Geometric Derivation of Quarks Magnetic Moments in Nucleons. Quark mass / Nucleon mass $\stackrel{?}{=}$ fine–structure constant

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Abstract: Spatial orientation and magnitude of quarks magnetic moments in nucleons are derived from geometric considerations. Magnetic moments for d and u quarks are estimated respectively at $11.5e^{-26}$ and $23e^{-26}$ J/T. From these findings, quark mass is found to be $\approx 1.22e^{-29}$ Kg (6.85 MeV) $\cong m_N\alpha$ where m_N is the nucleon mean rest mass and α the fine-structure constant.

1. Introduction

From the status of mathematical curiosity in the early 1960s to integration in the Standard Model of particle physics, quarks have been the subject of extensive research aiming at elucidating the fundamental nature of baryonic matter. However, and since 1977, all attempts to observe free quarks have failed [1]. This failure may be pointing out that quarks are not self-existent entities, but merely arising inside nucleons through internal energy rearrangement for nucleon-nucleon (NN) binding purposes. This process would be reversible, as suggested for example by the neutron β decay [2]. This conjecture would account for the non-detection of free quarks in high-energy particle collision experiments.

Nonetheless, inside nucleons quarks appear to behave like physical entities with intrinsic particle properties such as spin (S=± 1/2), fractional charge (Q=+2e/3, – e/3), mass (m_Q), or magnetic moment (μ_D and μ_U for respectively down and up quark).

In this article, magnetic moments of up and down quarks inside nucleons are derived from purely geometric considerations. Using those magnetic moments, quark mass is then estimated and found to be proportional to the nucleon mean rest mass by a factor ≅ α, the fine-structure constant.

2. Quarks magnetic moments in neutron

In Fig.1 are depicted the three co-planar quarks composing the neutron in the X,Z plane. The two down quarks contributing positively to the neutron magnetic moment ($\mu_n \approx 0.962e^{-26}$ J/T, CODATA 2018) are shown in the +Z direction, while the up quark contributing negatively to μ_n is depicted in the –Z direction. On the layout, the left down quark (μ_{DL}) is arbitrarily defined parallel to Z (angle β_{DL}=0). Therefore, the magnetic moments for the right down quark (μ_{DR}) and up quark (μ_U) are shown relative to μ_{DL} (and Z).

The three magnetic moments μ_{DL}, μ_{DR}, and μ_U are projections of the respective magnetic moments μ_D and μ_U in the X,Z plane. The projections μ_{DR} and μ_{DL} might be slightly different due to different angles between μ_D and the X,Z plan in each case. However, for steric hindrance and space optimization considerations, it is conjectured that those angles are close to zero, so that μ_D and μ_U are essentially in the X,Z plan as defined in Fig.1. Further, and to indicate the charge ratio 2:1 (in absolute values) between the up and down quarks, the line width for the up quark (black) is twice the line width of the down quarks (red) .

We can write:

$$\mu_n = \mu_{DLZ} + \mu_{DRZ} - \mu_{UZ} = \mu_{DL}\cos(0) + \mu_{DR}\cos(\beta_{DR}) - \mu_U\cos(\beta_U) \qquad (1)$$

For reasons explained above, we can write $\mu_{DL} \approx \mu_{DR} \approx \mu_D$

Further, and as a result of the charge ratio 2:1 between the up and the down quark, and estimating the quarks masses $m_{QD} \approx m_{QU}$, we can approximate $\mu_U \approx 2\mu_D$

