Extending the weak equivalence principle beyond massive bodies

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Abstract

The validity of the weak equivalence principle (WEP) for massive bodies has been empirically confirmed with extremely high precision. However, can this principle be applied beyond massive bodies? The WEP states that every massive body falls at the same rate in a gravitational field. Does a massless body fall at the same rate within the same field? Theoretically, a body can be massive, negative massive, or massless. We aim to extend the WEP beyond massive bodies. To accomplish this goal, for negative massive bodies, we experimentally derive that the WEP applies to them in the same way as massive bodies do. For massless bodies, we conclude that only a type of massless body–with zero net mass–has a role in the WEP. On the basis of this condition, we conclude that the WEP applies to massless bodies. Therefore, we extend the WEP to massless bodies and negative massive bodies satisfactorily.

Keywords: Equivalence principle; Weak equivalence principle; Extended weak equivalence principle; Massless body; Massless configuration; Negative mass

1. Introduction

The weak equivalence principle (WEP) emerged in the 17th century, when Galileo [1] experimentally derived that the acceleration of a massive body due to gravitation is independent of the amount of mass being accelerated.

In the 20th century, Einstein [2] used the following mathematical formulation to describe the WEP:

$$acceleration = \frac{gravitational\ mass}{inertial\ mass}(intensity\ of\ the\ gravitational\ field). \quad (1)$$

If the acceleration is always the same for a given gravitational field regardless of the nature and condition of a massive body, then the ratio of the gravitational mass to the inertial mass must likewise be the same for all massive bodies, usually one by a suitable choice of units. This principle is typically described as the gravitational mass of a body being equal to its inertial mass or the ratio of the gravitational mass of a body to its inertial mass being one. Experimentally, this principle was confirmed with extremely high accuracy by Eötvös [3] for the first time. Later experiments reconfirmed this finding or improved the precision further [4-11]. However, the principle states that it is only applicable to massive bodies.

Few studies have investigated the possibility of extending the WEP to massless bodies. General relativity (GR) has been used to study the motion of one type of massless body–photon [12-14]. However, photons always travel at the speed of light and possess kinetic energy. In theory, can the WEP be applied to a massless body? This principle indicates that every massive body falls at the same rate in a gravitational field regardless of the amount of mass being accelerated. Intuitively, if the amount approaches zero, the massive body becomes massless. Hence, the principle should also apply to a massless body.

We aim to extend the WEP to massless bodies. Nevertheless, what is the meaning of a massless body in the WEP? In classical physics, if a body is truly empty and has nothing, then there is no meaning in its role in the WEP. In this study, we consider a massless body as a body with zero net mass. In other words, massless bodies can have zero net mass, which makes it possible for them to consist of parts. However, if the net mass of all parts is zero, then the mass of some parts must be negative.

The concept of negative mass is relatively new. It is abstract and has a sign opposite to that of a normal mass. In his 1928 hypothesis of electrons and positrons, Dirac [15] speculated on the existence of negative energy in a vacuum. Later, modern investigations of negative mass began in the 1950s. Ferrell [16], within the framework of Newtonian mechanics, discussed a possible way to shield the gravitational effect by applying a negative mass. The idea behind this approach is that a negative mass could substitute for the mass in Newton's gravitational law and second law. He studied the interaction between mass and negative mass. One result was that mass could attract negative mass in the same way as mass. Bondi [17], within the framework of GR, studied uniform acceleration in a two-body system consisting of a body of normal mass and a body of negative mass separated by an empty space. He concluded that uniform acceleration did not violate GR. To date, there is no conclusive physical evidence that a negative mass exists in a vacuum. However, this has not hindered people's interest in studying it [18-25]. Recently, other possible applications of negative mass have been proposed in areas such as cosmology [26-27]. A conclusion from this research is that the existence of a negative mass does not violate the laws of Newtonian mechanics or GR. In our study, we adopt the concept of negative mass to construct massless bodies.

