On the Correlation in Bell-test Experiments

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Abstract:

This paper throws a new light on Bell-test experiments. It is analyzed what the results of Bell-test experiments exactly represent and how these results come to existance. It turns out that the 'violation of Bell's inequalities' can be accounted for in a logical way and that there is no need for 'spooky action at a distance'. A condition is that spin is a definite property of particles.

About this paper

This paper can be difficult meaning that the reader is asked (to be able) to see things in space. But I am confident that the reader can manage because the things are real things and the space is the normal 3-dimensional space. And I shall try to describe everything as logically as I possibly can.

Introduction

The deepest origin of Bell-test experiments stems from issues concerning the interpretation of Quantum Mechanics (QM). QM is a physical theory describing the smallest particles of the universe. It is a wonderful mathematical construction and it gives excellent accurate predictions about the behaviour of the 'atoms of the universe'. As we cannot see these particles we have to guess how they look like and how they behave and even where they are. The charasteristics of the particles are described by the wavefunction in wave equations (Schrödinger). It seems that the wavefunction doesn't tell anything about the state of a particle but it does predict the outcome of a interaction of the particle with another particle or a field (for example in an experiment). This impossibility to know the state of a particle made some physicists (Niels Bohr among others) say that particles have no properties at all before they interact. This reasoning is not valid, of course, as we will see in this paper. Another view is that particles are in 'superposition' before interaction. This means that they are in many states at the same time before interaction and at the moment of interaction they choose one particular state. Einstein did not agree on this idea of reality. These kind of problems makes the interpretation of QM very difficult: do particles have definite properties or do they not? Einstein was convinced they have. Bohr thought, and generally Copenhagen interpretation followers think, they have not.

In QM there is a principle called: the uncertainty principle of Heisenberg. This principle has nothing to do with the meaning of the wavefunction but it is sometimes wrongly related to the wavefunction. The principle states that the product of the variation in position and the variation of the impuls momentum of a particle is equal to or bigger than Planck's constant (h). The principle is a kind of energy law, comparable with Einstein's energy law: $E = mc^2$.

Then there is the matter of wave-particle duality, meaning that particles also have wave properties and that waves also have particle properties. So waves and particles have complementary properties: they have both wave- and particle properties. Bohr had an, in my opinion, strange view on this complementarity. He said that particles show either wave properties or particle properties but never at the same time. This is not very uncommon because what particles show, is what the circumstances ask them for. But Bohr didn't expect particles to have any properties at all until they interact. I should think they have both wave- and particle properties as dubble-slit experiments clearly show.

Allow me to spend a few lines on dubble-slit experiments. In dubble-slit experiments particles, passing the slits one by one, produce an interference pattern all the same. That is hard to explain. The question is: what is causing the interference pattern? Is it the trajectory of the particle or is it the mathematical QM description of the possible trajectories of the particle? If it is the QM description of the trajectories (in whatever way) then the QM description must have an interference pattern as outcome. That is possible. But then the particle must have in its movementcharasteristics something that causes the interference pattern otherwise the particle just obeys QM and that can not be real. After all it is the particle passing the slits, not the equations. So there must be something that has to do with the movement of the particle that causes an interference pattern. We can call this ´something´ as well: the wave properties of the particle.

One could imagine that QM describes other phenomena than wave phenomena that yet cause an interference pattern. In that case particles don't need wave properties. But with just particle properties particles will not manage to produce interference patterns. So the particles must have more properties than just particle properties. If those properties are not wave properties, what are they? Again: whatever they are, we can still call them as well wave properties. This means that particles have wave properties as well as particle properties at the same time.

So you see: the opinions concerning the properties of particles were very confusing and chaotic. Fortunately in the quantum world exists the phenomenon of entanglement. This means that in certain circumstances pairs of particles can be produced in a way that the particles have opposite properties: they have opposite charge, they have opposite spin and they move in opposite directions. Entanglement only tells about the production of a pair of particles and the fact that their properties are opposite. Entanglement has nothing to do with a connection or an interaction between the particles on a later moment, when they are at a large distance from each other. Heisenberg's uncertainty relation principle is being used to state that it is impossible to know the impuls momentum of a particle when its position is measured and it is impossible to know the position of a particle when its impuls momentum is measured. The phenomenon of entanglement made Einstein, Podolsky and Rosen, (EPR 1935), think of a way to get to know the properties of one particle by measuring the other. This way there would not be a disturbance of the first particle because it is not actually measured. The idea was that if the position of particle A is being measured, then the position of particle B is known so B must have had the 'property of position' and if the impuls momentum of A is measured then the impuls momentum of B is known and B must have had the property of impuls momentum. This way EPR showed that particles have definite properties and they can be determined without being touched. This means that Heisenberg's uncertainty relation principle goes for only one particle, not for a pair of entangled particles.

