Stabilized quantum field theory

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An analysis of the action of elementary charges on the vacuum leads to a resolution of divergence issues in QFT without mass and charge renormalization. For an irreducible self-interaction amplitude Ω, infinite field actions split the vacuum into positive and negative self-energy components such that its net mass-energy remains zero for free particles. For each particle mass in a loop, two dressed mass states including vacuum energy, are constructed for fermion and boson self-energy processes. For electroweak interactions, the stabilized amplitude $\hat{\Omega} = \Omega - \overline{\Omega}$ includes a correction for a vacuum energy deficit within a point-like, near-field region, where $\overline{\Omega}$ is given by an average of Ω over dressed mass levels. For QCD, strong interactions redistribute vacuum energy so that there is an energy surplus in the near-field with a corresponding deficit in the confinement region resulting in a sign reversal of $\hat{\Omega}$ relative to QED and asymptotic freedom. Stabilized amplitudes agree with renormalization for radiative corrections in Abelian and non-Abelian gauge theories. Renormalization is only required in standard QFT because it neglects near-field vacuum energy changes in violation of energy conservation.

I. INTRODUCTION

A long-standing enigma in particle physics is how an elementary charged particle such as an electron can be stable in the presence of its own electromagnetic field [20, 26]. Critical accounting for system stability is essential since radiative corrections in quantum field theory (QFT) involve self-interactions that appear to change the mass and charge of a particle. This analysis identifies the underlying physics that stabilizes a particle such that its mass and charge retain their physically observed values in radiative processes.

The agreement between renormalization theory and experiment confirms the effect of vacuum fluctuations on the dynamics of elementary particles to astounding accuracy. For example, electron anomalous magnetic moment calculations currently agree with experiment to about 1 part in a trillion [1, 16]. This achievement is the result of seven decades of effort since the relativistically invariant form of the theory took shape in the works of Feynman, Schwinger, and Tomonaga; see Dyson's unified account [9]. The agreement leaves little doubt that QFT predictions are correct; however, the renormalization technique [2, 28] used to overcome divergence issues in radiative corrections offers little insight into the underlying physics behind charge stability in the high-energy regime. Recall that divergent integrals occur in scattering amplitudes for self-energy processes and arise in sums over intermediate states of arbitrarily high-energy virtual particles. This stymied progress until theoretical improvements were melded with renormalization to isolate physically significant parts of radiative corrections by absorbing infinities into the electron mass and charge.

Although the renormalization method used to eliminate ultraviolet divergences yields numerical predictions in remarkable agreement with experiments, infinite renormalization of fundamental physical constants remains a very undesirable feature of the current theory. The forgoing objections are captured in concerns of early investigators including developers of the theory [11, 29]; for example, Feynman referred to renormalization as an "awkward process", and Schwinger stated QFT was "incomplete". Largely due to elegant and persuasive renormalization group arguments [3, 12, 36], many workers in the field no longer believe that the divergences in QFT and the renormalization procedure used to overcome them are issues requiring further consideration. Nevertheless, we contend there is a flaw in the theory that led to renormalization; in a nutshell, evaluation of radiative corrections fails to account for important changes in vacuum energy that oppose self-interaction energies: this omission violates the law of conservation of energy, and that is why the theory is "incomplete" and needs the simple correction proposed in this paper.

Our main purpose is to develop a mathematically and physically complete alternative to renormalization in QFT. A minimal requirement for this proposal is that it reproduce the successes of the accepted theory: These include the successful higher-order multiloop calculations of quantum electrodynamics (QED), the modern understanding of QED as a part of non-Abelian electroweak theory [13, 27, 33], and asymptotic freedom predictions [15, 24] in quantum chromodynamics (QCD). Starting with the classical self-energy problem in Section II, we define an energetically stable system (charge plus vacuum) and develop a stabilized amplitude applicable to all radiative processes and particles of the Standard Model. Our primary results for electroweak and QCD scattering amplitudes are presented in Sections II C and II D: Scattering matrix corrections for stability are simply constructed using unrenormalized (core) amplitudes from the literature, involve two additional Feynman diagrams associated with dressed mass states, and account for all changes in vacuum energy. General arguments are given

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to demonstrate that net S-matrix corrections in QFT for vacuum polarization, fermion self-energy, and vertex processes are finite and agree with renormalization theory.

