# Theory about rational prime numbers.

**Juan Millas**

**Zaragoza (Spain)**

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**0- Abstract:**

Using products of rational numbers and the Eratosthenes method we can find a solution to the problem of rational prime numbers. This kind of numbers is a subset of the rationals and the problem has variations for decimals, centesimals, etc.

**1- Introduction:**

A natural number is called prime if it is grater than one and can not be expressed as product of two smaller natural numbers. The set of the primes (P) is the result of the intersection between the set of the natural numbers $\mathbb{N}$ and the set A minus the number 1, where:

(1) $A=m\cdot n \quad \forall(m,n)\in\mathbb{N}-\{1\}$

We can write the set prime numbers (P):

(2) $P=(\mathbb{N}\cap A)-1$

In other words, the primes are the numbers that cannot be expressed in the form:

(3) $\frac{a}{b}=c \quad \forall(a,b,c)\in\mathbb{N}-\{1\}$

**2- Integer-decimal prime numbers:**

The integer-decimal prime numbers are the numbers that can not be expressed in the following fraction:

(4) $\frac{a_0,a_1}{b}=c_0,c_1 \quad \forall(a_0,a_1)(c_0,c_1)\geq 1{,}0 \quad \forall b\geq 2$

**2.1- Examples:**

We are going to use the Erastothenes method applied to the decimal numbers. In this case, analyzing all decimal numbers less than 10,0.

Using first the number 2:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2x1,0=2,0 | 2x1,1=2,2 | 2x1,2=2,4 | 2x1,3=2,6 | 2x1,4=2,8 | 2x1,5=3,0 | 2x1,6=3,2 | 2x1,7=3,4 | 2x1,8=3,6 | 2x1,9=3,8 |
| 2x2,0=4,0 | 2x2,1=4,2 | 2x2,2=4,4 | 2x2,3=4,6 | 2x2,4=4,8 | 2x2,5=5,0 | 2x2,6=5,2 | 2x2,8=5,4 | 2x2,8=5,6 | 2x2,9=5,8 |
| 2x3,0=6,0 | 2x3,1=6,2 | 2x3,2=6,4 | 2x3,3=6,6 | 2x3,4=6,8 | 2x3,5=7,0 | 2x3,6=7,2 | 2x3,7=7,4 | 2x3,8=7,6 | 2x3,9=7,8 |
| 2x4,0=8,0 | 2x4,1=8,2 | 2x4,2=8,4 | 2x4,3=8,6 | 2x4,4=8,8 | 2x4,5=9,0 | 2x4,6=9,2 | 2x4,7=9,4 | 2x4,8=9,6 | 2x4,9=9,8 |
| 2x5,0=10,0 | | | | | | | | | |

Using secondly the number 3:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3x1,0=3,0 | 3x1,1=3,3 | 3x1,2=3,6 | 3x1,3=3,9 | 3x1,4=4,2 | 3x1,5=4,5 | 3x1,6=4,8 | 3x1,7=5,1 | 3x1,8=5,4 | 3x1,9=5,7 |
| 3x2,0=6,0 | 3x2,1=6,3 | 3x2,2=6,6 | 3x2,3=6,9 | 3x2,4=7,2 | 3x2,5=7,5 | 3x2,6=7,8 | 3x2,7=8,1 | 3x2,8=8,4 | 3x2,9=8,7 |
| 3x3,0=9,0 | 3x3,1=9,3 | 3x3,2=9,6 | 3x3,4=9,9 | | | | | | |

It is not necessary to use the number 4 as it is usual in the Eratosthenes method, we must continue to do the method in number 5 and 7:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5x1,0=5,0 | 5x1,1=5,5 | 5x1,2=6,0 | 5x1,3=6,5 | 5x1,4=7,0 | 5x1,5=7,5 | 5x1,6=8,0 | 5x1,7=8,5 | 5x1,8=9,0 | 5x1,9=9,5 |
| 5x2,0=10,0 | | | | | | | | | |
| | | | | | | | | | |
| 7x1,0=7,0 | 7x1,1=7,7 | 7x1,2=8,4 | 7x1,3=9,1 | 7x1,4=9,8 | | | | | |

With this information we can apply the method:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2,1 | 2,2 | 2,3 | 2,4 | 2,5 | 2,6 | 2,7 | 2,8 | 2,9 |
| 3 | 3,1 | 3,2 | 3,3 | 3,4 | 3,5 | 3,6 | 3,7 | 3,8 | 3,9 |
| 4 | 4,1 | 4,2 | 4,3 | 4,4 | 4,5 | 4,6 | 4,7 | 4,8 | 4,9 |
| 5 | 5,1 | 5,2 | 5,3 | 5,4 | 5,5 | 5,6 | 5,7 | 5,8 | 5,9 |
| 6 | 6,1 | 6,2 | 6,3 | 6,4 | 6,5 | 6,6 | 6,7 | 6,8 | 6,9 |
| 7 | 7,1 | 7,2 | 7,3 | 7,4 | 7,5 | 7,6 | 7,7 | 7,8 | 7,9 |
| 8 | 8,1 | 8,2 | 8,3 | 8,4 | 8,5 | 8,6 | 8,7 | 8,8 | 8,9 |
| 9 | 9,1 | 9,2 | 9,3 | 9,4 | 9,5 | 9,6 | 9,7 | 9,8 | 9,9 |
| 10 | | | | | | | | | |

So as we can see easy, the integer-decimal primes less than 10,0 are: 2,1; 2,3; 2,5; 2,7; 2,9; 3,1; 3,5; 3,7; 4,1; 4,3; 4,7; 4,9; 5,3; 5,9; 6,1; 6,7; 7,1; 7,3; 7,9; 8,3; 8,9; 9,7.

**3- Integer-centesimal prime numbers:**

The integer-centesimal prime numbers are the numbers that can not be expressed in the following fraction:

(5) $$\frac{a_0,a_1a_2}{b}=c_0,c_1c_2 \qquad \forall(a_0,a_1a_2)(c_0,c_1c_2)\geq 1{,}00 \qquad \forall b\geq 2$$

## 3.1 Examples:

We can apply in this case the Eratosthenes method too. We start to do the products 2x1,00=2,00; 2x1,01=2,02; 2x1,02=2,04… And then 3x1,00=3,00; 3x1,01=3,03; 3x1,02=3,06… And do this method as long as we want.

## 4.1 Integer-n-esimal prime numbers:

The integer-n-esimal prime number are those that can not be expressed in a fraction like the following:

(6) $$\frac{a_0,a_1a_2...a_n}{b}=c_0,c_1c_2...c_n \quad \forall(a_0,a_1a_2...a_n)(c_0,c_1c_2...c_n)\geq 1,\overbrace{00...0}^{n} \quad \forall b\geq 2$$

## 4.2- Proposition I: Integer-n-esimal primes are infinite for all n.

I could not do a properly demonstration of this proposition, it needs other paper.