The c² Gravitational Potential Limit

and its implications for G as well as general relativity

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Abstract

The relationship between gravitational potential, black holes and squared light speed c^2 is examined as well as the implications of the presented findings for the gravitational constant G and general relativity theory. It is common knowledge that the velocity limit in our universe is defined by light speed c and as shown in this work c^2 plays a similar role for the gravitational potential since c^2/G is linked to the mass density of black holes, our local Hubble sphere and the overall universe. Furthermore, it is demonstrated that the rift between cosmology and quantum physics can be reconciled by acknowledging the physical meaning of the Planck units which proposedly define the characteristics of quantized space-time. This notion is also supported by the presented logarithmic relationships between the cosmological scale and the quantum scale. Finally, the presented findings allow uncovering a physical relationship between the constituents of Dirac's contested large number hypothesis.

Keywords: light speed; gravity; gravitational constant; gravitational potential; gravitational force; general relativity; black hole; Schwarzschild; Kerr; Newton; Einstein; Dirac; Hubble; sphere; space-time; quantization; Planck units; strong force; Euler number; Sommerfeld constant

1 Introduction

The term c^2 is mostly known for its appearance in the famous energy mass equivalence $E = mc^2$, which was discovered by Albert Einstein in 1905. A contemporary of Einstein, Erwin Schrödinger, noticed in 1925 that c^2 can also be expressed in the physical units of gravitational potential, i.e. $\mathrm{J/kg}$ or equivalently $\mathrm{N\,m/kg}$, besides the "raw" units $\mathrm{m^2/s^2}$ as pointed out by Alexander Unzicker in (2). Such a coincidence of units may point towards an important physical relationship, like it was the case with the units of Planck's constant h which can be interpreted as $\mathrm{J/Hz}$ or as the units of angular momentum, i.e. $\mathrm{kg\,m^2/s}$. Niels Bohr scrutinized this congruence in 1913 which then led him to the discovery of quantized electron shells in atoms. The purpose of this paper is to examine the gravitational potential interpretation of c^2 for hidden physical meanings and their implications for the gravitational constant G as well as general relativity theory. To achieve this objective some standard equations are repeated first for reference in the upcoming sections.

2 Prerequisites

The force of Newtonian gravity for two masses m_1 and m_2 is usually presented as

$$F_{gm} = G\frac{m_1 m_2}{d^2} \tag{2.1}$$

whereby d denotes the distance between their center of mass. Using $E_1 = m_1\,c^2$ and $E_2 = m_2\,c^2$ it is also possible to express Newtonian gravity with respect to energy:

$$F_{ge} = \frac{G}{c^4}\frac{E_1 E_2}{d^2} \tag{2.2}$$

The associated gravitational potential energy can subsequently be expressed in the following two ways:

$$U_{gm} = -G\frac{m_1 m_2}{d} \qquad \text{with} \qquad F_{gm} = -\frac{\mathrm{d}\,U_{gm}}{\mathrm{d}\,d} \tag{2.3}$$

$$U_{ge} = -\frac{G}{c^4}\frac{E_1 E_2}{d} \qquad \text{with} \qquad F_{ge} = -\frac{\mathrm{d}\,U_{ge}}{\mathrm{d}\,d} \tag{2.4}$$

The gravitational potential is subsequently given by the following two equations, whereby equation 2.5 describes gravitational potential energy per unit mass (i.e. J/1 kg)

$$V_{gm} = \frac{U_{gm}}{m_2} = -G\frac{m_1}{d} \tag{2.5}$$

and equation 2.6 describes gravitational potential energy per unit energy (i.e. J/1 J):

$$V_{ge} = \frac{U_{ge}}{E_2} = -\frac{G}{c^4}\frac{E_1}{d} \tag{2.6}$$

The previous two equations are later on also referred to as ”gravitational potential” for brevity.

Please note that the G/c^4 term in equation 2.2, 2.4 and 2.6 is also present in the Einstein constant κ, which is used in the equations of general relativity theory.

$$\kappa = 8\pi\frac{G}{c^4} \tag{2.7}$$

This correlation already demonstrates the relatedness of Newtonian gravity with general relativity theory and it’s unfortunate that this obvious connection is seemingly not presented in physics literature. The implicit presence of 4π in the Einstein constant indicates a relationship with spherical geometry as shown hereafter in section 4. Moreover, it’s noteworthy that c^4/G has the physical units of force, i.e. N or J/ m. The physical units of the gravitational constant G itself are N m^2 kg^{-2}, whereby its strikingly difficult to make sense of G’s ”raw” units m^3 s^{-2} kg^{-1} until regarding equation 3.2 rearranged for G.

This paper also uses some key results of general relativity theory, in particular the mass for a static Schwarzschild black hole with radius r_s

$$m_s = \frac{r_s}{2}\frac{c^2}{G} \tag{2.8}$$

and the mass of an extreme Kerr black hole with radius r_k which rotates with light speed c at its equatorial ring:

$$m_k = r_k\frac{c^2}{G} \tag{2.9}$$

It’s noteworthy that these two types of black hole mass are simply related by a factor of two for an identical radius. Kerr black holes with other equatorial velocities are not relevant for this paper.