The idea of EPR needed proof by way of an experiment. It was David Bohm who got the idea that such an experiment is possible by measuring spin of particles in three directions in a plane. These experiments became known as Bell-test experiments.

Spin

Copenhagen interpreters define spin as an 'intrinsic' property of particles that cannot be imagined. When it is being measured, it is always 'up' or 'down', nothing in between. To me this doesn't mean very much. I need to imagine things and to be able to do so, I look in the every day world for examples. I consider spin as the rotation of a particle. Spin then can be represented by the axis of rotation. It may have to do with electric charges and magnetic momentum but that is not very important in this explanation. It is like the earth spinning around her axis. The direction of the axis is completely arbitrary and is totally independent from the direction of the movement of the earth. But it has always the same direction in space. The earth' axis always points in the direction of the polar star. The 'far away' stars make space absolute, as it were.

In the same way spin of a particle doesn't depend on the direction in which the particle moves (line of motion). Spin can be represented by a vector, an axial vector. Looking in one direction along the axis, the rotation can be right way around or left way around and when it is right way around then that is the direction of the vector representing spin of the particle.

This vector also keeps its direction in space, exactly like the axis of the earth. The earth orbits around the sun in a plane. Suppose an enormous planet comes nearby the earth, without collision, and makes the earth orbit in another plane around the sun. This incident doesn't change the rotation direction of the earth: her axis still points at the polar star. In the same way the spinvector of a particle keeps its direction in space as long as the particle doesn't interact. I consider a spinvector as an arrow with a fixed direction in space. It is possible to detect the component of spin of a particle in the direction of the magnetic field, by using a Stern Gerlach device.

Stern Gerlach devices

A Stern Gerlach device consists of two magnets and a detector. The magnets have different shapes and because of that the magnetic field they pruduce between the poles is inhomogenous. That means that the strength of the magnetic field varies in a certain direction between the poles. The detector is a glass plate at the end of the device and able to detect the particles. The particles used in these experiments must have a magnetic dipole momentum so the (vertical) inhomogenous field forces the particles, moving between the poles, up- or downwards. The line of motion of the particles is chosen to be the direction of reference. When the particles leave the space between the magnets, they are being detected by the detector. The detector is placed perpendicularly on the line of motion. The magnets can be adjusted in an arbitrary angle meaning they can be rotated around the line of motion in a plane perpendicular on the line of motion. In that way the direction of the magnetic field can have any direction between vertical and horizontal. J.S. Bell (see ref. 1)) describes the experiment perfectly.

Schematic a Stern Gerlach device can be considered as a direction of the magnetic field and a central plane perpendicular to that field direction. When a number of particles is sent through the device, 50% of them will arrive at the detector above the central perpendicular plane and 50% will arrive beneath it. The particles are expected to have their spin in an arbitrary direction. So a large number of particles will have their spinvector in all possible directions. When a device is not adjusted in the vertical direction but in another direction at angle φ in respect of the vertical then it is to be expected to get the same results and this is what is actually being seen.

Many physicists believe that spin of partcles is quantized. If this means that the direction of spin of a particular particle is only possible in certain directions and not in others, then this is nonsense. Stern Gerlach experiments clearly show that the ratio of spinresults is always 50% 'up' and 50% 'down', in whatever angle the device is adjusted. This means that spinvectors of particles have a totally random direction in space.

In some experiments two Stern Gerlach devices are placed one behind the other, the second at an angle φ in respect of the first. Only the particles with spin 'up' leaving the first device are sent through the second device. Some of the particles, leaving the second device, show spin 'down'. It appears that the number of spin 'down' particles is proportional to $\sin^2(\varphi/2)$. Why this is the case I will explain in the explanation of Bell-test experiments.