The equality of the gravitational mass and the inertial mass for a negative massive body was assumed in the studies above. That is, the WEP was assumed to be applicable to negative massive bodies. Nevertheless, in the following sections, we find that the WEP applies to negative massive bodies in the same manner as massive bodies do. We further find that the WEP applies to massless bodies that consist of masses and negative masses. Finally, we revisit Einstein's thought experiment regarding the indistinguishable behaviors of massive bodies within an accelerating frame and in a gravitational field. Instead of massive bodies, we elaborate the effects of massless bodies and negative massive bodies.

2. Methodology

One assumption in previous studies of negative mass was the equality of the negative gravitational mass and the negative inertial mass, even if their signs were negative. Plugging both into Eq. (1) results in the conclusion that negative massive bodies fall at the same rate as massive bodies in a gravitational field.

In this study, we use decomposition and composition methods to extend the WEP to negative massive and massless bodies. In addition to recognizing the assumption above, from experiments, we deduce that negative massive bodies fall at the same rate as massive bodies within a gravitational field. Next, from first principles, we find that massless bodies fall at the same rate as massive bodies within a gravitational field.

2.1. Decomposing a Massive Body

Let us analyze a two-body system. Assume that a massive system S consists of two equal massive bodies (M). The two bodies rotate around their common barycenter, as shown in Fig. 1. The distance from the barycenter–marked as a dot in the center–to each body is r.

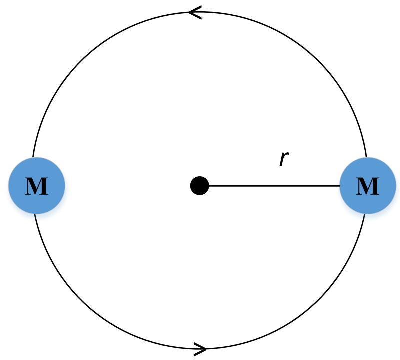

Fig. 1 A system consisting of two massive bodies (***M***). The two bodies rotate around their common barycenter (the dot in the center), and they are always on opposite sides of the common orbit. The distance from the barycenter to each body is ***r***. The total mass of the system is less than the sum of the two constituent masses because of the negative total energy inside the system

Fig. 1 shows a configuration in which two equal massive bodies (M) move around their common barycenter in a circular orbit. During revolution, they are always on opposite sides of the orbit. The mass of the system S is not the same as the sum of the masses of its constituents. In addition to the two masses, there are internal energies inside$S$.

The total internal energy of S includes the gravitational potential energy between the two bodies and the kinetic energy each body possesses.

$$E_t = 2T + U, \tag{2}$$

where E_t is the total energy of S, T is the kinetic energy of each body, and U is the gravitational potential energy between the two bodies. The potential energy is obtained as follows:

$$U = -G\frac{MM}{2r}, \tag{3}$$

in classical physics, it is known that for this configuration, the total kinetic energy is half of the potential energy but with the opposite sign:

$$2T = -\frac{U}{2} = G\frac{MM}{4r}, \tag{4}$$

therefore, the total energy E_t is:

$$E_t = -G\frac{MM}{4r}. \tag{5}$$

According to the mass-energy equivalence principle, E_t is equivalent to a mass $\Delta M = \frac{E_t}{c^2}$. Since E_t is negative, ΔM is also negative. The following equation represents the total mass of system S:

$$M_s = 2M + \Delta M, \tag{6}$$

where M_s is the mass of system S and where ΔM is the equivalent negative mass caused by the negative total energy of S.

We can interpret Eq. (6) as S consisting of three constituents: two masses (M) and a negative mass (ΔM) corresponding to the negative energy inside S. As a whole, S is a massive body, as is M. According to the WEP, S falls at the same rate as its two constituents (M) in a gravitational field.

If the other constituent, negative mass (ΔM), does not fall at the same rate as Ms in the same field, then ΔM would separate from both Ms over time. This means that S would become unstable over time in a gravitational field during its fall. However, that conjecture contradicts the experiments performed to verify the WEP.

