV_0$ irrespective of the value of the mixing parameter. The mass of the electron neutrino is therefore,

$m \approx 2m_e^2/V_0$ = 2.12eV. We took 0.5 for q_2 in Eq. (19) assuming a possibility that $q_2 \approx q_0$.

Exact mass of the electron neutrino is, (with $sin^2\vartheta_W = 0.2254$),

$$\mathrm{m} = 2.12098\ eV \ . \tag{23}$$

The above value appears correct. In Eq. (10) also the very last factor involving the constants $\left[\frac{a_2^2}{h_1} + h_1\right]$ must as well be a function of gauge constants and from Ref. [5], it can be noted that,

$$m_\mu^2 = m_1 V_0 q_1 = m_1 V_0 \frac{(g_V/g_A)^4_{\nu\mu}}{(g_V/g_A)^4_{e\mu}} \left[1 + \left\{1 - \left({}^{g_V}/{}_{g_A}\right)^4_{e\mu}\right\}^{1/2}\right] , \tag{24}$$

From the above the muon neutrino mass m_1 is given by,

$$m_1 = 2.1254 eV \ . \tag{25}$$

While computing the mass of the Tau-neutrino, we used the possibility in Eq. (20), that $q_2 \approx q_1 = 21.33239 \times 10^3$, which is $q_1$is from Eq. (24)

In place of $(g_V/g_A)^2_{e\mu}$ we take $(g_V/g_A)^2_{\tau}$ in both Eqs. (21) & (24) and find that [6],

$$m_\tau^2 = m_2 V_0 \frac{(g_V/g_A)^4_{\nu\tau}}{(g_V/g_A)^4_{\tau}}\left[1 \pm \left\{1 - \left({}^{g_V}/_{g_A}\right)^4_\tau\right\}^{1/2}\right] = m_2 V_0\, q_2 \ . \qquad (26)$$

If, $(g_V/g_A)^2_\tau = (-1 + 4sin^2\vartheta)^2_\tau = 1,$ (27)

the expression after m_2V_0 in the middle is just one and so $q_1 = 1$, that means that the mixing parameter for the electroweak model of the tau lepton is 0.5. It is SU(2)LXU(1) gauge model with a different mixing parameter. The e−μ−τ universality is no longer valid. Thus, the experimental determination of Tau-neutrino mass is very crucial [7, 8, 9] and of course we are sure that, $q_2 = 1$, and

$$m_2 = 12.825\ MeV\ . \qquad (28)$$

Experiments suggest that the theoretical estimations here of all the neutrino masses are right. The assumption that the neutrino mass eigen state exists and obtains its mass through its interaction with the same Higgs field through which the corresponding charged lepton obtains its mass is a departure from the Standard Model. But

$$m_e^2 = V_0^2 h\left[\frac{a_1^2}{h} + h\right] = V_0^2[a_1^2 + h^2]\ . \qquad (29)$$

In which we shifted the neutrino mass constant. Suppose h^2 is very small compared to a_1^2, then we can assume that the neutrino has no interaction at all and our neutrino is massless and left-handed, but the electron mass is given by its interaction with the Higgs field. In a similar way the muon mass can be arranged and the muon neutrino has no mass apparently and it is left-handed like the electron neutrino.

This is what is happening in the electroweak model. Once we know the mass m_i of the neutrino from experiment then,

$$\frac{M_i^2}{V_0 m_i} = q_i \quad , \tag{30}$$

where M_i is the mass of the corresponding charged lepton and V_0 is the VEV which is known. From m_i the parameter h can be found and then the corresponding a_i can as well be calculated and it fixes the CP transformation.

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