Bell-test experiments and probabilities

In some Bell-test experiments Stern Gerlach devices are used to detect spin of particles. In these experiments pairs of entangled particles are produced. The particles of a pair move in opposite directions, for example: to the left and to the right. So two Stern Gerlach devices are needed to detect the particles: one at the left and one at the right. The devices are placed perpendicular on the line of motion of the particles. Suppose one of the devices (A) is adjusted vertical and the other (B) is adjusted at an angle φ in respect of A. Entangled particles also have opposite spin directions. But although the spin direction of one particle is perfectly opposite to the spin direction of its counterpart, the common axis of the spin directions has a completely random direction in space. This means that, as previously is explained, when a large number of pairs is being produced and detected, both detector A and detector B will show a series of spinresults consisting of 50% 'up' and 50% 'down'. But they are not the same series, of course, because B detect in an other angle in respect of A. When the series are being compared to each other there appear to have been pairs with combinations of opposite spinresults and pairs with combinations of equal spinresults. The part of the total number of pairs with the combination of equal spinresults appears to be proportional to $\sin^2(\varphi/2)$. We have to keep in mind that that part is a ratio of numbers, it is a probability. So the combined series of results, produced by the detectors, don't represent numbers of pairs with equal spin, because there aren't any pairs with equal spin. All pairs of entangled particles have opposite spin. The combined series of results represent probabilities for combinations of equal spinresults.The part of the total number of pairs with the combination of opposite spinresults is proportional to $\cos^2(\varphi/2)$.

Correlation

These probabilities are predicted by QM. They are used to calculate the correlation. The correlation is defined as the number of combinations of equal spinresults subtracted by the number of combinations of opposite spinresults and the outcome divided by the total number of pairs. So correlation is calculated from probabilities. Correlation (C) calculated from QM probabilities is: $C(QM) = \sin^2(\varphi/2) - \cos^2(\varphi/2) = - \cos\varphi$. I have to explain this correlation, or rather the probabilities.

Vectorspaces

As we have seen previously we can consider a Stern Gerlach device as a direction and a central perpendicular plane. In Bell-test experiments two such devices, A and B, are placed on the line of motion. A is in the vertical position and B is adjusted in an angle φ. As the angle between A and B is φ, the angle between their central perpendicular planes also is φ. The two planes, when imagined stretched to infinity, divide space in four spaceparts: two by two opposite of each other (see Fig.1). We can consider these four spaces as vectorspaces. The particles move along the line of motion, having their spinvector in one of these four vectorspaces. We shall call the opposite spaces between the planes that make an angle φ: spaces E (equal) and the other two opposite spaces: spaces O (opposite). As spinvectors of an entangled pair are always opposite they either belong both to E or both to O. When both spinvectors of a pair belong to E, it is not difficult to see that the two vectors are either in the upper hemisphere of both detectors or they are in the lower hemisphere of both detectors, giving combinations of equal spinresults in either case. In the same way spinvector pairs in O give combinations of opposite spinresults because the vectors are in opposite hemispheres of the detectors. As spinvectors are equally distributed in space, the number of spinvectors in E is proportional to the size of E and so the probability for a random vector to be in E is proportional to $\varphi/180°$. And the probability for a random vector to be in O is proportional to $(180° - \varphi) / 180°$. Calculating the correlation from these probabilities Bell got the result of $C(Bell) = (\varphi / 90°) - 1$
He calculated the correlation in several ways but never obtained the QM result $C(QM) = - \cos\varphi$. So Bell thought that this QM correlation was not possible in a normal local-real universe. The correlations he found always were proportional to φ, never proportional to cos φ (see diagram).

Fig.1)