It is possible that ΔM and the other two Ms can be bundled so that S would not fall apart during its movement. However, if ΔM falls in a different rate than that of the other two Ms, then the combined rate would be different than that of individual M. Again, this conjecture contradicts the experiments performed to verify the WEP. Therefore, ΔM must fall at the same rate as a massive body within the same gravitational field, bundled or not.

We decompose a massive two-body system to demonstrate that a negative mass exists because of the negative total energy inside the system. This type of negative mass (ΔM) is not unique to two-body systems bound by gravity. A nucleus has a mass defect, which is the difference between the total mass of a nucleus and the sum of the masses of all its constituent nucleons. The nucleus can be interpreted as consisting of nucleons and a negative mass.

Based on the mass-energy equivalence principle, internal negative energy is a form of negative mass. Of course, this type of negative mass is internal inside a body. Nevertheless, from the point of view of the composition of the body, the body consists of various constituents, including negative masses. Hence, the negative mass can be considered independently in the context of the WEP. We can generalize it to any form of negative mass, internally or externally. Therefore, if a negative massive body is studied independently in a gravitational field, it would fall at the same rate as a massive body. We summarize the first conclusion of the extended WEP as follows:

> *Conclusion* 1: Negative massive bodies fall at the same rate as massive bodies do in a gravitational field.

The conclusion above indicates that the acceleration of a negative massive body solely depends on the gravitational field. From Eq. (1), the ratio of its (negative) gravitational mass to its (negative) inertial mass is one. That is, the equality of the gravitational mass and the inertial mass is true for a negative massive body. In other words, *Conclusion* 1 is consistent with the assumption made in previous studies regarding negative mass.

2.2. Composing a massless body

Next, let us explore the application of the WEP to massless bodies–with zero net mass. Theoretically, can such a massless body exist? In the following thought experiment, we construct a massless body S_0, which consists of two negative masses and a mass, as depicted in Fig. 2.

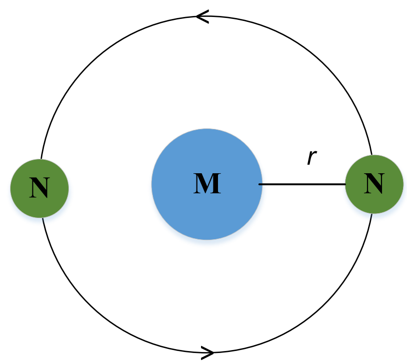

Fig. 2 A body consists of two negative masses (***N***) and a mass (***M***). The two negative masses rotate around the common barycenter of the body, which is located at the position of the mass. During their revolution, they are always on opposite sides of the orbit. With a particular distance *r* between a negative mass and the mass, the body is massless in its entirety

Fig. 2 shows a configuration in which a mass (M) is at the center, two negative masses (N) move around it in a circular orbit, and *r* is the distance between a negative mass and the mass. The two negative masses are always on opposite sides of the orbit. The barycenter of the entire body is located at the position of the mass. The repulsion exerted on one negative mass by the other is $F_{NN} = G\frac{-N(-\mathrm{N})}{4r^2}$, and the attraction exerted on each negative mass by the mass is $F_{MN} = G\frac{M(-N)}{r^2}$. If $M > \frac{N}{4}$, then $|F_{MN}| > |F_{NN}|$. The centripetal acceleration of each negative mass toward the mass keeps it in circular motion.

Now, let us determine whether there exists a distance *r* with which S_0 becomes massless in its entirety. In addition to the mass and negative masses, there is total energy E_t inside S_0. From the mass-energy equivalence principle, E_t can be converted into mass. The massless condition of S_0 requires its net mass to be zero:

$$M - 2N + \frac{E_t}{c^2} = 0. \tag{7}$$

Furthermore, the total energy E_t of S_0 includes the total gravitational potential energy among the mass and negative masses and the total kinetic energy of the negative masses. The total potential energy is as follows:

$$-2G\frac{M(-N)}{r} - G\frac{-N(-N)}{2r} = G\frac{4MN-NN}{2r}, \tag{8}$$

in classical physics, the total kinetic energy can be derived as half of the total potential energy but with the opposite sign. Therefore, the total energy is:

