

## Proof of Goldbach Conjecture

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### Abstract

In this paper, the author explores the proof of Goldbach Conjecture.

### Objective:

If  $\{1$  is also a prime number $\}$  is true, then any even number greater than 0 can be written as the sum of two prime numbers.

### Method:

Triangular lattice

### Result:

An even  $a$  can be written as  $T(a)$  sums of two prime numbers

$$T(a) \sim \left( \frac{a/2}{2} - \frac{a/2}{\ln(a/2)} \right) \left( \frac{(a-1)/2 - (a-1)}{\ln(a-1)} - \frac{(a/2-1)/2 - (a/2-1)}{\ln(a/2-1)} \right) - \left( \frac{(a/2-1)/2 - (a/2-1)}{\ln(a/2-1)} \right) \left( \frac{(a-3)/2 - (a-3)}{\ln(a-3)} - \frac{(a/2-2)/2 - (a/2-2)}{\ln(a/2-2)} \right) \left( \frac{(a/4)/2 - (a/4)}{\ln(a/4)} \right) \left( \frac{(3*a/4)/2 - (3*a/4)}{\ln(3*a/4)} - \frac{(a/2)/2 - (a/2)}{\ln(a/2)} \right) \left( \frac{(a/4)/2 - (a/4)}{\ln(a/4)} \right) \left( \frac{(3*a/4)/2 - (3*a/4)}{\ln(3*a/4)} - \frac{(a/2)/2 - (a/2)}{\ln(a/2)} \right) + \left( \frac{(a/2)/2 - (a/2)}{\ln(a/2)} - \frac{(a/4)/2 - (a/4)}{\ln(a/4)} \right) \left( \frac{a/2 - a}{\ln(a)} - \frac{(3*a/4)/2 - (3*a/4)}{\ln(3*a/4)} \right) + \left( \frac{(a/2-1)/2 - (a/2-1)}{\ln(a/2-1)} \right) \left( \frac{(a/2-1)/2 - (a/2-1)}{\ln(a/2-1)} + 1 \right) / 2 - \left( \frac{(a/2-2)/2 - (a/2-2)}{\ln(a/2-2)} \right) \left( \frac{(a/2-2)/2 - (a/2-2)}{\ln(a/2-2)} + 1 \right) / 2 + a / \ln(a) - a/4$$

### Conclusions:

If  $\{1$  is also a prime number $\}$  is true, then any even number greater than 0 can be written as the sum of two prime numbers.

If  $\{1$  is also a prime number $\}$  is true, then any even number greater than 2 can be written as the sum of two different prime numbers.

If  $\{1$  is also a prime number $\}$  is false, then any even number greater than 5 can be written as the sum of two different prime numbers.

Key words: Goldbach; Euler.

## 1 Structure

### 1.1 Concept

Set of natural numbers is denoted as  $N$ ,  $N = \{n\}$ .

If one variable belongs to  $N$ , then it is denoted as  $n$ .

If two variables belong to  $N$ , then they are denoted as  $n_1$  and  $n_2$ .

Set of even numbers is denoted as  $A$ ,  $A = \{a | a = 2 * n\}$ .

If one variable belongs to  $A$ , then it is denoted as  $a$ .

If two variables belong to  $A$ , then are denoted as  $a_1$  and  $a_2$ .

Set of odd numbers is denoted as  $B$ ,  $B = \{b | b = 2 * n + 1\}$ .

If one variable belongs to  $B$ , then it is denoted as  $b$ .

If two variables belong to  $B$ , then they are denoted as  $b_1$  and  $b_2$ .

Set of odd composite numbers is denoted as  $C$

$C = \{c | c = (2 * n_1 + 1) * (2 * n_2 + 1), n_1 \text{ is not } 0 \text{ and } n_2 \text{ is not } 0.\}$

If one variable belongs to  $C$ , then it is denoted as  $c$ .

If two variables belong to  $C$ , then they are denoted as  $c_1$  and  $c_2$ .

Set of prime numbers is denoted as  $D$

$D = \{d | d \text{ belongs to } B \text{ and } d \text{ does not belong to } C\}$

If one variable belongs to  $D$ , then it is denoted as  $d$ .

If two variables belong to  $D$ , then they are denoted as  $d_1$  and  $d_2$ .