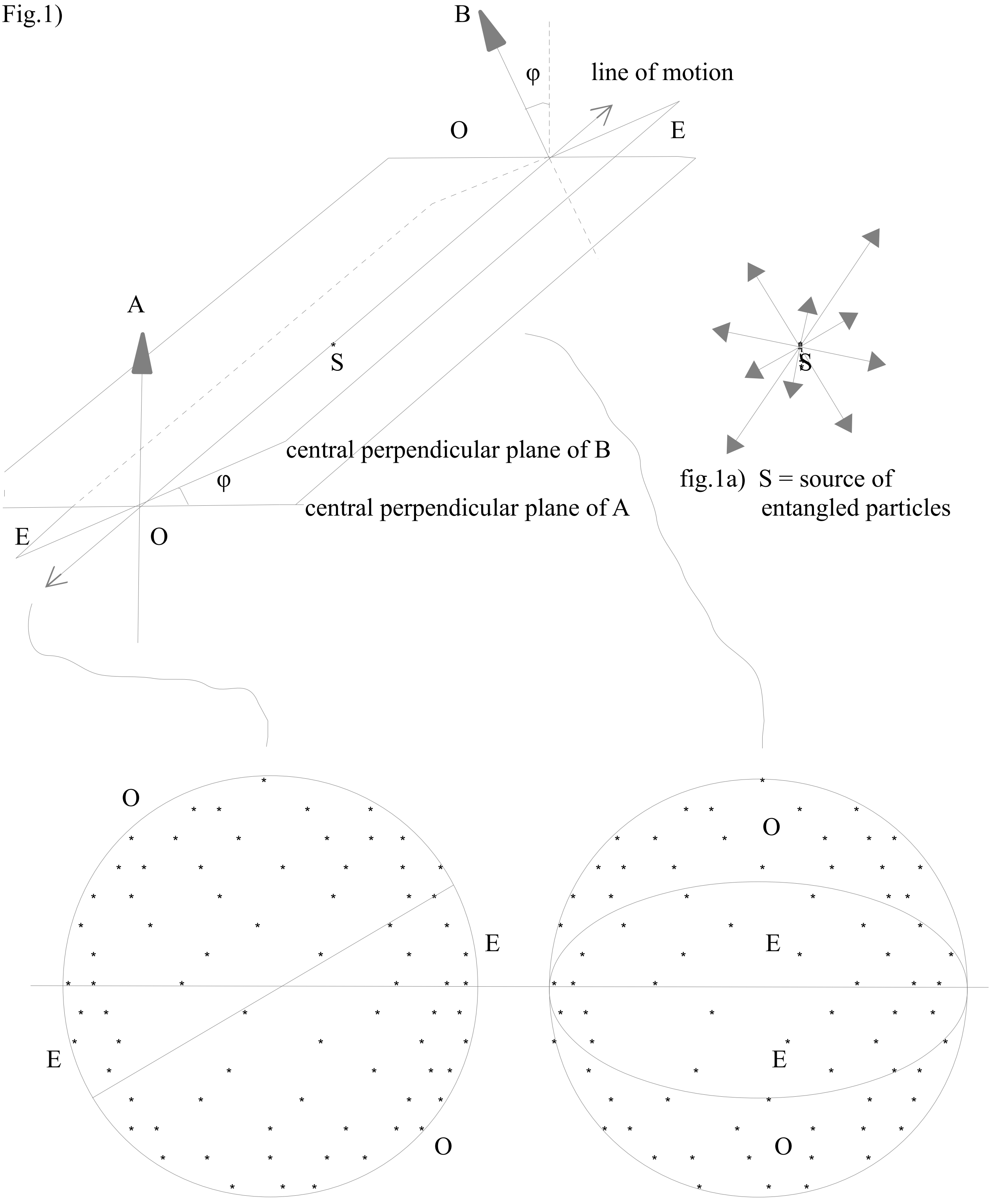
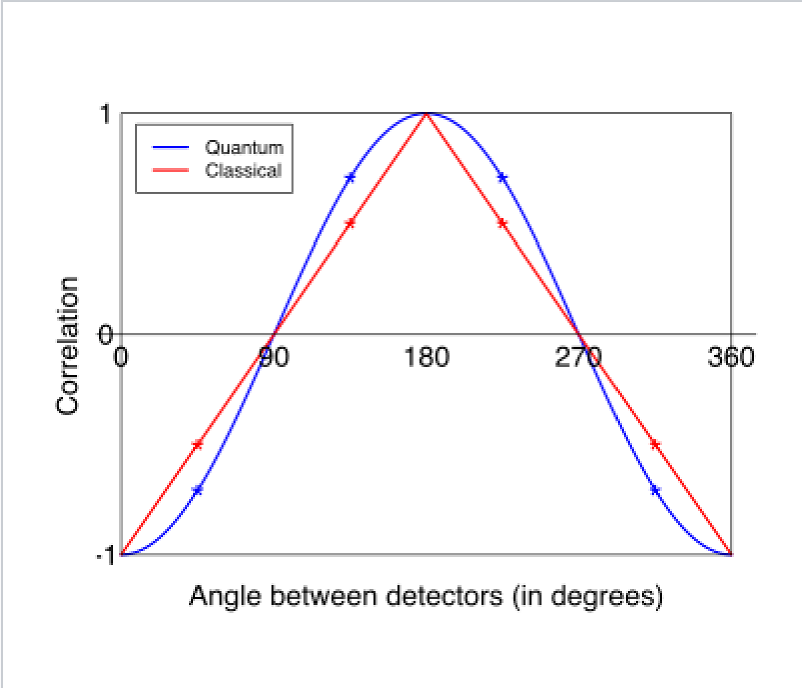


fig.1a) S = source of entangled particles

fig.1b) Bell's probabilities
Projection of the vectorspaces E and O from the perspectives of the detectors.