3 The c^2 Limit

The value of c^2 expressed as gravitational potential energy per unit mass is given by:

$$c^2 = 8.988 \times 10^{16}\ \mathrm{J\,/\,kg} \tag{3.1}$$

Since light speed c is considered to be the maximum possible velocity in our universe and because c^2 also has an extraordinarily high value it is sensible to postulate that c^2 denotes the magnitude of maximum gravitational potential energy per unit mass in our universe. Utilizing this assumption by setting equation 2.5 equal to $-c^2$ then gives:

$$G\frac{m}{d} = c^2 \tag{3.2}$$

Finding the maximum value for the gravitational potential energy per unit energy is not that straightforward, but comparing equation 2.6 with 3.2 reveals that the appropriate value is $-c^2/c^2 = -1$. This expression evaluates to a pure number but the appropriate physical units can be added without violating any rules.

$$\frac{G}{c^4}\frac{mc^2}{d} = 1\frac{\mathrm{J}}{\mathrm{J}} \tag{3.3}$$

Surprisingly, the last two equations result in a well known relationship when rearranging either of them for mass m:

$$m = \frac{dc^2}{G} \tag{3.4}$$

This relation equals the extreme Kerr black hole mass equation 2.9 when interpreting distance d as radius r_k. Due to its spherical symmetry a black hole’s mass can be treated as being compressed into its centre for a classical gravitational potential calculation. Doing such a calculation for the distance of a black hole's boundary to its centre then leads to the equality of d with r_k and equation 2.9 with 3.4. Consequently, these findings suggest that black holes form when the gravitational potential for a spherical region of space is between $-c^2/2$, like for a Schwarzschild black hole, and $-c^2$. The primary characteristic of regions with such a strong gravitational potential is that they cannot be destroyed by spinning them up since this would require a tangential velocity greater than light speed c, as explained in more detail in section 6.

It's worth emphasizing here that the presented findings mean that the existence of black holes can be inferred from Newtonian gravity when also acknowledging a limiting velocity c and a limiting gravitational potential $-c^2$, but there is no necessity for assuming any space curvature. Since an extreme Kerr black hole poses a limit to the amount of energy for a spherical volume of space, and angular momentum absorption makes it grow, the gravitational potential of $-c^2$ must represent a general upper limit, i.e.

$$-V_{gm} \leqq c^2 \quad \text{and} \quad -V_{ge} \leqq 1\text{J}/\text{J} \tag{3.5}$$

is always given. Presumably, electrically charged black holes cannot violate these constraints either.

4 Our Hubble Sphere

The expansion of our universe creates an invisible and intangible spherical boundary centred around our location in space at which objects are moving away from us with light speed c and objects outside of this boundary are receding even faster. The current rate of this expansion is denoted by the Hubble constant H_0 which can also be used to calculate the distance to this boundary, whereby the enclosed volume is called the Hubble sphere whose radius is in turn denoted as Hubble radius r_H.

$$H_0 \cong 74.3\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \tag{4.1}$$

$$r_H = \frac{c}{H_0} \cong 1.25 \times 10^{26}\,\text{m} \tag{4.2}$$

The correct value of H_0 is still disputed, but the value used here seems to fit well with the following calculations.

Calculating the mass density of a Schwarzschild black hole that has the extent of our Hubble sphere gives the following mass density

$$\rho_H = \frac{m_H}{V_H} = \frac{m_s(r_s = r_H)}{4\pi r_H^3/3} = \frac{3H_0^2}{8\pi G} = 1.0 \times 10^{-26}\,\text{kg}/\text{m}^3 \tag{4.3}$$

whereby m_H denotes the Hubble sphere mass and V_H denotes the Hubble sphere volume. Surprisingly, that result matches with the so called "critical density" which denotes a flat universe according to general relativity theory. This, in turn, also suggests that a Schwarzschild black hole possesses flat space and consequently doesn't have a central singularity - a notion which also matches with the findings of the previous section. NASA reports that the mass density of our universe is around $1.0 \times 10^{-26}\,\text{kg}/\text{m}^3$ (see footnote 1) and thus our local Hubble sphere is either a Schwarzschild black hole or very close to being one. This makes sense since black holes set an upper limit for the amount of energy and information that can be contained in a particular volume of space. Moreover, nothing can leave our Hubble sphere, due to the expansion of space, which also matches with the accepted presumption that nothing can leave a black hole.

Since our local Hubble sphere seems to qualify as Schwarzschild black hole it makes sense to apply equation 2.8 to it. Moreover, that equation can also be multiplied by c^2 and rearranged so that the result coincides with the gravitational potential equation 2.6 adapted for the case of a Schwarzschild black hole:

$$\frac{G}{c^4}\left(\frac{m_H c^2}{r_H}\right) = \frac{1\text{J}}{2\text{J}} \tag{4.4}$$

Interestingly, Erwin Schrödinger was already proposing a similar relationship for analysis in 1925, although there was much less knowledge about the observable universe back then (6). The interesting part of the last equation is the term in brackets which has the physical units of J/m and can be dissected to analyse the Hubble sphere's internal energy distribution:

$$\frac{1\text{J}}{2\text{J}}\frac{c^4}{G} = \frac{m_H c^2}{r_H} = \frac{\rho_H V_H c^2}{r_H} = \frac{4\pi}{3}\rho_H r_H^2 c^2 = \frac{2}{3} \times 2\pi\rho_H r_H^2 c^2 \tag{4.5}$$