The eq. (1) now simplifies:

$$\mu_n = \mu_D\,[1 + \cos(\beta_{DR}) - 2\cos(\beta_U)] \qquad (2)$$

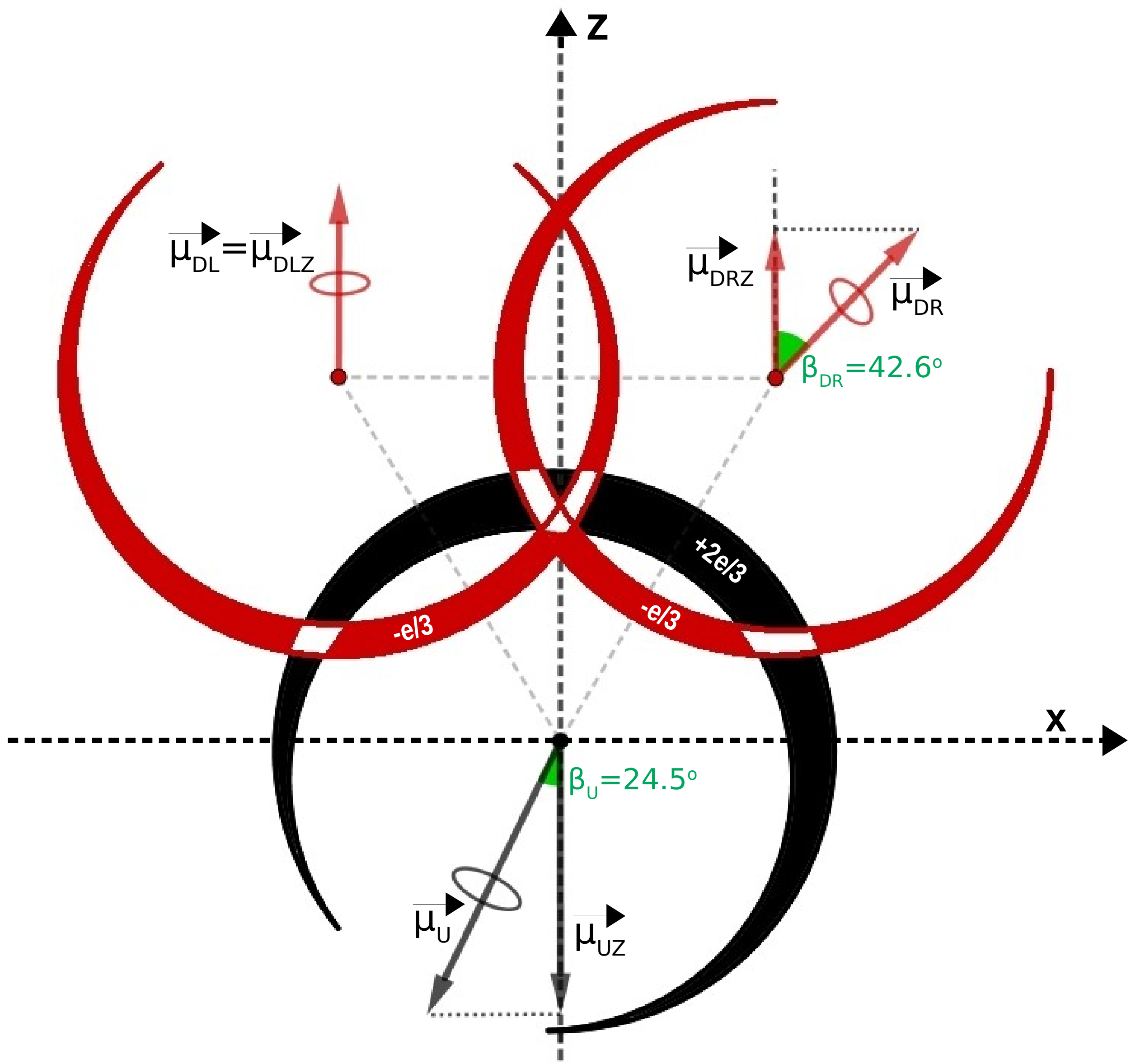
The eq. (2) has three unknowns and the following three constraints:

$$\left.\begin{array}{l} a)\ 2\cos(\beta_U) > 1+\cos(\beta_{DR}) \\ b)\ 0\leq\beta_U\leq\pi/2 \\ c)\ 0\leq\beta_{DR}\leq\pi/2 \end{array}\right\} \quad (3)$$