fig.1c) QM's probabilities
Projection of the same vectorspaces E and O from the perspective of the particles.



The best possible local realist imitation (red) for the quantum correlation of two spins in the singlet state (blue), insisting on perfect anti-correlation at 0°, perfect correlation at 180°. Many other possibilities exist for the classical correlation subject to these side conditions, but all are characterized by sharp peaks (and valleys) at 0°, 180°, and 360°, and none has more extreme values (±0.5) at 45°, 135°, 225°, and 315°. These values are marked by stars in the graph, and are the values measured in a standard Bell-CHSH type experiment: QM allows $\pm 1/\sqrt{2} = \pm 0.7071\ldots$, local realism predicts ±0.5 or less.

Source: Wikipedia
red = straight line, sharp peak
blue = – cos

The probabilities that Bell found for combinations of equal and opposite spinresults can be seen as the projection in the direction of the line of motion of space E and space O onto a plane (the detector). The results of the projections of E and O onto the detector don't correspond to the results of the experiments. The experiments clearly show probablities of $\sin^2(\varphi/2)$ for combinations of equal spinresults and probabilities of $\cos^2(\varphi/2)$ for combinations of opposite spinresults.

Remarkably there is a way to obtain the QM probabilities. These probabilities can be obtained by projecting the exact same identical spaces E and O in a direction perpendicularly to the line of motion onto an imaginary plane along the line of motion. This is a very strong hint that the problem (of explaining the QM probabilities) is of a logical / mathematical / physical nature and that there is no need to look for the solution in considering a non-local universe.

Perspective

Now that we found the correct QM probabilities, we have to explain why the vectorspaces have to be projected in a totally unexpected direction. This has to do with perspective. Perspective has to do with directions and rotations. Perspective is the direction in which the universe is observed. When you turn around you see the universe in a different perspective. Perspective is one of the most difficult phenomena in physics. It is difficult because it is not about the questions: 'how do you see the universe?' or 'how do I see the universe?, as one would expect, but it is about the question: 'how do you see the universe from my perspective?' It is extremely difficult to grasp the meaning of this question. But fortunately it is easy to find the answer to the question: one only carefully needs to take into account all rotations. Taking into account all rotations means that when one observer rotates in respect of another one, he has to imagine the universe to rotate along with him. It then is as if the observer didn't rotate at all but as if the other observer rotated the other way round. In that way the perspectives of both observers correspond with each other. The perspectives become relative to one another. I tried to demonstrate this in a video (see ref. 2)). I strongly recommand to watch it.

But what has perspective to do with Bell-test experiments? We remember that the line of motion is chosen as reference direction. We also remember that the detectors are being placed perpendicularly on the line of motion. Now ask yourself: how has this been done? There is only one way to do this, if not at random. One has to start putting the detectors in a position along the line of motion and then rotate them 90° to the perpendicular position. By rotating the detectors 90° in respect of the line of motion, the perspectives of the detectors become different from the perspectives of the particles. Because QM describes the particles, we have to adapt the perspectives of the detectors by projecting the vectorspaces (E and O) in a 90° rotated direction, or, what is equivalent, by imaginary rotating the vectorspaces 90° before projecting them onto the detectors.

Second chances

We have seen that there are different ways to calculate different probabilities for one and the same vectorspace. The combinations of spinresults in Bell-test experiments can be considered as 'second chances', chances of chances, analogous to 'second derivative'. A Bell probability is the probability for a particular vector to belong to a certain vectorspace. A QM probability is the probability for a particular vector to belong to one of the vectors in that vectorspace. It is that probability that emerges when the two series of spinresults of a Bell-test experiment are being compared to one another.