### 1.2 $N(a)$

$a = a/2 + a/2$ ,  $a > 0$ .

If  $a/2$  belongs to  $A$ , then  $a = [(a/2 - 1) - 2n] + [(a/2 + 1) + 2n]$

$n < (a - 2)/4$ ,  $\text{Card}(n) = a/4$ .

$(a/2 - 1) - 2n$  is denoted as  $b_L$ ,  $(a/2 + 1) + 2n$  is denoted as  $b_R$ .

If  $a/2$  belongs to  $B$ , then  $a = (a/2 - 2n) + (a/2 + 2n)$

$n < a/4$ ,  $\text{Card}(n) = (a + 2)/4$ .

$a/2 - 2n$  is denoted as  $b_L$ ,  $a/2 + 2n = b_2$  is denoted as  $b_R$ .

$\text{Card}(n)$  is one function of  $a$ , it is denoted as  $N(a)$ .

### 1.3 $e = b_R - b_L$

Set one increasing positive even sequence, it corresponds to  $\{a | a > 0\}$ .

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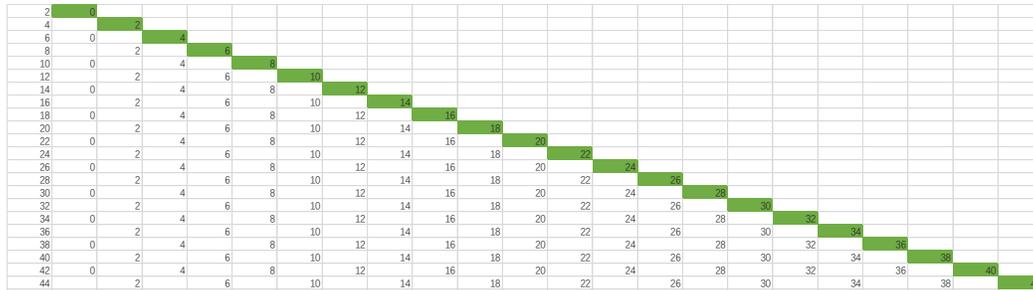
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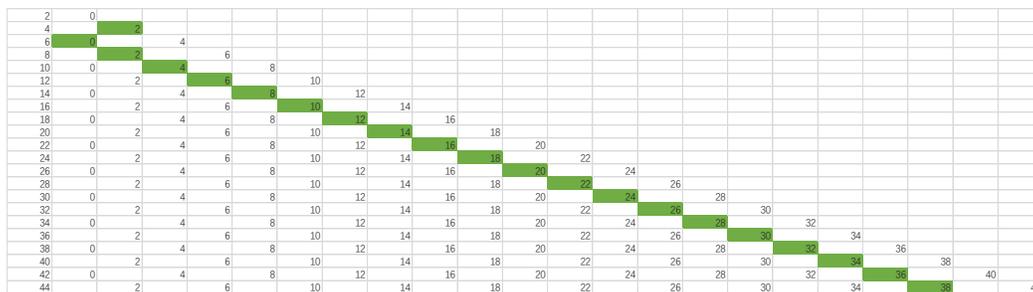


e is one function of a when g is invariable, any odd composite number in (0, a) corresponds to one cell in {L=a}. Equation is  $e=|(a-g)-g|$ ,  $a>g$ .

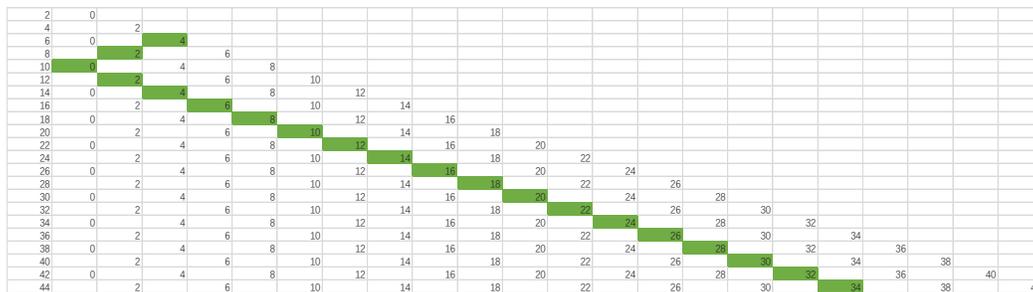
$e=|(a-1)-1|$ ,  $a>1$ .