The calculations in this document usually treat masses as point particles, but here it is necessary to consider the effect of a changing gravitational force inside the Hubble sphere. Therefore the factor $2/3$ appears on the

1 http://map.gsfc.nasa.gov/universe/uni_matter.html

right side of equation 4.5 which is characteristic for the gravitational potential of a sphere with uniform energy density. The remaining part fits with the ideas of Mach, Dicke, Sciama and Schrödinger who speculated that all masses in the observable universe and their distance to us should be causal for the gravitational constant G (2), which seems to be the case when the appropriate scaling and conversion factors are considered:

$$\frac{3}{2}\frac{1\text{J}}{2\text{J}}\frac{c^4}{G} = 2\pi r_H^2 \rho_H c^2 = \int_0^{r_H} \frac{4\pi r^2 \rho_H c^2 \,\text{d}r}{r} = \lim_{\delta r \to 0} \sum_{r_i=0}^{r_H} \frac{4\pi r_i^2 \rho_H c^2 \,\delta r}{r_i} = \sum_{i=0}^{\infty} \frac{m_i c^2}{r_i} \tag{4.6}$$

It is assumed here that the mass density of our Hubble sphere can be treated as being approximately homogeneous on large scales. This is why the last equation divides our Hubble sphere into a series of spherical shells, which ideally should be infinitely thin, whereby $m_i = 4\pi r_i^2 \rho_H \,\delta r$ denotes the mass of one such spherical shell whose distance to us is given by r_i and whose thickness is δr. The resulting sum term on the right side of equation 4.6 is what typically appears in a gravitational potential equation when several masses are involved and equation 4.4 can thus also be expressed as follows:

$$\frac{G}{c^4}\left(\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i c^2}{r_i}\right) = \frac{1\text{J}}{2\text{J}} \tag{4.7}$$

The findings which were presented above suggest that gravitational potential, G, c^2 and the physical parameters of our local Hubble sphere are all inter-connected. This leads to the notion that the gravitational constant G can be regarded as a result of those relationships, in particular of our local Hubble sphere's energy density and its associated gravitational potential of $-c^2/2$. Consequently, the gravitational constant G can be defined as an emergent constant in the following ways

$$G = \frac{c^2}{2}\Big/\left(\frac{m_H}{r_H}\right) = \frac{c^2}{2}\Big/\left(\frac{4\pi r_H^2}{3}\rho_H\right) = \frac{c^2}{2}\Big/\left(\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i}{r_i}\right) \tag{4.8}$$

in case our Hubble sphere really qualifies as a Schwarzschild black hole. On the same grounds the Einstein constant can also be rewritten with respect to the parameters of our local Hubble sphere:

$$1/\kappa = \frac{1}{8\pi}\frac{c^4}{G} = \frac{1}{4\pi}\frac{m_H c^2}{r_H} = \frac{1}{4\pi}\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i c^2}{r_i} = \frac{1}{3}\rho_H r_H^2 c^2 \tag{4.9}$$

The implications of these expressions will be treated later on in the discussion section.

The emergent constant notion for the gravitational constant G implies that equation 2.1 to 2.6 are connected to cosmological quantities. Equation 2.5 and 2.6, for example, can subsequently be rewritten in a form without G as follows, whereby $E_1 = m_1 c^2$, $E_i = m_i c^2$ and $E_H = m_H c^2$:

$$V_{gm} = -G\frac{m_1}{d} = -\frac{3}{4}\frac{m_1 c^2}{d}\sum_{i=0}^{\infty}\frac{r_i}{m_i} = -\frac{c^2}{2}\frac{m_1/m_H}{d/r_H} \tag{4.10}$$

$$V_{ge} = -\frac{G}{c^4}\frac{E_1}{d} = -\frac{3}{4}\frac{E_1}{d}\sum_{i=0}^{\infty}\frac{r_i}{E_i} = -\frac{1}{2}\frac{E_1/E_H}{d/r_H} = -\frac{1}{2}\frac{m_1/m_H}{d/r_H} \tag{4.11}$$

These equations reveal that the presence of G conceals a normalization relationship for mass m_1 and distance d with respect to the Hubble mass m_H and the Hubble radius r_H, respectively. Put another way, the mass gradients m_1/d and m_H/r_H are both contributing to the gravitational potential functions V_{ge} and V_{gm}. The calculation results obtained from these functions would consequently change inversely proportional with an alteration in the mass density of our local Hubble sphere, in case such a density change would not also affect the quantities of length, time, light speed or mass - which might be the case though (see section 6 for more details).