Considered afterwards it is logical that the experiments can only yield these QM probabilities because one detector can't perceive a pair of entangled particles. One detector can only perceive a series of particles, with spin in arbitrary directions. Each particle stems from an entangled pair and has a 50 % chance to have its spinvector 'up'- or 'down'wards. A detector can't represent the probability for a particular particle to have its spinvector in space E or in space O. Even two detectors cannot do that. But the combined results of A and B can represent the probability for a particular particle that its spinvector belongs to one of the vectors in space E or space O. This probability is the QM probability, resulting in the cosine correlation. And if a spinvector of a particular particle is in space E, the spinvector of its counterpart is in the other part of space E. The same goes for space O. It has been demonstrated what projection is needed to find the QM probablities. From the perspective of the particles this is all perfectly correct.

Conclusion

In Bell-test experiments two Stern Gerlach devices are being used. The position and adjustment in respect of one another and in respect of the line of motion of the particles determine vectorspaces. Such a vectorspace can be projected in different directions, or, what is equivalent, looked at from different directions. These projections yield different probabilities for one and the same vectorspace. So the differences of the probabilities don´t depend on the volume of the vectorspace or on the number of spinvectors it contains but they depend on the differences in area of the projections. Vectorspaces projected onto the detector yield Bell´s probabilities and the same vectorspaces looked at in a direction perpendicular to the line of motion, yield QM probabilities. These directions of projection represent the perspectives of the detectors and of the particles. The line of motion of the particles is chosen as reference direction. The detectors have been rotated 90°

in relation to this reference direction.

QM describes the particles and because the particles haven´t changed their perspective in relation to the reference direction, the results of the experiments correspond to QM. The results of the experiments don´t correspond to Bell´s probabilities because he calculated the probabilities from the perspectives of the detectors which differ from the perspective of the particles. So Bell obtained probabilities that were different from the probabilities of QM and so his correlations didn´t match that of QM.

A probability is a ratio of numbers, it is not a number of something. (There are no entangled pairs with equal spin). Bell-test experiments yield probabilities as result, no numbers. These probabilities therefore emerge by comparing the lists of results of the detectors. And correlations are calculated using the probabilities. Because Bell used probabilities that differed from QM´s probabilities he obtained correlations that were different from QM's correlation. But also the probabilities of QM are local and real. They are only calculated from a different perspective.

Summerized: Bell's probabilities are the spaces E and O, determined by the central perpendicular planes of the detectors, perceived from the perspectives of the detectors. The detectors cannot represent these probabilities. A detector represent spinresults of a series of particles, each particle stemming from a pair of entangled particles, and having a 50 % chance of 'up' or 'down' spin. QM probabilities are the same spaces E and O perceived from the perspective of the particles. The detectors also cannot represent these probabilities. These probabilities are represented by the combinations of the results of A and B.

The fact that correlation in Bell-test experiments can be accounted for in a classical physical way, based on definite properties of particles, means that the interpretation of the wavefunction should be revised. But that too may be a matter of perspective.

References:

1) J.S. Bell, Bertlmann's socks and the nature of reality. Archives ouverte HAL, 1981

2) https://www.youtube.com/watch?v=g1quDMTEIFE (video)

Addendum

I would like to reflect a bit on direction. The role of direction in physics is of the same significance as the role of time. In certain circumstances direction and time are even exchangeble among themselves. When a force is working on a moving object, in the direction of the movement, the object will accellerate, meaning that the movement changes in time, not in direction. When the same force is working on the same moving object, but in a perpendicular direction, the object will not accellerate at all, but it will deflect, meaning that the movement changes in direction, not in time. So direction is important and it is strange that there is no dimension for it in the way that the dimension for time is the second. Such a dimension could be useful, at least for polar vectors. The direction of the field between the magnets of a Stern Gerlach device can be considered as a polar vector. Someone proposed the 'cycle' as dimension for direction. That sounds very good to me.

A circulair movement becomes a vibration when looked at it from another direction and plotted against time it becomes a wave. It is all a matter of perspective.

I mentioned the analogy between 'second chances' and second derivatives. When a second derivative to time is calculated from a function of position then a function of accelleration is obtained. In a comparable way one could imagine that when a vector is being projected two times, one obtaines the probabilities that emerge from experiments with two Stern Gerlach devices (?) When an arrow is being projected, in the direction of the line of motion, onto a detector and subsequently is being projected, perpendicularly, onto the diameter in the field direction of that detector, then the exact QM probability emerges.