II. FORMULATION

A. Physical model

Regarding an electron as a point particle [21], the classical electrostatic self-energy $e^2/2a \equiv \alpha\Lambda_\circ$ diverges linearly as the shell radius $a \to 0$, or energy cutoff $\Lambda_\circ \to \infty$, where $-e$ is the charge and $\alpha = e^2/4\pi$ is the fine-structure constant. However, Weisskopf [34, 35] showed using Dirac's theory [7] that the charge is effectively dispersed over a region comparable in size to the Compton wavelength of the electron, $\lambda_c(m) = 1/m$, due to pair creation in the vacuum, and the self-energy only diverges logarithmically. Feynman's calculation [10] in covariant QED yields an electromagnetic mass

$$m_{em}^+ = \frac{3\alpha m}{2\pi}\left(\ln\frac{\Lambda_\circ}{m} + \frac{1}{4}\right), \tag{1}$$

where $m = g_e v/\sqrt{2}$ is the observed mechanical mass generated via interaction between the fermion and Higgs fields in electroweak theory, g_e is a coupling constant, and v is the ground state vacuum energy. In the absence of a compensating negative energy, (1) signals an energetically unstable electron. This is the fermion self-energy (FSE) problem, whose general resolution will suggest a solution for boson self-energy (BSE) processes as well resulting in finite amplitudes for all radiative corrections. In this section we derive a vacuum stability condition and a complete set of mass states for an electron.

To ensure that the total electron mass is its observed value, renormalization theory posits that a negatively infinite "bare" mass must exist to counterbalance m_{em}^+. For lack of physical evidence, negative matter is naturally met with some skepticism; see Dirac's discussion [8] of the classical problem, for example. Nevertheless, energies that stabilize a charge must be negative to conserve energy, and we can understand their origin by first considering the source for the electrical energy required to assemble a classical charge from infinitesimal parts in the rest frame. Since the agents that do the work must draw energy $\mathcal{E}_{em}^+$ from an external energy source (well), the well's energy is depleted and the total energy

$$\mathcal{E} = m + \mathcal{E}_{em}^+ + \mathcal{E}_w \tag{2}$$

of the system including matter, electromagnetic field $\mathcal{E}_{em}^+$, and energy well $\mathcal{E}_w$ is constant.

Assume that the depleted energy well is part of the vacuum, elementary charges are inherently stable, and consider an electron and its neighboring vacuum as two distinct systems that can act on one another. In particular, suppose that an electron redistributes vacuum energy into positive and negative energy parts such that (2) is satisfied with

$$\mathcal{E}_w \to \mathcal{E}_{em}^- = -\mathcal{E}_{em}^+ \tag{3}$$

for a free particle; therefore, we have a stability condition

$$m_{em}^+ + m_{em}^- = 0\,, \tag{4}$$

where $\mathcal{E}_{em}^\pm = m_{em}^\pm$ in natural units. Since the positive energy $\mathcal{E}_{em}^+$ in the surrounding region corresponds to the observed electric field, the deficit (3) must reside infinitesimally close to the electron: within a near-field region of radius $r \lesssim \lambda_c(m_{em}^+)$, as will become evident. Thus, an electrically charged particle effectively acts as a sink for negative energy, and the deficit must be taken into account to uphold energy conservation.

In addition to the core mass m, (4) suggests that a stable electron includes two electromagnetic masses $m_{em}^\pm$ that are assumed large in magnitude, but finite, until the final step of the development. We can think of $m_{em}^\pm$ as components of an electromagnetic vacuum (zero net energy) which are tightly bound to the core mass and inseparable from the core and each other, at least for finite field actions. From (2), either mass can be associated with m. Considering all non-vanishing masses constructed from the set $\{m,\, m_{em}^+,\, m_{em}^-\}$, we define a complete set of mass levels $m + \lambda M$, where $\lambda = \{0, \pm 1\}$ and $M \equiv |m_{em}^\pm|$. For $\lambda = \pm 1$, an electromagnetically dressed core mass (DCM) refers to a composite particle with mass levels

$$m_d = m + \lambda M\,. \tag{5}$$

For a particle of four-momentum p, the dressed momentum is $p_d = p + \lambda M$.