The gravitational potential of a static Schwarzschild black hole is $-c^2/2$ according to equation 4.4. Since our local Hubble sphere may qualify as Schwarzschild black hole this in turn suggests that the overall universe, which contains our local Hubble sphere, is a rotating extreme Kerr black hole which has a gravitational potential of $-c^2$. This notion is gaining some support from the possibility to express Hubble's constant as an angular frequency:

$$H_0 \cong 2.41 \times 10^{-18}\ \text{rad/s} \tag{4.12}$$

Frequencies are always linked to some kind of energy and thus it must be possible to calculate the Hubble sphere's mass and energy by explicitly using the Hubble constant H_0. Using equation 2.8 and 4.2 the Hubble mass m_H can indeed be calculated from H_0 as follows

$$m_H = m_s(r_s = r_H) = \frac{r_H}{2}\frac{c^2}{G} = \frac{\hbar}{2}\frac{1}{H_0\, l_l^2} \tag{4.13}$$

whereat the so called Planck length $l_l = \sqrt{\hbar G/c^3} = 1.62 \times 10^{-35}$ m appears naturally in the resulting equation as well as Planck's constant h. The corresponding Hubble energy E_H can be expressed as follows

$$E_H = m_H c^2 = \frac{\hbar}{2}\frac{1}{H_0\, t_l^2} = \frac{E_l}{4\pi}\frac{t_H}{t_l} = \frac{E_l}{2}\frac{r_H}{l_l} \tag{4.14}$$

whereby $t_l = l_l/c = 5.4 \times 10^{-44}$ s is the Planck time, $t_H = 2\pi/H_0 = 2.61 \times 10^{18}$ s is the rotation period of H_0 when interpreted as angular frequency and $E_l = c\hbar/l_l$ denotes the so called Planck energy.
The natural appearance of the super short Planck length and Planck time in equation 4.13 and 4.14 suggests that the cosmological scale is linked to the quantum scale via the Planck units - a notion which is explored further in section 6.

5 The Proton and c²

A remarkable coincidence appears when dividing the gravitational potential of $-c^2$ by the hypothetical gravitational potential of a proton with mass m_p.

$$\vartheta = c^2 \Big/ \left(G\frac{m_p}{r_p}\right) = 1.693 \times 10^{38} \cong \sqrt{\exp(1)} \times 10^{38} \tag{5.1}$$

Since contemporary physics claims that the strong force is about 10^{38} times stronger than the gravitational force the last equation suggests that the strong force is actually the near field behaviour of gravity *(note: the same conclusion was also reached in (1) with a different calculation approach)*. This result was achieved by using the proton's reduced Compton wavelength $r_p = \hbar/(m_p\, c)$ as particle radius but using the experimental proton radius of 0.842 fm does not affect this result too since both values are related by a factor of 4.0, a difference which is negligible compared to 10^{38}. Moreover, equation 5.1 implies that the proton's gravitational potential energy per unit mass is seemingly $-c^2$ in close vicinity and thus the proton should qualify as an extreme Kerr black hole that rotates with light speed c at its equatorial ring. This presumption can be succinctly expressed as follows:

$$m_p = \frac{m_k(r_k = r_p)}{\vartheta} \tag{5.2}$$

The appearance of the approximate square root of Euler's number $\exp(1)$ in equation 5.1 indicates that the value of the proton mass m_p is not coincidental. Surprisingly, applying the natural logarithm function $\ln$ to ϑ even results in an integer number (to 3 significant figures):

$$\ln(\vartheta) = \ln\left(\frac{m_k(r_k = r_p)}{m_p}\right) = 88.0 \tag{5.3}$$

This result is clearly related to the material presented hereafter in section 7, in particular equation 7.2 and 7.19.

6 Quantized Space

Quantum physics revealed that physical quantities are often discrete and therefore it also makes sense to assume that space is not infinitely divisible. A sensible candidate for the smallest possible length in our universe is the so called Planck length l_l which already appeared in equation 4.13.

$$l_l = \sqrt{\frac{G}{c^2}\frac{\hbar}{c}} = 1.62 \times 10^{-35}\ \text{m} \tag{6.1}$$

Interestingly, a hypothetical extreme Kerr black hole which has a radius of one Planck length l_l and rotates with light speed c at its equatorial ring would have a mass of one Planck mass m_l:

$$m_l = m_k(r_k = l_l) = \sqrt{\frac{c^2}{G}\frac{\hbar}{c}} = 2.18 \times 10^{-8}\ \text{kg} \tag{6.2}$$

The Planck mass can also be used to define the Planck energy $E_l = m_l c^2$. It was already suggested in (1) that the Planck units define the properties of the proposed quanta of space. This proposal makes sense since equation 6.1 and 6.2 only contain fundamental quantities, i.e. light speed c, Planck's constant h, the gravitational constant G and the gravitational potential limit c^2, whereby both equations contain a G/c^2 term which also suggests a relationship to the Hubble sphere according to equation 4.8. This leads to the notion that every single quanta of space contains, or mirrors, the characteristic properties of our universe in its own "micro-verse" *(moreover, each space quanta presumably possesses a positive or negative Planck charge)*. If this notion is appropriate then equation 2.9 is not only valid on cosmological scales but also for the quantized structure of space itself, which leads to the following equalities:

$$\frac{c^2}{G} = \frac{m_k}{r_k} = \frac{m_l}{l_l} \tag{6.3}$$

$$\frac{c^4}{G} = \frac{m_k c^2}{r_k} = \frac{E_l}{l_l} \tag{6.4}$$

Consequently, equation 2.5 and 4.10, which describe gravitational potential energy per unit mass, as well as equation 2.6 and 4.11, which describe gravitational potential energy per unit energy, can also be expressed without the gravitational constant G as follows:

$$V_{gm} = -G\frac{m_1}{d} = -c^2\frac{m_1/m_l}{d/l_l} \tag{6.5}$$

$$V_{ge} = -\frac{G}{c^4}\frac{E_1}{d} = -\frac{E_1/E_l}{d/l_l} = -\frac{m_1/m_l}{d/l_l} \tag{6.6}$$

The last two equations demonstrate that gravitational potential can also be defined using only the properties of local space, which elegantly solves the problem how all the masses in our observable universe can influence local gravitational interactions. Colloquially speaking: as above so below.