And the numerical values found that fit equation (2) with constraints (3) are the following:

$$\left.\begin{array}{ll} \beta_{DR} & = 42.6^o \\ \beta_U & = 24.5^o \\ \mu_D & = 11.5\ \text{x}10^{-26}\ \ J/T \\ \mu_U & \approx 2\mu_D = 23.0\ \text{x}10^{-26}\ \ J/T \end{array}\right\} \quad (4)$$

Figure 1: Configuration of quarks magnetic moments in Neutron



3. Configuration of the quarks magnetic moments in proton

Similarly, the configuration of the quarks magnetic moments in proton may be geometrically estimated, more particularly in light of the data provided in paragraph 2. For the proton (Fig.2), the magnetic moment of the down quark μ_D is arbitrarily chosen parallel to the Z axis, and therefore μ_{UL} and μ_{UR} are defined relative to μ_D (and Z). Then, we can posit:

$$\mu_P = \mu_{DZ} - \mu_{ULZ} - \mu_{URZ} = \mu_D \cos(0) - \mu_{UL} \cos(\beta_{UL}) - \mu_{UR} \cos(\beta_{UR}) \quad (5)$$

For reasons described in paragraph 2, we can simplify:

$$\mu_P \approx \mu_D - \mu_U [\cos(\beta_{UL}) + \cos(\beta_{UR})] \quad (6)$$

$$\Rightarrow \quad \cos(\beta_{UL}) + \cos(\beta_{UR}) = (\mu_p - \mu_D) / (-\mu_U) \quad (7)$$

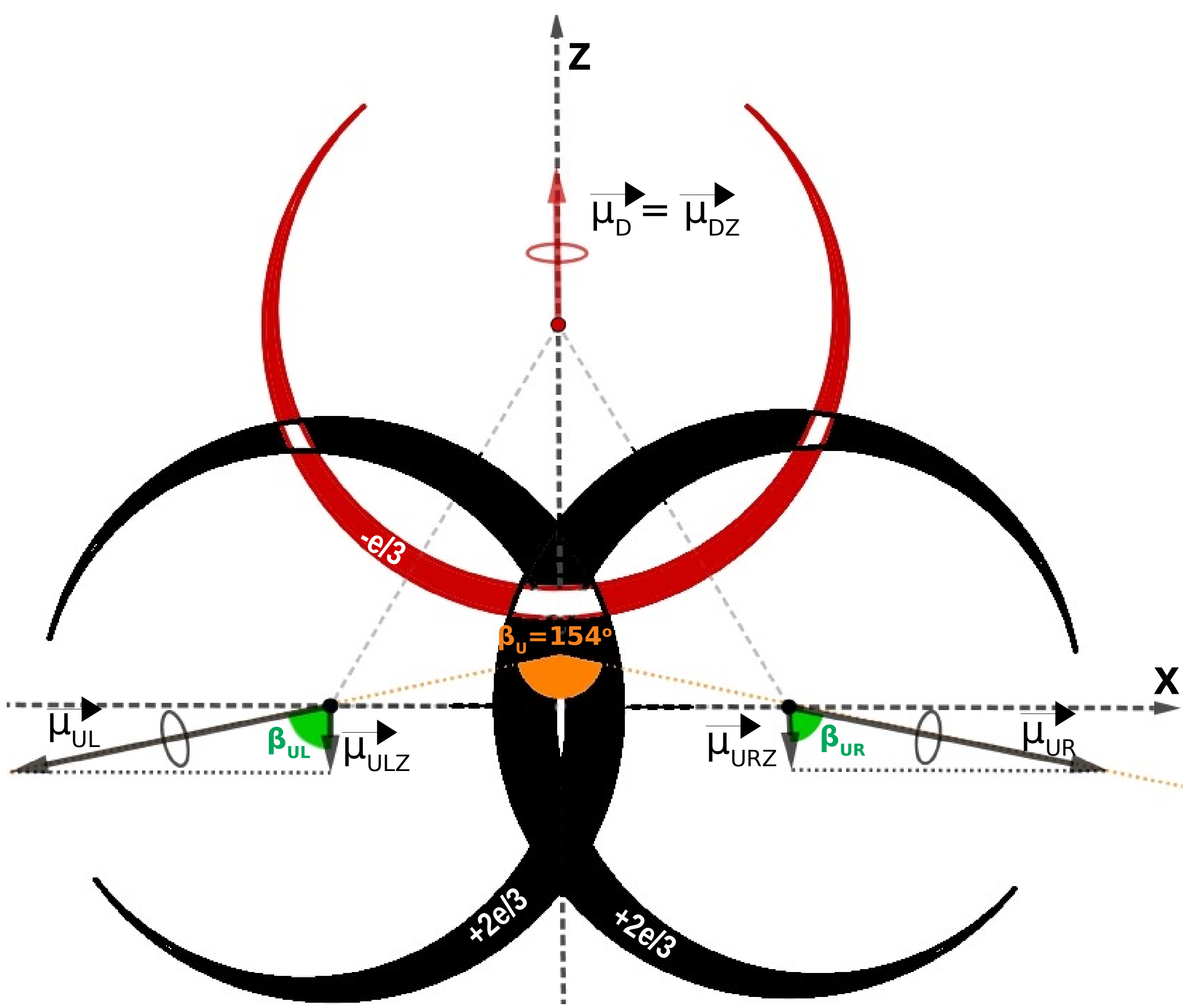
If we define $\mu_p=1.41e^{-26}$ J/T , $\mu_D=11.5e^{-26}$ J/T , and $\mu_U=23e^{-26}$ J/T