$e=|(a-3)-3|$ ,  $a>3$ .



$e=|(a-5)-5|$ ,  $a>5$ .



...

1.5  $U(a)-T(a)=S(a)-N(a)$

If bL or bR belongs to C, then color the cell.



If bL and bR belong to C, then color it black.



## 2.1 $H(a) \sim H(a-2)$

Maximum error is denoted as  $Or(a)$ ,  $Or(a) \sim 0$  when  $a > a_0$ .

$M = \{(cL, cR) | cL + cR \text{ belongs to } (0, a]\}$ ,  $Card(cL, cR)$  is denoted as  $M(a)$ .

$M(a) = X(a) + Y(a)$ ,  $U(a) = M(a) - M(a-2)$ .

Let  $U(a) = S(a) - N(a)$ ,  $H(a) \sim (S(a) - N(a) - X(a) + X(a-2)) / (Y(a) - Y(a-2))$ .

$H(a) \sim (a/4 - a/\ln(a) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * ((a/2-1)/2 - (a/2-1)/\ln(a/2-1) + 1)/2 + ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)) * ((a/2-2)/2 - (a/2-2)/\ln(a/2-2) + 1)/2 / (((a/2)/2 - (a/2)/\ln(a/2)) * (((a-1)/2 - (a-1)/\ln(a-1)) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1))) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * (((a-3)/2 - (a-3)/\ln(a-3)) - ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)))$

Let  $U(a) = S(a) - N(a) + 1$ ,  $H(a) \sim (S(a) - N(a) + 1 - X(a) + X(a-2)) / (Y(a) - Y(a-2))$ .

Let  $U(a) = S(a) - N(a) + 2$ ,  $H(a) \sim (S(a) - N(a) + 2 - X(a) + X(a-2)) / (Y(a) - Y(a-2))$ .

## 2.2 $H(a) \sim J(a) / (J(a) + K(a))$

Maximum error is denoted as  $Oe(a)$ ,  $Oe(a) \sim (W(a) - (J(a) + K(a))) / W(a) \sim 1/2$ .

$J = \{(cL, cR) | cL \text{ belongs to } (0, a/4] \text{ and } cR \text{ belongs to } (a/2, 3*a/4]\}$ ;

$Card(cL, cR)$  is denoted as  $J(a)$ ,  $J(a) = S(a/4) * (S(3*a/4) - S(a/2))$ .

$K = \{(cL, cR) | cL \text{ belongs to } (a/4, a/2] \text{ and } cR \text{ belongs to } (3*a/4, a]\}$ ;

$Card(cL, cR)$  is denoted as  $K(a)$ ,  $K(a) = (S(a/2) - S(a/4)) * (S(a) - S(3*a/4))$ .

$J(a) / (J(a) + K(a)) \sim (((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - (a/2)/\ln(a/2))) / (((((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - (a/2)/\ln(a/2)))) + (((a/2)/2 - (a/2)/\ln(a/2)) - ((a/4)/2 - (a/4)/\ln(a/4))) * ((a/2 - a/\ln(a)) - ((3*a/4)/2 - (3*a/4)/\ln(3*a/4))))$

## 2.3 $H(a) \sim (J(a) + p_1 + p_2) / (J(a) + K(a) + p_1 + p_2 + q_1 + q_2)$

Maximum error is denoted as  $O_2(a)$ ,  $O_2(a) \sim 1/4$ .

$P_1 = \{(cL, cR) | cL \text{ belongs to } (0, a/8] \text{ and } cR \text{ belongs to } (3*a/4, 7*a/8]\}$ ;

$Card(cL, cR)$  is denoted as  $p_1$ ,  $p_1 = S(a/8) * (S(7*a/8) - S(3*a/4))$ .

$P_2 = \{(cL, cR) | cL \text{ belongs to } (a/4, 3*a/8] \text{ and } cR \text{ belongs to } (a/2, 5*a/8]\}$ ;

$Card(cL, cR)$  is denoted as  $p_2$ ,  $p_2 = (S(3*a/8) - S(a/4)) * (S(5*a/8) - S(a/2))$ .