Dressed mass states are important for radiative corrections because they provide additional degrees of freedom needed to compute near-field corrections to scattering amplitudes which stabilize the system. Introduction of a bare mass or charge in renormalization theory does not account for all possible mass states in radiative processes; consequently, the underlying physics is concealed, and the theory is rendered more complicated: For example, introducing a bare mass results in an asymmetry which requires wave field renormalization in the electron self-energy problem.

Consider a free particle state $|p,\, m\rangle$ satisfying $p^2 = m^2$. Spin is omitted in $|p,\, m\rangle$ since it is inessential to the subsequent development, and the rest mass is included because it is the fundamental particle characteristic which becomes dressed with vacuum energy in stability corrections to the S-matrix. For radiative corrections, assume that dressed states

$$|p + \lambda M\,,\, m + \lambda M\rangle \tag{6}$$

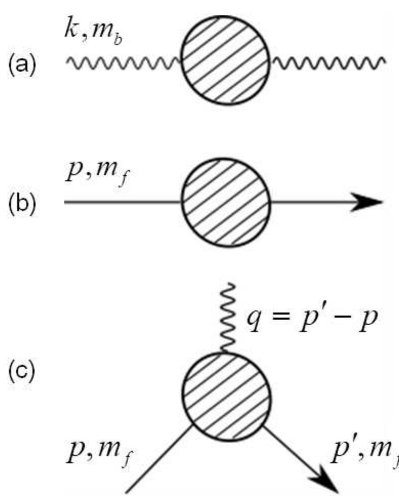

Figure 1. Generic self-energy and vertex diagrams: (a) BSE, (b) FSE, and (c) vertex.

may be created with equal probability in intermediate states with infinitesimally small lifetimes

$$\Delta t \simeq \frac{\hbar}{(m + \lambda M)\, c^2}$$

in accordance with Heisenberg's uncertainty principle [17]. Scattering amplitudes for low-energy processes are unaffected because the energies are insufficient to excite dressed states (6). In fact, we shall see that finite parts of radiative corrections vanish in the limit $\eta \equiv M/m \to \infty$.

B. General dressed mass states

This section generalizes DCM rules to all elementary particles and radiative processes of the electroweak Standard Model; in particular, we derive a rule for dressing particles in BSE processes with vacuum energy.

External lines for processes in Fig. 1 involve (a) gauge bosons $b \in \{\gamma, W, Z, H\}$, and (b,c) fermions f. Blobs in Fig. 1 contain irreducible insertions, which in general, may involve photons and other particles in the Standard Model mass set

$$\mathbb{M} = \{m_f, m_W, m_Z, m_H\} \ .$$

On the mass shell, scattering amplitudes $\Sigma^f(p)$ and $\Sigma^b(k^2)$ define fermion and boson self-energy functions [18, 30]

$$M_f = \Sigma^f(p)\big|_{\not p = m_f} \ , \text{ and} \tag{7}$$

$$M_b^2 = \mathrm{Re}\left[\Sigma^b(k^2)\right]_{k^2 = m_b^2} \ . \tag{8}$$

In the proposed model, M_f and M_b each represent energy borrowed from the vacuum within a near-field region of radius

$$r_\circ \simeq \lambda_c \left(M_f | M_b\right)$$

to create a configuration of high-energy virtual particles in the far-field: $r > r_\circ$. In order to have well defined amplitudes for FSE and BSE processes, negative probability (depletion) amplitudes that oppose (7) and (8) are required, thereby ensuring conservation of energy. Loosely, one may think of (8) as a squared energy borrowed from the vacuum and $-M_b^2$ as a deficit.

In general, DCM levels for fermions in FSE processes and massive bosons $b \in \{W, Z, H\}$ in BSE processes are defined by requiring that averages of free field Lagrangian densities over dressed mass levels for each particle class give the undressed value. Since Yukawa and Higgs densities, $\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}}(m_f)$ and $\mathcal{L}_{\mathrm{H}}(m_b^2)$, involve sums of terms linear in m_f and m_b^2, we have

$$\frac{1}{2}\sum_{\lambda=\pm1} \mathcal{L}_F^{Yukawa}(m_f + \lambda M_f) = \mathcal{L}_F^{Yukawa}(m_f) \ , \text{ and}$$

$$\frac{1}{2}\sum_{\lambda=\pm1} \mathcal{L}_H\left(m_b^2 + \lambda M_b^2\right) = \mathcal{L}_H\left(m_b^2\right) \ .$$

