On the Orbital Frequency of Coalescing Stars

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Abstract

Simple estimates, utilizing Newton's theory of gravitation, for the limiting orbital frequencies of merging high-density stars reveal, that a interpretation in terms of constant average density of the stars yields much more satisfactory agreement with experimental results than does a interpretation in terms of black holes.

Estimates

In an earlier note we presented a simple estimate for the limiting frequency, ν, of two (equal mass) coalescing high density stars [1]:

$$\nu^2 = \frac{G \rho \alpha^3}{12 \pi} \quad , (\alpha \to 1), \tag{1}$$

where G is the gravitational constant, ρ the mass-density of the stars (assumed constant), and α the ratio between the radius of the stars and their distance from the center of mass. At the moment of coalescence α approaches a value of one from below. At that moment this expression only depends on the stars density ρ. For neutron stars with a density of the order of atomic nuclei the resulting frequency lies in the range of a couple of hundred Hertz.

Common wisdom, derived from the General Theory of Relativity, states that such stars cannot exist whenever their radius falls below the Schwarzschild radius, r_s,, associated with their mass M

$$r_s = \frac{2 G M}{c^2} \quad , \tag{2}$$

where c is the speed of light. This would limit the mass range of neutron stars to below a few solar masses. Higher-mass objects are supposed to be black holes with a radius equals to r_s.

With an effective density of black holes, ρ_{BH} [1] of

$$\rho_{BH} = \frac{3}{8\pi} \frac{c^2}{G r_s^2} \quad , \tag{3}$$

the estimate (1) for a merger (α = 1) of two black holes would read

$$\nu=\frac{c}{4\sqrt{2}\pi r_s}=\frac{c^3}{8\pi\sqrt{2}G\,M}=\frac{M_s}{M}5712\,Hz \quad , \tag{4}$$

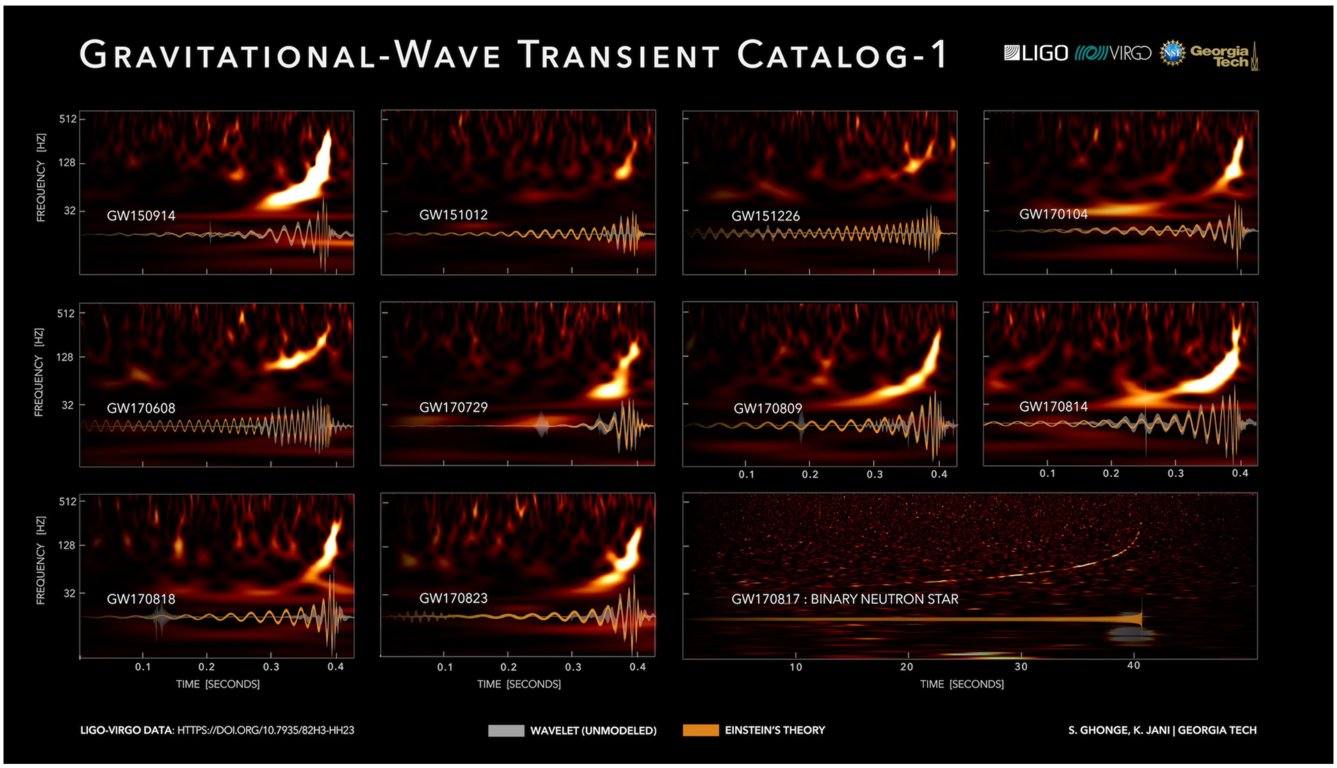
where M_s is the solar mass. One can estimate the speed of the masses in the center of mass system in Newtons approximation as

$$v_{limit}=2\pi\,\nu\,r_s=\frac{c}{2\sqrt{2}}=0.354\,c \quad , \tag{5}$$

which tells us that relativistic corrections of order $(v/c)^2$ would amount to just some 13%.

Experimental Situation

We reproduce here a compilation of transients gravitational waves by Ghonge and Jani from the LIGO/VIRGO collaboration [2].



The displayed frequency analyses make evident that the limiting frequency at the merger is almost constant around 300 to 400 Hz. However, the (geometrical) average masses of the participant stars vary between 1.4 (GW170817) and 41.6 (GW170729). Quite obviously, the findings shown in the picture strongly favor an interpretation in terms of fixed-density stars (roughly nuclear density) according to expression (1) over one in terms of black holes for which one would expect a variation of a factor of four from expression (4). This factor of four excludes GW170817, rated as a neutron-star merger, which, however, shows the same limiting frequency!

References

1. Gerber P. R., (2017), viXra:1710.0252
2. Ghonge S. and Jani K., LIGO/VIRGO collaboration