$Q_1 = \{(cL, cR) | cL \text{ belongs to } (a/8, a/4] \text{ and } cR \text{ belongs to } (7*a/8, a]\}$ ;

$Card(cL, cR)$  is denoted as  $q_1$ ,  $q_1 = (S(a/4) - S(a/8)) * (S(a) - S(7*a/8))$ .

$Q_2 = \{(cL, cR) | cL \text{ belongs to } (3*a/8, a/2] \text{ and } cR \text{ belongs to } (5*a/8, 3*a/4]\}$ ;

$Card(cL, cR)$  is denoted as  $q_2$ ,  $q_2 = (S(a/2) - S(3*a/8)) * (S(3*a/4) - S(5*a/8))$ .

## 2.4 $H(a) \sim (J(a) + p_1 + \dots + p_6) / (J(a) + K(a) + p_1 + \dots + p_6 + q_1 + \dots + q_6)$

Maximum error is denoted as  $O_6(a)$ ,  $O_6(a) \sim 1/8$ .

$P_3 = \{(cL, cR) | cL \text{ belongs to } (0, a/16] \text{ and } cR \text{ belongs to } (7*a/8, 15*a/16]\}$ ;

$Card(cL, cR)$  is denoted as  $p_3$ ,  $p_3 = S(a/16) * (S(15*a/16) - S(7*a/8))$ .

$P_4 = \{(cL, cR) | cL \text{ belongs to } (a/8, 3*a/16] \text{ and } cR \text{ belongs to } (3*a/4, 13*a/16]\}$ ;

$Card(cL, cR)$  is denoted as  $p_4$ ,  $p_4 = (S(3*a/16) - S(a/8)) * (S(13*a/16) - S(3*a/4))$ .

$P_5 = \{(cL, cR) | cL \text{ belongs to } (a/4, 5*a/16] \text{ and } cR \text{ belongs to } (5*a/8, 11*a/16]\}$ ;

$Card(cL, cR)$  is denoted as  $p_5$ ,  $p_5 = (S(5*a/16) - S(a/4)) * (S(11*a/16) - S(5*a/8))$ .

$P_6 = \{(cL, cR) | cL \text{ belongs to } (3*a/8, 7*a/16] \text{ and } cR \text{ belongs to } (a/2, 9*a/16]\};$   
 Card(cL, cR) is denoted as  $p_6, p_6 = (S(7*a/16) - S(3*a/8)) * (S(9*a/16) - S(a/2)).$   
 $Q_3 = \{(cL, cR) | cL \text{ belongs to } (a/16, a/8] \text{ and } cR \text{ belongs to } (15*a/16, a]\};$   
 Card(cL, cR) is denoted as  $q_3, q_3 = (S(a/8) - S(a/16)) * (S(a) - S(15*a/16)).$   
 $Q_4 = \{(cL, cR) | cL \text{ belongs to } (3*a/16, a/4] \text{ and } cR \text{ belongs to } (13*a/16, 7*a/8]\};$   
 Card(cL, cR) is denoted as  $q_4, q_4 = (S(a/4) - S(3*a/16)) * (S(7*a/8) - S(13*a/16)).$   
 $Q_5 = \{(cL, cR) | cL \text{ belongs to } (5*a/16, 3*a/8] \text{ and } cR \text{ belongs to } (11*a/16, 3*a/4]\};$   
 Card(cL, cR) is denoted as  $q_5, q_5 = (S(3*a/8) - S(5*a/16)) * (S(3*a/4) - S(11*a/16)).$   
 $Q_6 = \{(cL, cR) | cL \text{ belongs to } (7*a/16, a/2] \text{ and } cR \text{ belongs to } (9*a/16, 5*a/8]\};$   
 Card(cL, cR) is denoted as  $q_6, q_6 = (S(a/2) - S(7*a/16)) * (S(5*a/8) - S(9*a/16)).$

2.5  $O_{126}(a) < 1/64$  when  $a > a_0$

$H(a) \sim (J(a) + p_1 + \dots + p_{14}) / (J(a) + K(a) + p_1 + \dots + p_{14} + q_1 + \dots + q_{14})$

Maximum error is denoted as  $O_{14}(a), O_{14}(a) \sim 1/16.$

$H(a) \sim (J(a) + p_1 + \dots + p_{30}) / (J(a) + K(a) + p_1 + \dots + p_{30} + q_1 + \dots + q_{30})$