Defining a common scaling factor η such that $M_f \equiv \eta m_f$ and $M_b \equiv \eta m_b$, dressed masses are generated by

$$m_f \to m_f(1 + \lambda\eta) \ , \text{ and} \tag{9}$$

$$m_b^2 \to m_b^2\left(1 + \lambda\eta^2\right) \ . \tag{10}$$

External momenta, $p = m_f + \delta p_{os}$ and $k^2 = m_b^2 + \delta k_{os}^2$, in Fig. 1 become dressed in the blobs using (9) and (10), where δp_{os} and δk_{os}^2 are off-shell terms. For the purpose of defining general stability corrections for radiative processes, we can focus on the mass dependence of core amplitudes.

To ensure consistency when fermions and bosons mix in FSE and BSE processes, DCM levels for all $m \in \mathbb{M}$ in the blobs of Fig. 1 are defined by the replacement

$$m^n \to m^n(1 + \lambda\eta^n) \ , \tag{11}$$

where

$$n = \begin{cases} 1 & \text{FSE/vertex} \\ 2 & \text{BSE} \end{cases} \tag{12}$$

for irreducible FSE, vertex, and BSE diagrams in electroweak theory. The regulation of infrared singularities for soft photon emissions [10] provides a simple example: For FSE and vertex processes, a fermion m_f and a small photon mass μ mix in terms of form $\ln\frac{m_f}{\mu}$; for consistency, we require

$$\mu \to \mu(1 + \lambda\eta) \ , \tag{13}$$

then $\ln\frac{m_f}{\mu}$ is invariant under (9) and (13).

Vertex factors, including the weak mixing angle, charge, and neutral current coupling constants are all stationary under (11). However, propagators involving massive particles are not stationary under DCM transforms, and dressed amplitudes constructed from them are either driven to zero or a stabilizing correction for finite tree or divergent loop processes, respectively.

Since $m \propto v$ in Higgs mass formulae, dressed mass levels correspond to vacuum displacements: $\Delta v = \lambda\eta v$ and $\Delta v^2 = \lambda\eta^2 v^2$ for FSE and BSE processes, respectively.

C. Electroweak scattering amplitude

Generally, if an irreducible radiative process represented by $\mathit{\Omega}$ borrows energy from the vacuum creating a deficit, then an opposing amplitude is required to ensure conservation of probability and energy. For the moment, assume dimensional regularization is used to tame improper integrals. To account for the deficit and include all possible intermediate mass states, the total amplitude is defined by

$$\hat{\mathit{\Omega}} = \mathit{\Omega}\left(\mathbb{M}\right) - \overline{\mathit{\Omega}}\left(\mathbb{M}\right) \,, \tag{14}$$

where $\mathit{\Omega}$ accounts for self-interaction effects involving physical masses in $\mathbb{M}$, and

$$\overline{\mathit{\Omega}}\left(\mathbb{M}\right) = \frac{1}{2}\lim_{\eta\to\infty}\sum_{\lambda=\pm1}\mathit{\Omega}\left(\mathbb{M}_d = \eta_\lambda\mathbb{M}\right) \tag{15}$$

is a subtrahend for vacuum depletion; from (11), we have

$$\eta_\lambda \equiv \begin{cases} 1+\lambda\eta & \text{FSE/vertex} \\ \sqrt{1+\lambda\eta^2} & \text{BSE} \end{cases} \,. \tag{16}$$

For any $m \in \mathbb{M}$, the dressed mass is

$$m_d = \eta_\lambda m \,. \tag{17}$$

In addition to m_b or m_f, $\mathit{\Omega}$ depends on external momenta $\{k,\, p\}$ for Feynman diagrams in Fig. 1 which may be on- or off-shell. For notational simplicity, any dependence on external momentum parameters has been suppressed during construction of $\overline{\mathit{\Omega}}$ because $\{k,\, p,\, q\}$ are implicitly dependent on associated core masses.

In dimensional regularization we have a singular function [19, 22]

$$D\left(\Delta,\sigma\right) = \frac{1}{\sigma} - \ln\Delta - \gamma \,. \tag{18}$$

where $\sigma = 2 - \mathrm{d}/2$ with spacetime dimension $\mathrm{d} \lesssim 4$; Δ depends on $\mathbb{M}$, momentum parameters external to the loop, and integration variables; and $\gamma = 0.577...$ is the Euler–Mascheroni constant. Divergent terms involving $1/\sigma$ cancel in (14), and the net amplitude is well defined since it involves a factor $-\ln|\Delta/\Delta_\circ|$, where

$$\Delta_\circ = \lim_{\eta\to\infty}\eta_\lambda^{-2}\Delta(\eta_\lambda\mathbb{M}) \tag{19}$$

is derived in Appendix B of Ref. [5].

