Incremental forms of Time and Special Relativity

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Abstract

In this paper, I will give the concluding remarks and corrections in the formalism of incremental forms of time. Incremental form of time popped out from an primitive claim in the field of classical mechanics. The claim is based on the inequality in total time spend by an observer and total time spend in observing a point like particle moving with uniform velocity in space. we will reformulate and revise the entire formalism. The paper will be complete in it's own way.

1 Introduction

Consider a point particle in space, let us limit ourself in x direction. Say we are the observer's and sitting at the origin. Assume that we observe a object placed at a position given by x. Say we can precisely measure time by a very good clock well placed at origin. We start the time as we send the light pulse towards the object,and read the clock when the light pulse reflects back from the object and reaches us. Along with that we start another clock which measures 'observed' time. The difference between the time in the two clocks is precisely the time took by light to travel a distance x. At any given time t in 'observers' clock (the one which begins by sending a light pulse), the coordinate x can be represented as,

$$x = \frac{c}{2}\left(t - t_0\right) \tag{1}$$

The number 2 is written under c as we measure the two way speed of light. t_0 is the time for which the object was observed. Now here comes the claim.

Claim: As long as object under observation remains at rest, the increments in t and t_0 are equal. In differential form

$$\frac{dt_0}{dt} = 1$$

Proof: let us say δx, δt and δt_0 are increments in x, t and t_0 respectively.The resulting equation is,

$$x + \delta x = \frac{c}{2}(t + \delta t - t_0 - \delta t_0) \tag{2}$$

obviously only the increments will be left. we now divide the equation by δt and take the limit of δt tending to zero. I.e.

$$\lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \lim_{\delta t \to 0} \frac{c}{2}\left(1 - \frac{\delta t_0}{\delta t}\right)$$

If x is stationary, there is nothing to prove. If x is not stationary we recognize the limit at LHS to be velocity of particle. Denote the velocity by u and rearranging the equation gives:

$$\frac{dt_0}{dt} = \left(1 - \frac{2u}{c}\right) \tag{3}$$

Observe that $dt_0 = dt$ only if $u = 0$ that is velocity of particle is zero.
Hence the proof.

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2 Special Relativity

What we understand is this, in the moving frame equation (3) is valid. we may wonder what happens in rest frame. To see this we will assume that there exist a frame moving with velocity v. By using lorentz transformations we will go from the unprimed coordinates to primed coordinates. To do that, in equation (3) we do the following changes.

First, we use $dt = (dt' + \frac{v}{c^2}dx')\gamma$ where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ secondly we have to use the velocity transformation equation as

$$u = \frac{u' + v}{1 + vu'/c^2}$$

putting this all we get the dt_0 in terms of prime coordinates as:

$$\frac{dt_0}{dt'} = \left(1 + \frac{vu'}{c^2} - \frac{2}{c}(u' + v)\right)\gamma \tag{4}$$

Thus we have established the incremental forms of time.

3 Specific Cases

Equations (3) and (4) are correct and are corrections to the equations as derived in previous papers. Now we will look at certain specific cases.

3.1 $u = 0$

when $u = 0$ we observe that $dt = dt_0$ and $dt' = \gamma dt_0$

3.2 $u' = 0$

when $u' = 0$ we observe that $dt\left(1 - \frac{2v}{c}\right) = dt_0$ and $dt'\left(1 - \frac{2v}{c}\right)\gamma = dt_0$

4 conclusions

The Increments provide another way to see how theory of relativity affects clocks. It shows how two clocks designed to count two distinct 'times' vary. This variation in their rates can be calculated in a form of derivative. Further it would be interesting to see if this formalism has any applications and how it can be extended to accelerated frames of reference. It should be Noted that the equations stated in the previous two papers were incorrect and had some inconsistencies[2][1]. A question arises, is this only one clock which varies this way? can we find more such clocks? can we generalize the clocks? In this formalism the clock was a function of the form $f(u)$ where u is velocity of object under observation. Is it possible to have a clock of the form $f(x, u, \dot{u})$ or a clock which remains invariant? is it possible to remove the dependence of clocks?

References

[1] Suraj Deshmukh. "Dynamics under increments of time". In: (). DOI: `http://doi.org/10.5281/zenodo.3945198`.

[2] Suraj Deshmukh. "On observation of incremental form of time". In: (). DOI: `http://doi.org/10.5281/zenodo.3824448`.