The proposed "mirroring" becomes even more apparent when expressing the Planck length and Planck mass in terms of Hubble sphere parameters (*please note that the following two equations are only exact in case our Hubble sphere really qualifies as a Schwarzschild black hole*):

$$l_l = \sqrt{\frac{1}{2}\frac{\hbar}{c}\frac{r_H}{m_H}} \tag{6.7}$$

$$m_l = \sqrt{2\frac{\hbar}{c}\frac{m_H}{r_H}} \tag{6.8}$$

These two relations suggest that a change in our Hubble sphere's density can result in a different Planck length and Planck mass, but a definitive conclusion on that matter is difficult since light speed c and Planck's constant h may also be affected by such a density change.

The term c^4/G, which has the physical unit of force, has even more sensible physical meanings than previously discussed. This becomes obvious when dissecting that term into an acceleration part and a mass part:

$$\frac{c^4}{G} = a \times m = \frac{c^2}{r_k}m_k = \frac{c^2}{l_l}m_l \tag{6.9}$$

The last equation has physical meaning for the macro as well as the micro scale of space.

- On the macro scale: objects can be disintegrated by spinning them up to the point where they can overcome their gravitational self attraction when flying apart. For everyday objects this is not a surprising feature but this process is theoretically also possible with large objects. For example, if a planet could be spun up enough it could disperse itself in space permanently when finally breaking apart. The same process could theoretically be tried with a Schwarzschild black hole which would gradually turn into an extreme Kerr black hole until its equatorial ring velocity reaches light speed c and its centripetal acceleration becomes c^2/r_k, but since light speed c cannot be exceeded it is therefore not possible to destroy an extreme Kerr black hole through spinning it up. Thus for a spherical object with an arbitrary radius r_k the maximum possible centripetal acceleration is given by c^2/r_k and the upper limit for that object's mass is m_k.
- On the micro scale: The acceleration term $c^2/l_l = 5.56 \times 10^{51}\,\mathrm{m/s^2}$, which is also known as the Planck acceleration a_l, presumably denotes the maximum possible rotational & translational acceleration in our universe. The case is clear for rotation: there is a limit for centripetal acceleration which results from the circular motion equation v^2/r and the meaning of c as well as l_l as limit for their respective domain. For comprehending maximum translational acceleration it is necessary to realize that the minimal amount of time needed to travel the fundamental distance l_l is $t_l = l_l/c$. Consequently, the maximum possible translational acceleration is also given by:

$$a_l = \frac{\delta v}{\delta t} = \frac{c - 0\,\mathrm{m/s}}{l_l/c} = \frac{c^2}{l_l} \tag{6.10}$$

Thus every time G/c^4 appears in an equation its usage either conceals a normalization with respect to some macro limits of space-time, i.e. c^2/r_k and m_k, or alternatively a normalization with respect to the acceleration limit a_l and the presumed mass of a quanta of space, i.e. the Planck mass m_l.

7 Logarithmic Relations

Hartmut Müller brought up the idea that the mass ratios of fundamental particles are given by exponential relationships of Euler's number $\exp(1)$, whereby the relation's exponent is always close to an integer number or an integer number plus one half. For example, proton mass m_p and electron mass m_e are related as follows,

$$\ln\left(\frac{m_p}{m_e}\right) = 7.5 \qquad \text{or} \qquad m_e \times \exp(7) \times \sqrt{\exp(1)} \cong m_p \tag{7.1}$$

whereby all logarithm results which are presented in this section are rounded to one decimal place. Müller thinks that exponential Euler number relationships which follow the stated exponent scheme avoid destructive gravitational resonance, because $\exp(1)$ is an irrational as well as transcendental number, and this adherence then in turn stabilizes fundamental particles and even planetary orbits (7). Müller also showed that such Euler number relationships exist with respect to the Planck mass m_l:

$$\ln\left(\frac{m_l}{m_p}\right) = 44.0 \qquad \text{or} \qquad m_p \times \exp(44) \cong m_l \tag{7.2}$$

$$\ln\left(\frac{m_l}{m_e}\right) = 51.5 \qquad \text{or} \qquad m_e \times \exp(51) \times \sqrt{\exp(1)} \cong m_l \tag{7.3}$$

Due to the properties of logarithms the last three equations are related numerically, i.e. $51.5 - 44.0 = 7.5$. This has been a small selection of Müller's findings. In addition to the presented examples it can be shown that the Hubble sphere also has similar exponential relationships to the proton, electron and Planck mass:

$$\ln\left(\frac{m_H}{m_l}\right) = 139.5 \qquad \text{or} \qquad m_l \times \exp(139) \times \sqrt{\exp(1)} \approxeq m_H \tag{7.4}$$

$$\ln\left(\frac{m_H}{m_p}\right) = 183.5 \qquad \text{or} \qquad m_p \times \exp(183) \times \sqrt{\exp(1)} \approxeq m_H \tag{7.5}$$

$$\ln\left(\frac{m_H}{m_e}\right) = 191.0 \qquad \text{or} \qquad m_e \times \exp(191) \approxeq m_H \tag{7.6}$$

These results have intervals which match with earlier results, i.e. $191.0 - 183.5 = 7.5$, $183.5 - 139.5 = 44.0$ and $191.0 - 139.5 = 51.5$.