If an energy cutoff $\Lambda_\circ$ is assumed in lieu of dimensional regularization, then we must include $\Lambda_\circ$ in the arguments of $\mathit{\Omega}$. The cutoff scales in the same way as (17); that is,

$$\Lambda_d = \eta_\lambda\Lambda_\circ \,. \tag{20}$$

Cutoff scaling is required for consistent definition of integrals in $\mathit{\Omega}$ and $\overline{\mathit{\Omega}}$: it synchronizes the cutoff to $\Lambda_\circ$, and yields a well defined limit as $\eta \to \infty$ in (15). Divergent integrals occurring in $\mathit{\Omega}$ and $\overline{\mathit{\Omega}}$ are invariant under (17) and (20); for electron self-energy, this is evident from the argument of the logarithm in (1), and again, divergent terms in (14) cancel.

In contrast to the regulator technique of Pauli and Villars [23], the above method employs physically meaningful dressed mass levels, albeit virtual only, and it applies to all radiative processes in QFT without introduction of auxiliary constraints.

From the functional form in (15), $\overline{\mathit{\Omega}}$ represents the same basic physical process as $\mathit{\Omega}$, but it is distinguished by sticky collisions between core masses and vacuum energy components. Since $-\overline{\mathit{\Omega}}$ opposes $\mathit{\Omega}$, it represents a negative probability correction, thus Feynman's suspicions [11] that negative probabilities might be used " ... to solve the original problem of infinities in quantum field theory" were well founded.

D. QCD scattering amplitude

The foregoing DCM rules apply to QCD as well since its Lagrangian is invariant under an average over dressed mass states. As with electroweak, all vertex factors are independent of mass and are therefore DCM invariant. However, two modifications are required:

First, the sign of $\hat{\mathit{\Omega}}$ must be reversed. Since the Callan-Symanzik [4, 31] beta function depends on coefficients of divergent terms only [14], the sign reversal is expected from opposing signs in the first two terms of (18). However, the sign reversal admits a much more enlightening physical interpretation if one assumes, in contrast to an electrical charge, that a color charge redistributes vacuum energy so there is a deficit in the region of confinement which is balanced by a surplus in the near-field as suggested in Fig. 2: For an electric or color charge, we have a generalized stability condition

$$\mathcal{E}^+ + \mathcal{E}^- = 0 \,. \tag{21}$$

Resulting colored quarks and gluons are enveloped in a negative energy color field, and the quasi-probability of observing them is likewise negative, at least for low energy probes. Since positive and negative energy regions in Fig. 2 for color charges are interchanged relative to electrical charges, the stabilized amplitude in QCD is

$$\hat{\mathit{\Omega}}_{QCD} = \lambda_s\hat{\mathit{\Omega}} \,, \tag{22}$$

where $\hat{\Omega}$ from (14) employs the usual Feynman rules, and

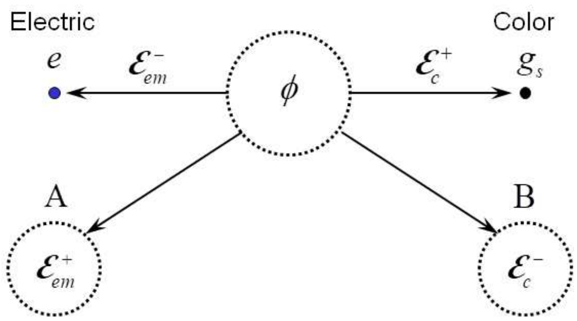

Figure 2. Intrinsically stable electrical and color charges $\{e, g_s\}$ effectively draw {negative, positive} energy from the vacuum ϕ leaving an energy {surplus, deficit} in surrounding (far-field) regions {A, B}.

$$\lambda_s = -1 \tag{23}$$

is a switching factor. For physically meaningful interpretation of the amplitudes, unobservable quark and gluon states must have negative norm and spacelike momenta.