$$E_t = G\frac{4MN-NN}{2r} - \frac{1}{2}G\frac{4MN-NN}{2r} = G\frac{4MN-NN}{4r}, \tag{9}$$

plugging Eq. (9) into Eq. (7), we have

$$M - 2N + G\frac{4MN-NN}{4rc^2} = 0, \tag{10}$$

solving Eq. (10), we have

$$r = \frac{G}{4c^2}.\frac{4MN-N^2}{2N-M}, \tag{11}$$

if $2N > M > \frac{N}{4}$, then $r > 0$. We can conclude that with some values of M and N, there exists a distance *r* between each negative mass and the mass to make S_0 massless in its entirety.

From Eq. (7), we can interpret that S_0 consists of a mass (M), two negative masses ($-N$), and an equivalent mass of $\frac{E_t}{c^2}$ corresponding to the total energy inside S_0. $\frac{E_t}{c^2}$ is just a mass–according to the mass-energy equivalence principle–so that it falls at the same rate as the mass (M) in a gravitational field. *Conclusion* 1 indicates that the negative masses ($-N$) fall at the same rate as the mass (M) within the same field. Therefore, S_0 must fall at the same rate as all its constituents in the same field.

S_0 is not unique in the context of the WEP. We can generalize this reasoning to any massless body that consists of masses and negative masses. Therefore, we conclude that if a massless body consists of masses and negative masses, then the massless body falls at the same rate as a massive body in a gravitational field. We summarize the second conclusion of the extended WEP as follows:

> *Conclusion* 2: Massless bodies fall at the same rate as massive bodies do in a gravitational field.

3. Results

We have studied all possible types of bodies falling in a gravitational field. Using massive bodies as a base, we have derived that negative massive bodies fall in the same way within the same field. Considering that massless bodies consist of masses and negative masses, we have derived that massless bodies fall in the same manner within the same field. Table 1 summarizes the extended WEP for a body with different amounts of mass.

Table 1. The extended WEP for a body

Amount of mass	Massive	Massless	Negative massive
Fall at the same rate as a massive body in a gravitational field	Yes	Yes	Yes
Ratio of the gravitational mass to the inertial mass	1	1	1

Since a massless body has zero gravitational and inertial mass, the ratio of the gravitational mass to the inertial mass is undermined. *Conclusion* 2 indicates that the acceleration of a massless body solely depends on the gravitational field. From Eq. (1) we can define this ratio as one. Table 1 summarizes that the ratio of the gravitational mass to the inertial mass for a body is one, regardless of whether it is massive, massless, or negative massive. The extended WEP indicates that all the bodies fall at the same rate in a gravitational field.

4. Discussion

Following Einstein's original thought experiment, let us explore the application of the extended WEP in thought experiments conducted in a uniformly accelerating frame and in a uniform gravitational field.

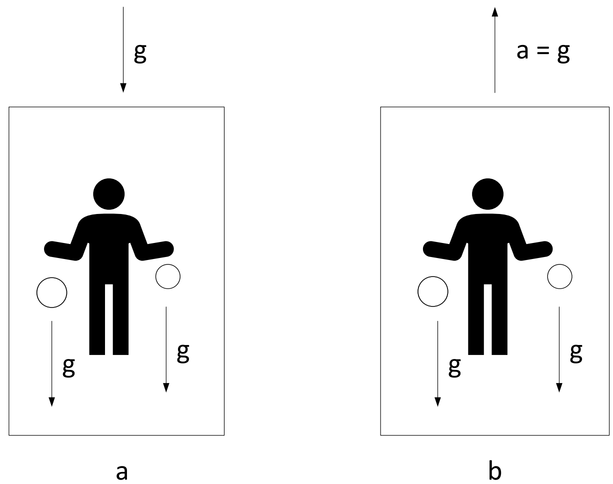

Fig. 3 An observer performing mechanical experiments by dropping massless objects within two frames. (a) An observer is in a closed case on Earth, with a downward acceleration of $\boldsymbol{g}$. (b) The same observer is in an identical closed case, far from any gravitational field, accelerating upward in a straight line at a rate of $\boldsymbol{a} = \boldsymbol{g}$. For the same observer, the trajectories of the massless objects are measured in the same way in both cases