Surprisingly, similar exponential relationships exist for the proton and electron with respect to the Sommerfeld constant $\alpha \cong 1/137.036$, whereby these relationships also exhibit integer or integer plus one half exponents:

$$\log_\alpha\left(\frac{m_e}{m_p}\right) = \log_\alpha\left(\frac{r_p}{r_e}\right) = 1.5 \qquad \text{or} \qquad m_p \times \alpha \times \sqrt{\alpha} \cong m_e \tag{7.7}$$

Here r_p and r_e denote the reduced Compton wavelength for the proton and electron, i.e. $r_p = \hbar/(m_p\, c)$ and $r_e = \hbar/(m_e\, c)$. Interestingly, these wavelengths exhibit logarithmic relationships of α with hydrogen's radius a_o, aka the Bohr radius, as well as the Hubble radius r_H:

$$\log_\alpha\left(\frac{r_e}{a_0}\right) = 1.0 \qquad \text{or} \qquad a_0 \times \alpha = r_e \tag{7.8}$$

$$\log_\alpha\left(\frac{r_p}{a_0}\right) = 2.5 \qquad \text{or} \qquad a_0 \times \alpha^2 \times \sqrt{\alpha} \cong r_p \tag{7.9}$$

$$\log_\alpha\left(\frac{r_e}{r_H}\right) = 18.0 \qquad \text{or} \qquad r_H \times \alpha^{18} \approxeq r_e \tag{7.10}$$

$$\log_\alpha\left(\frac{r_p}{r_H}\right) = 19.5 \qquad \text{or} \qquad r_H \times \alpha^{19} \times \sqrt{\alpha} \approxeq r_p \tag{7.11}$$

Further noteworthy $\log_\alpha$ relationships exist with respect to the Planck mass m_l and Planck length l_l:

$$\log_\alpha\left(\frac{m_e}{m_l}\right) = \log_\alpha\left(\frac{l_l}{r_e}\right) = 10.5 \qquad \text{or} \qquad m_l \times \alpha^{10} \times \sqrt{\alpha} \approxeq m_e \tag{7.12}$$

$$\log_\alpha\left(\frac{m_p}{m_l}\right) = \log_\alpha\left(\frac{l_l}{r_p}\right) = 8.9 \qquad \text{or} \qquad m_l \times \alpha^{9} \approxeq m_p \tag{7.13}$$

These results are also related numerically, i.e. $2.5 - 1.0 = 19.5 - 18.0 = 10.5 - 9 = 1.5$. Another logarithmic relationship which should not be missing in this line-up is the exact $\sqrt{\alpha}$ relationship of the fundamental charge e with the Planck charge $q_l = \sqrt{2\epsilon_0 hc} = 1.88 \times 10^{-18}$ C:

$$\log_\alpha\left(\frac{e}{q_l}\right) = 0.5 \qquad \text{or} \qquad q_l \times \sqrt{\alpha} = e \tag{7.14}$$

Comparing the presented $\log_\alpha$ relations with the $\ln$ relations, e.g. equation 7.1 with 7.7, begs the question if Euler's number is related mathematically to the Sommerfeld constant. Their direct logarithmic relation is $\log_\alpha(\exp(1)) = 7.5/1.5 = 1/5 = 0.2$ with a deviation of around 1.6%. Further interesting relations are $\log_{10}(\exp(1)/\sqrt{\alpha}) = 1.5$, $\log_{\sqrt{2}}(\exp(1)/\sqrt{\alpha}) = 10.0$ and $\ln(\sqrt{2}/\sqrt{\alpha}) = 2.8 \cong 2\sqrt{2}$, whereby these three relations all have a deviation around 0.2%. Although not mathematically exact those relationships might still have physical relevance.

The proton has even more noteworthy exponential relationships which is indicative of its extraordinary role in our universe. Besides its exponential relations with $\exp(1)$ and α it also exhibits an exponential relationship with $\sqrt{2}$ which again follows the previously mentioned exponent scheme.

$$\log_{\sqrt{2}}\left(\frac{m_l}{m_p}\right) = \log_{\sqrt{2}}\left(\frac{r_p}{l_l}\right) = 127.0 \quad \text{or} \quad m_p \times \sqrt{2}^{127} \cong m_l \tag{7.15}$$

$$\log_{\sqrt{2}}\left(\frac{m_H}{m_p}\right) = 529.5 \quad \text{or} \quad m_p \times \sqrt{2}^{529} \times \sqrt{\sqrt{2}} \cong m_H \tag{7.16}$$

$$\log_{\sqrt{2}}\left(\frac{r_H}{r_p}\right) = 277.5 \quad \text{or} \quad r_p \times \sqrt{2}^{277} \times \sqrt{\sqrt{2}} \cong r_H \tag{7.17}$$

The fact that the results of all the logarithms which were presented in this section are close to an integer number, or an integer number plus one half, cannot be the result of chance and is certainly indicative of an underlying physical mechanism that presumably is related to gravitational anti-resonance.