Second, for gluon self-energy diagrams in the pure gauge sector, we lack a mass reference; the solution is to introduce a small gluon mass μ_g via $k^2 \to k^2 - \mu_g^2$ in gluon propagators when constructing amplitudes. Define

$$m_d^2 = \mu_g^2 + \lambda M_g^2\big|_{\mu_g=0} \;\; (M_g \equiv \eta\mu_\circ) \; , \tag{24}$$

where M_g is a near-field energy surplus, and $\mu_\circ$ is an arbitrary unit of mass measure: $\mu_\circ$ can be related to a standard reference mass M_s, usually chosen as m_Z, such that the polarization function vanishes on the mass shell. Introduction of mass terms of the form (24) does not break gauge invariance since the sum over mass levels in the Yang-Mills Lagrangian is zero. See Appendix D of Ref. [5] for details.

III. VERIFICATION

This section briefly justifies rules (14) and (22) for computing stabilized amplitudes. See case study [5] for detailed verification.

A. Electroweak Interactions

For three classes of diagrams in Fig. 1, (14) is verified for electroweak processes focusing on mass and momentum external to the loop. The complete set of Feynman diagrams and expressions for unrenormalized amplitudes are given in Hollik [18] and cited references. Results below agree with [6, 18].

Depletion amplitudes $-\overline{\Omega}$ for diagrams in Fig. 1 meet basic requirements for a vacuum energy deficit: they have negative energy localized at a point (in the limit $\eta \to \infty$), and account for additional mass states (11).

For BSE processes in Fig. 1 (a), (14) gives

$$\hat{\Sigma}^b(s) = \Sigma^b(s) - \overline{\Sigma}^b(s) \; , \tag{25}$$

where the dependence on $s = k^2$ in included, and $\mathbb{M}$ is omitted to simply notation. Since $\Sigma^b(s)$ is linear in s and m^2 with $m \in \mathbb{M}$, and factors involving mass ratios are invariant under (11), the dressed amplitude

$$\overline{\Sigma}^b(s) = \Sigma^b\left(m_b^2\right) + \left.\frac{\partial \Sigma^b}{\partial s}\right|_{s=m_b^2} \left(s - m_b^2\right) \tag{26}$$

includes only the first two terms of a Taylor series expansion of $\Sigma^b(s)$. In the expansion of $\Sigma^b(s)$, $s - m_b^2$ is invariant under (10), and dressed higher order derivatives vanish in the limit $\eta \to \infty$. Since (25) satisfies expected mass shell conditions

$$\hat{\Sigma}^b\left(m_b^2\right) = 0 \tag{27}$$

and

$$\left.\frac{\partial \hat{\Sigma}^b(s)}{\partial s}\right|_{s=m_b^2} = 0 \, , \tag{28}$$

it is finite and agrees with renormalization.

For FSE processes in Fig. 1 (b), we have

$$\hat{\Sigma}^f(p) = \Sigma^f(p) - \overline{\Sigma}^f(p) \; . \tag{29}$$

Since $\Sigma^f(p)$ is linear in p and m_f, its series expansion yields a dressed amplitude

$$\overline{\Sigma}^f(p) = \Sigma^f(p)\big|_{\not{p}=m_f} + \left.\frac{\partial \Sigma^f}{\partial \not{p}}\right|_{\not{p}=m_f} \left(\not{p} - m_f\right) \tag{30}$$

similarly to (26). In the expansion of $\Sigma^f(p)$, $\not{p} - m_f$ is invariant under (9), and dressed higher order derivatives again vanish in the limit $\eta \to \infty$. For a free particle

$$\hat{\Sigma}^f(p)\Big|_{\not{p}=m_f} = 0 \, , \tag{31}$$

and

$$\left.\frac{d\hat{\Sigma}(p)}{d\not{p}}\right|_{\not{p}=m_f} = 0 \tag{32}$$

ensures that $i = \sqrt{-1}$ is the residue of the propagator pole. The second term in (30) eliminates any need for wave field renormalization, and (29) is indeed the desired finite amplitude.

Equation (14) is easily verified for vertex diagrams in Fig. 1 (c): From Ward's identity [32], the vertex function $\Lambda^\mu(q)$ is dimensionless; consequently, the dressed amplitude $\overline{\Lambda}^\mu(q)$ involves only the first (divergent) term in an expansion of $\Lambda^\mu(q)$ about the origin since $q_d = q$, and dressed derivatives vanish as $\eta \to \infty$. Therefore, $\overline{\Lambda}^\mu(q) = \Lambda^\mu(0)$, and

$$\hat{\Lambda}^\mu(q)\Big|_{q=0} = 0 \tag{33}$$

in agreement with renormalization.