Suppose that an observer stands in a closed case on Earth, as depicted in frame (a) of Fig. 3. The case is in a downward gravitational field of g. If the observer drops massless objects, according to the extended WEP, the objects fall toward the floor at an acceleration of g. Next, the same observer moves into an identical closed case, frame (b) of Fig. 3, far from any gravitational field. By means of a rocket engine, the case is accelerating upward in a straight line at a rate of $a = g$. If the observer drops the same massless objects, then owing to the upward acceleration of g for the case, the objects–have no attachment to the case–appear to fall toward the floor at an acceleration of g. Hence, the trajectories of the massless objects in both experiments appear to be the same to the observer. The observer cannot distinguish a uniform gravitational field from a uniformly accelerating frame by performing mechanical experiments involving massless objects.

A similar thought experiment can be performed in which an observer drops negative massive objects within the same two frames, as depicted in Fig. 4. The result is the same. An observer cannot distinguish a uniform gravitational field from a uniformly accelerating frame by performing mechanical experiments involving negative massive objects.

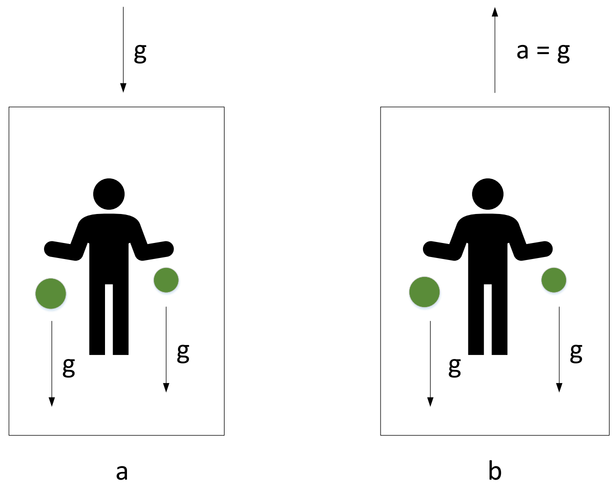

Fig. 4 An observer performs mechanical experiments by dropping negative massive objects within two frames. (a) An observer is in a closed case on Earth, with a downward acceleration of $\boldsymbol{g}$. (b) The same observer is in an identical closed case, far from any gravitational fields, accelerating upward in a straight line at a rate of $\boldsymbol{a} = \boldsymbol{g}$. To the same observer, the trajectories of the negative massive objects are measured the same in both cases

Both of these thought experiments, alongside Einstein's original thought experiment, imply that in small enough regions of spacetime, the motion of freely falling objects is the same in a gravitational field and in a uniformly accelerated frame. These objects can be massive, massless, or negative massive.

5. Conclusion

The WEP has played a key role in the formation of GR. However, many studies have focused only on how massive bodies obey the WEP. In this work, we have satisfactorily extended the WEP to massless bodies and negative massive bodies. Based on the extended WEP, we conclude that distinguishing a uniformly accelerated frame from a uniform gravitational field by performing mechanical experiments involving massive, massless, or negative massive bodies is impossible.

The WEP implies that once gravity is included, every observer of any reference frame has equal footing to study the motion of massive bodies. Furthermore, the extended WEP implies that for any type of body, regardless of whether it is massive or not, any observer can have equal footing to study its motion.

We feel satisfied that the extended WEP applies to all bodies and does not contradict GR as well. Naturally, we ask the following question: can the Einstein Equivalence Principle also be extended to all types of bodies? We would like to further investigate how massless bodies fit into the Einstein equivalence principle.

Declarations

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Data availability statement

The datasets used and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Author contribution statement

All the authors contributed to the study conception and design.

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