Moreover, there are further noteworthy logarithmic relations which involve the square root of two:

$$\ln\left(\frac{529.5}{127.0}\right) = \ln\left(\frac{183.5}{44.0}\right) = 1.428 \cong \sqrt{2} \tag{7.18}$$

$$m_l \times \sqrt{2}^{127} \cong m_k(r_k = r_p) = m_p \times \vartheta \cong 2\sqrt{2} \times 10^{11}\ \text{kg} \tag{7.19}$$

Similar relations even show up in the time domain whereby $t_l = l_l/c$, $t_H = 2\pi/H_0$ and $t_p = 2\pi/\omega_p = r_p/(2\pi c)$.

$$\ln\left(\frac{t_H}{t_l}\right) = 142.0 \approxeq \sqrt{2} \times 10^2 \tag{7.20}$$

$$\ln\left(\frac{t_H}{t_l}\right) \approxeq \sqrt{2} \times \ln\left(\frac{t_H}{t_p}\right) \tag{7.21}$$

8 Large Number Coincidences

The previous sections proposed that the very large and the very small are interconnected. Weyl and Dirac were among the first physicists who noted numerical coincidences in certain fundamental ratios that curiously also evaluated to extremely large numbers, which led them to the presumption that these coincidences cannot be the result of chance. Dirac, for example, provided the following coincidence

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} = 2.269 \times 10^{39} \tag{8.1}$$

$$\frac{r_H}{4 r_p} = 1.48 \times 10^{41} \tag{8.2}$$

whereby both results are in the vicinity of 10^{40}. Here m_p denotes the proton's mass, r_p denotes the proton's reduced Compton wavelength, m_e denotes the electron's mass, e denotes the fundamental electric charge and ϵ_0 denotes the electric field constant. The last two equations can be brought into much closer alignment when replacing e with the Planck charge q_l and doubling the equation which involves r_H.

$$\frac{q_l^2}{4\pi\epsilon_0 G m_p m_e} = \frac{e^2}{4\pi\epsilon_0 \alpha G m_p m_e} = 3.1 \times 10^{41} \tag{8.3}$$

$$\frac{1}{2}\frac{r_H}{r_p} = 3.0 \times 10^{41} \tag{8.4}$$

These two results are remarkably close, although the correct value for the Hubble radius r_H is still somewhat in dispute. Furthermore, using the reduced Compton wavelength of the proton and electron it's possible to reformulate equation 8.3 into something surprisingly simple:

$$\frac{q_l^2}{4\pi\epsilon_0 G m_p m_e} = \frac{r_p r_e}{l_l^2} \tag{8.5}$$

Relating equation 8.4 with 8.5 then leads to a geometric relationship which demonstrates that the properties of the most fundamental objects in our universe are related to each other in an orderly fashion:

$$\frac{r_p}{l_l} \cong \sqrt{\frac{1}{2}\frac{r_H}{r_e}} \tag{8.6}$$

It's worth emphasizing that this remarkable connection can only be obtained when utilizing the Planck units. Moreover, it is also possible to express the last equation in terms of frequencies using equation 4.2 and the angular frequency relationship $\omega = c/r$:

$$\frac{\omega_l}{\omega_p} \cong \sqrt{\frac{1}{2}\frac{\omega_e}{H_0}} \tag{8.7}$$

This relation features the Hubble constant H_0 again, which can also be expressed as an angular frequency.

9 Discussion

The previous sections uncovered several noteworthy physical relationships which seemingly contradict general relativity theory. These conflicts mainly revolve around the following three issues:

1. What is the meaning of the Einstein constant?
2. Is space curvature physically real and do gravitational singularities exist?
3. How can dark energy and dark matter be incorporated into the presented ideas?

Regarding issue 1: it can be argued that general relativity theory should have got rid of the gravitational constant G since its usage leads to the situation that general relativity theory predicts black holes but unknowingly already uses the physical reality of black holes in disguise of a G/c^4 term, which is contained in the Einstein constant κ (equation 2.7), and this circumstance makes general relativity theory somewhat circular conceptually. Einstein also would have liked general relativity theory to be more in line with Ernst Mach's thinking, i.e. that local gravity is related to the relative relationships with all the masses in our universe. Replacing the Einstein constant κ with Hubble sphere parameters (see equation 4.9) would have been a step in that direction, but back then there was much less knowledge about cosmology. Einstein also was not able to unify general relativity theory with quantum physics. It was proposed in this paper that the Planck units are the key to this unification, as the Planck units define the properties of the quanta of space, which essentially also are micro black holes that implicitly "encode" the gravitational constant. This notion ultimately leads to a thermodynamic understanding of quantum gravity, whereby thermodynamic gravity is not dealing with the states of atoms or molecules but instead it operates on basis of the proposed quanta of space (1).