then $(\mu_p - \mu_D) / (-\mu_U) = 0.439 \approx \cos(64^o)$ (8)

$\Rightarrow \quad 64^o \leq \beta_{UL}, \beta_{UR} \leq 90^o$ <u>and</u> $\beta_U = 154^o$ (9)

As a result, both angles β_{UL} and β_{UR} need to be between [64°– 90°]. In addition, and most importantly, the angle β_U between $\vec{\mu}_{UL}$ and $\vec{\mu}_{UR}$ must be kept ≈ 154°.

Figure 2: Configuration of quarks magnetic moments in Proton



4. $\frac{\textbf{Quark mass}}{\textbf{Nucleon mass}} = \alpha$?

From the magnitude of quarks magnetic moments derived in paragraph 2, and defined in (4), it is now possible to estimate the quark mass through the simple formula:

$$\mu = (g)\frac{QS}{2m} \Rightarrow m = (g)\frac{QS}{2\mu} \qquad (10)$$

As for quarks, we can reasonably estimate a g factor ≈1 and calculate the mass of the down (m_{QD}) and up (m_{QU}) quark from the following:

$$m_{QD} = \frac{e/3\ .\ \hbar/2}{2\mu_D} \quad \text{and} \quad m_{QU} = \frac{2e/3\ .\ \hbar/2}{2\mu_U} \qquad (11)$$

Hence, the values found are: $m_{QD} \approx m_{QU} = m_Q = 1.22\ e^{-29}$ Kg (12)

which gives $$\frac{m_Q}{m_N} = 0.00729 \cong \alpha \qquad (13)$$

The results found in (12) and (13) reveal that the mass of the up and down quarks are "very close or equal to" (≅) $m_N\alpha$, with m_N and α being respectively the nucleon mean rest mass (≈1.673 775 e^{-27} Kg) and the fine-structure constant. These results also corroborate the approximations made in paragraph 2.

Finally, the quark mass found in this study is of the same order of magnitude as that determined by QCD theory [3]. It is not known at this time which Quark/Nucleon pair the eq. (13) has specific relevance to.

5. References

[1] PARTICLE PHYSICS BOOKLET page 26 (Available online)- Extracted from the Review of Particle Physics; M. Tanabashi et al (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

[2] Bruno R Galeffi; From 10^{-8} to 10^{-33} m: The Interplay of a Wide Range of Scales in the Neutron β Decay; https://vixra.org/abs/2010.0200

[3] PARTICLE PHYSICS BOOKLET page 25 (Available online)- Extracted from the Review of Particle Physics; M. Tanabashi et al (Particle Data Group), Phys. Rev. D 98, 030001 (2018)