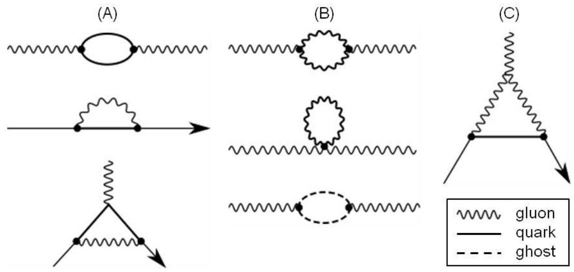

Figure 3. Feynman diagrams for strong interaction processes in QCD: (A) BSE, FSE, and vertex; (B) BSE for gluons in pure gauge sector; and (C) quark/3-gluon vertex. For accretion amplitude $\overline{\Omega}$, particle lines between dots become dressed with vacuum energy.

B. Strong Interactions

For non-Abelian QCD, Ref. [5] applies (22) to one-loop diagrams in Fig. 3. The arguments in Sec. III A are general: stabilized QCD amplitudes are finite and agree with asymptotic freedom predictions [14, 25] only if $\lambda_s = -1$. Equations (27)-(28) for bosons and (31)-(32) for fermions represent free particle vacuum stability conditions for all primitively divergent diagrams.

Physically, we expect that color charges will tend to cluster to maximize overlap of negative energy regions; however, as one probes a charged gluon cloud, the effective color coupling decreases as expected from QCD anti-screening effects. From Eq. (D23) in Ref. [5],

$$\overline{\alpha}_s\left(\rho_s\right) = \frac{\alpha_s}{1-\lambda_s \frac{\alpha_s}{4\pi}\left(11-\frac{2}{3}n_f\right)\ln\rho_s}\,, \tag{34}$$

where $\alpha_s = \frac{g_s^2}{4\pi}$ is the strong coupling constant, n_f is the number of quarks, and $\rho_s = -\frac{k^2}{M_s^2}$ with spacelike momentum k. From (34) the corresponding beta function is given by

$$\beta_{QCD} = 2\frac{\partial\,\overline{\alpha}_s}{\partial\ln\rho_s}\bigg|_{\rho_s=1} = \lambda_s\frac{\alpha_s^2}{2\pi}\left(11-\frac{2}{3}n_f\right)\,. \tag{35}$$

For Abelian and non-Abelian theories, the stability method is expected to yield finite results to all orders in perturbation theory since it is applied repeatedly in any complex Feynman diagram to each irreducible radiative correction, working from the innermost loop outward; a detailed proof for Abelian QED is given in Ref. [5].

IV. CONCLUDING REMARKS

In this paper, we developed a model for stable charges, where the vacuum is split into negative and positive energy components. For an electrical or color charge, respectively, we have a vacuum energy deficit or surplus in a point-like, near-field region with opposing energies in the far-field. The net electromagnetic or color self-energy of a free charged particle is zero. The model was generalized to apply to all Standard Model interactions by defining mass states dressed with both positive and negative vacuum energy for fermion and boson self-energy processes: These new intermediate mass states have infinitesimally short duration and are the key to computing near-field vacuum energy corrections which stabilize the theory. Concise rules for constructing stabilized S-matrix corrections were developed and applied to resolve divergence issues in Abelian QED and non-Abelian QCD and electroweak theories without renormalization.

Particle observability is a consequence of how a charge redistributes vacuum energy; compared to an electrical charge, a color charge acts on the vacuum to create a negative energy density in the far-field region of confinement. Thus, in strong interactions, the dressed amplitude $\overline{\Omega}$ corresponds to an accretion of vacuum energy in the near-field, and the physical masses are cloaked in the depletion part $-\Omega$ of the stabilized amplitude.

In summary, infinite mass, charge, and wave-field renormalizations are required in the standard (unstabilized) theory because core amplitudes do not account for near-field vacuum energy corrections in violation of the law of conservation of energy. Stabilized amplitudes are finite, agree with renormalized QFT, and are uniquely determined in contrast to multiple renormalization schemes.

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