Regarding issue 2: space curvature is a consequence of the modelling approach chosen by general relativity theory and its physical reality is not proven unequivocally, which also implies that gravitational singularities may not be physically real. Few people know that Albert Einstein was initially conceiving a gravitational theory with a variable speed of light which doesn't require curved space (2). This approach doesn't necessarily violate special relativity theory either, as Alexander Unzicker notes in his book "Einstein's lost key". Light speed c would still be a general upper limit in the absence of a gravitational field and (non-accelerating) observers in an approximately homogeneous gravitational field would all measure the same speed of light. These observers would not even be aware of the reduced speed of light because time is also slowed down correspondingly in their gravitational field. However, in a varying gravitational field the subsequent change in the speed of light results in a bent trajectory. Back in the day Einstein was even able to derive an optical refraction index for light whose trajectory is bent by a central mass m which was only off from the correct result by a factor of two. Therefore it's rather incomprehensible why Einstein did not pursue this idea again later. Dicke, however, was able to derive the correct refraction index n_l in 1957 with a different calculation approach (5), seemingly without knowing Einstein's prior work on that topic (see Unzicker's book for details).

$$n_l = 1 + \frac{G}{c^2}\frac{2m}{d} = 1 + \frac{G}{c^4}\frac{2mc^2}{d} \qquad \text{result in range } [1,2] \text{ for static spheres} \tag{9.1}$$

Since the optical refraction index is defined as $n = c/v$ and $n_l \geqq 1$ the speed of light decreases to $v_l = c/n_l$ in the vicinity of a uniform spherical mass, whereby the predicted slowdown is still small close to our sun (less than 0.4%). Moreover, it's obvious that the last equation is related to the gravitational potential analysis that is presented in this paper since the occurring mathematical terms are very similar (see equation 2.6, for example). In case equation 9.1 doesn't apply to rotating black holes the theoretical maximum value of n_l evaluates to 2 in close vicinity to a static Schwarzschild black hole using equation 2.8. If equation 9.1 is also valid for rotating black holes then the theoretical maximum value of n_l would be 3.
Dicke was remarkably far sighted as he even speculated that the $1+$ part of equation 9.1 may be related to

the gravitational potential of our universe itself since the second term was identified by him as a gravitational potential whose value is also much smaller than 1 for any realistic application. Like it was the case in section three the number 1 may actually represent the gravitational potential limit $1\,\mathrm{J/J}$, which, in accordance with Dicke's presumption, gives equation 9.1 the meaning of a sum of gravitational potential magnitudes.
In addition to these findings a scientific paper from 1960 demonstrates that the mathematical framework of general relativity is not necessary to describe undeniable gravitational effects like gravitational lensing (4). The notion of time in general and special relativity theory is also problematic since no satisfying justification is given why time should be treated like a spatial dimension. Considering all these arguments it seems reasonable that the physical relationships which are presented in this work do not require the mathematics associated with space curvature.

Regarding issue 3: it has been shown that the gravitational potential energy per unit mass of our local Hubble sphere is $-c^2/2$ whereas the general gravitational potential limit is $-c^2$. The latter potential value is presumably related to a rotating universe in which our non-rotating but expanding local Hubble sphere is embedded in. In case this notion is appropriate the potential difference is due to our overall universe's rotational energy, which in turn should account for the major portion of the so called dark energy. The remaining dark energy portion could be related to the expansion of our Hubble sphere and/or to its translational kinetic energy in case it moves relative to the enclosing overall universe. Dark matter, on the other hand, might have a quite unspectacular explanation as Randell Mills suggests that dark matter is just made from interstellar clouds of a rather unreactive form of hydrogen which he calls hydrino (8).

Critics may argue that this paper represents the same equations in different ways, but this is unavoidable because there is an interconnected and repeating underlying physical core mechanism that can be viewed from different perspectives and at different scales, but which still leads to similar physical expressions. The starkest example of this interconnectedness is the uncovered relationship between the strong force and gravity (see section 5).

10 Conclusions

The main findings of this paper are:

- The gravitational potential energy per unit mass of our overall universe is $-c^2$ and this value also constitutes the general gravitational potential limit. This limit can also be expressed as gravitational potential energy per unit energy with a value of $-1\,\mathrm{J/J}$.
- Newtonian gravity already predicts rotating black holes when also considering the velocity limit of light speed c as well as a gravitational potential per unit mass between $-c^2/2$ and $-c^2$, whereby a gravitational potential of exactly $-c^2/2$ corresponds to a static black hole.
- The Newtonian gravity equation is not limited to calculations involving mass since it can be adapted to energy using $m = E/c^2$.
- Our local Hubble sphere might be a Schwarzschild black hole, or it is at least close to being one. Its gravitational potential per unit mass is subsequently close to $-c^2/2$.
- The gravitational constant G is related to all the masses in our observable universe as suspected by Dicke, Mach, Sciama and Schrödinger.
- The gravitational constant G can be regarded as an emergent constant that follows from from our Hubble sphere's energy density and it's gravitational potential per unit mass of $-c^2/2$.
- Gravitational effects on cosmological scales can be explained without space curvature. This suggests that space is perfectly flat, like Cartesian space, and that gravitational singularities don't exist.
- Space-time is quantized and mirrors cosmological properties on the quantum scale and vice versa.
- The properties of the quanta of space are given by the Planck units.
- The sizes and the masses of the fundamental objects in our universe are not coincidental. Underlying patterns exist, in particular various logarithmic relationships and a fractional relation were presented.
- The strong force is the disguised near field behaviour of gravity.
- Physical units are an important tool in physics and should not be omitted in calculations for brevity.
- Don't get lost in math and remember that physical theories are abstractions of the physical reality. This statement is especially true for all theories which emphasize mathematical fields and frames.

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