Maximum error is denoted as  $O_{30}(a), O_{30}(a) \sim 1/32.$

$H(a) \sim (J(a) + p_1 + \dots + p_{62}) / (J(a) + K(a) + p_1 + \dots + p_{62} + q_1 + \dots + q_{62})$

Maximum error is denoted as  $O_{62}(a), O_{62}(a) \sim 1/64.$

$H(a) \sim (J(a) + p_1 + \dots + p_{126}) / (J(a) + K(a) + p_1 + \dots + p_{126} + q_1 + \dots + q_{126})$

Maximum error is denoted as  $O_{126}(a), O_{126}(a) < 1/64$  when  $a > a_0.$

2.6 Conclusions

$S(a) = Ch * (a/2 - a/\ln(a)), Ch \sim 1$  when  $a > a_0.$

Error of  $S(a) \sim a/2 - a/\ln(a)$  is denoted as  $O(a), O(a) \sim 0$  when  $a > a_0.$

When  $a > a_1, (J(a) + p_1 + \dots + p_{126}) / (J(a) + K(a) + p_1 + \dots + p_{126} + q_1 + \dots + q_{126}) - (S(a) - N(a) - X(a) + X(a-2)) / (Y(a) - Y(a-2)) > 1/64.$

And endorse when  $a$  belongs to  $(0, a_1]$

Conclusion: If  $\{1 \text{ is also a prime number}\}$  is true, then any even number greater than 0 can be written as the sum of two prime numbers.

When  $a > a_2, (J(a) + p_1 + \dots + p_{126}) / (J(a) + K(a) + p_1 + \dots + p_{126} + q_1 + \dots + q_{126}) - (S(a) - N(a) + 1 - X(a) + X(a-2)) / (Y(a) - Y(a-2)) > 1/64.$

And endorse when  $a$  belongs to  $(2, a_2]$

Conclusion: If  $\{1 \text{ is also a prime number}\}$  is true, then any even number greater than 2 can be written as the sum of two different prime numbers.

When  $a > a_3, (J(a) + p_1 + \dots + p_{126}) / (J(a) + K(a) + p_1 + \dots + p_{126} + q_1 + \dots + q_{126}) - (S(a) - N(a) + 2 - X(a) + X(a-2)) / (Y(a) - Y(a-2)) > 1/64.$

And endorse when  $a$  belongs to  $(5, a_3]$

Conclusion: If  $\{1 \text{ is also a prime number}\}$  is false, then any even number greater than 5 can be written as the sum of two different prime numbers.

2.7 T(a)

$$J(a)/(J(a)+K(a)) \sim (J(a)+P(a))/(J(a)+K(a)+P(a)+Q(a))$$

$$T(a) \sim (Y(a)-Y(a-2)) * J(a)/(J(a)+K(a)) + X(a) - X(a-2) - S(a) + N(a)$$

$$\begin{aligned} T(a) \sim & (((a/2)/2 - (a/2)/\ln(a/2)) * (((a-1)/2 - (a-1)/\ln(a-1)) - ((a/2-1)/2 - (a/2-1)/\ln(a/2-1))) - \\ & ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * (((a-3)/2 - (a-3)/\ln(a-3)) - ((a/2-2)/2 - (a/2-2)/\ln(a/2- \\ & 2))) * (((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - \\ & (a/2)/\ln(a/2))) / (((a/4)/2 - (a/4)/\ln(a/4)) * (((3*a/4)/2 - (3*a/4)/\ln(3*a/4)) - ((a/2)/2 - \\ & (a/2)/\ln(a/2))) + (((a/2)/2 - (a/2)/\ln(a/2)) - ((a/4)/2 - (a/4)/\ln(a/4))) * ((a/2 - a/\ln(a)) - \\ & ((3*a/4)/2 - (3*a/4)/\ln(3*a/4))) + ((a/2-1)/2 - (a/2-1)/\ln(a/2-1)) * ((a/2-1)/2 - (a/2-1)/\ln(a/2- \\ & 1) + 1) / 2 - ((a/2-2)/2 - (a/2-2)/\ln(a/2-2)) * ((a/2-2)/2 - (a/2-2)/\ln(a/2-2) + 1) / 2 + a/\ln(a) - a/4 \end{aligned}$$