Could Newton's gravity law be reconciled with Einstein's general relativity theory?

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Abstract

I claim that Newton's gravity law can be reconciled with Einstein's general relativity. Newton's law is a good approximation in a special case.

Einstein's Field Equations (EFE) describe the relationship between the geometry of spacetime and the distribution of mass, energy, and momentum within it. The exact solution of EFE depends on the assumptions taken. The first who solved EFE was Schwarzschild in 1916. He assumed a mass that is spherically symmetric, and non-rotating and that spacetime outside the mass is static. Newton assumed the same assumptions.

It can be shown, mathematically, that Newton's law, in the weak gravitational field and slow-motion approximation, can be derived from the Schwarzschild solution of EFE. See:

https://www.quora.com/How-can-we-derive-Newtons-law-of-gravitation-from-Einsteins-theory-of-relativity?q=can%20Newton%27s%20gravitational%20law%20derived%20from%20general%20relativity

However, Schwarzschild's solution as well as Newton's gravitational law do not describe exactly the physical world. It is known that all celestial bodies spin on their axis and therefore have a considerable angular momentum. Schwarzschild's solution and Newton's gravitational law do not take into consideration the spinning mass angular momentum. Therefore, both are approximations.

The real-world solution using EFE was derived in 1963 by Roy Kerr. Kerr's solution takes into consideration the spinning of the mass and is suitable for strong gravitational fields and high-velocity motion. Kerr's solution describes also an additional phenomenon – the frame-dragging of space by the spinning mass. In this sense, Newton's law deviates from Kerr's solution.

I think that the majority of the readers will find the Kerr's solution derivation quite difficult. Therefore, in this paper, I suggest a simpler way to compare Newton's gravity and Kerr's solution. To this end, I use the final frame dragging equation derived from Kerr's solution of EFE, without relating to the way how it was derived.

Newton's gravitational law describes gravity as the force exerted between two bodies. The force is dependent on the masses of the two bodies and the distance between the centers of the two bodies. The formula is:

$$F = \frac{G \cdot M_1 \cdot M_2}{R^2}$$

Although it was his idea, Newton was baffled by the gravitational force and could not explain its origin. However, Newton's law has been proven to be valid and sufficiently accurate in our solar system. It is used in calculating the motion of the planets and trajectories of spacecraft being sent e.g., from Earth to Mars.

On the other hand, some observations cannot be explained by Newton's law, for example, the precession of Mercury, the bending of light by massive celestial bodies, or the dynamics near neutron stars or black holes. Einstein's GR explains these observations quite accurately. Einstein described gravity as a distortion of the fabric of spacetime by a celestial body. The trajectory of a small celestial body will be along the geodesics in space created by a bigger body.

The question now is how two theories that are based on different assumptions can be reconciled.

<u>**Hypothesis**</u>**: I claim that Newton's law is a good approximation to GR, except that the distance R between the centers of the two bodies is not a straight line connecting both centers, but should be replaced by the geodesic length derived from general relativity $L_{geodesic}$** The geodesic length for two celestial bodies is calculated below.

Therefore, Newton's modified equation should be:

$$F = \frac{G \cdot M_1 \cdot M_2}{{L_{geodesic}}^2}$$

To explain the geodesic-length I would like to relate to the phenomenon of frame-dragging of spacetime around any rotating celestial body. The rotational frame-dragging effect was first derived in 1918 by Lense-Thirring based on GR. Later, in 1963 frame dragging was also derived from the solution of a rotating Kerr black hole. This effect was validated in 2011 by the Gravity Probe B experiment. The effect was measured near Earth, and the results were minuscule but validated the frame-dragging effect.

The equation of frame-dragging around a spinning celestial body is given by: https://en.wikipedia.org/wiki/Frame-dragging

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{r_s \alpha r c}{\rho^2 \left(r^2 + \alpha^2\right) + r_s \alpha^2 r \sin^2 \theta}$$

Where Ω - angular velocity depends on the radius r and the colatitude θ.

For simplicity, I use the equation of the angular velocity in the equatorial plane of a celestial body:

$$\Omega(r)=\frac{R_H\cdot\alpha\cdot C}{r^3+\alpha^2\cdot r+R_H\cdot\alpha^2}$$ ....The angular velocity

Where:

$c=2.9979\cdot10^8\cdot\frac{m}{s}.$ ....Light velocity

$R_H=\frac{2\cdot G\cdot M}{C^2}.$ .... Schwarzschild's radius

$G=6.67\cdot10^{-11}\cdot\frac{m^3}{kg\cdot s}..$ ....Gravitational constant

$M\cdot kg$ .... Mass of celestial body

$m_{neutron}=1.674927\cdot10^{-27}\cdot kg.$ ....The Neutron's mass

$\hbar=1.054571\cdot10^{-34}\cdot J\cdot s.$ ....Reduced Planck constant

$J=\hbar\cdot\left(\frac{M}{m_{neutron}}\right)^{\frac{4}{3}}$ ......Angular momnetum of a celestial body

- according to Muradian (See Note 1)

$\alpha=\frac{J}{M\cdot c}$ ......Definition of spin parameter

Note 1: This equation is according to the primeval Hadron hypothesis:

1. R. M. Muradian (1980). “The primeval hadron: origin of stars, Galaxies, and astronomical Universe” https://lib-extopc.kek.jp/preprints/PDF/1979/7911/7911323.pdf
2. R. M. Muradian “SCALING LAWS IN PARTICLE PHYSICS AND ASTROPHYSICS” https://arxiv.org/ftp/arxiv/papers/1106/1106.1270.pdf

To verify the hypothesis two examples are given:

1) Frame-dragging around the Sun.

2) Frame-dragging around a Neutron star PSR J1748-2446ad

1. Frame – dragging around the Sun

Given:

$M_{sun} = 1.99 \cdot 10^{30} \cdot kg.$ ....The Sun mass

$R_{sun} = 696342 \cdot km.$ .... The Radius of the Sun

$T_{sun} = 27 \cdot day.$ .... Spin of Sun around its axis

$\mathrm{R}_{earth\_sun} = 149.5 \cdot 10^{6} \cdot km.$ ....Average Sun-Earth distance

$T_{earth\_sun} = 1 \cdot yr.$ .... Time of Earth rotation around Sun

$J_{sun} = \hbar \cdot \left(\frac{M_{sun}}{m_{neutron}} \right)^{4/3} = 1.327 \cdot 10^{42} \, J \cdot s$.....The angular momentum of Sun

$\alpha_{sun} = \frac{J_{sun}}{M_{sun} \cdot c} = 2.2439 \cdot km$ ....Spin parameter

$R_{H\_sun} = \frac{2 \cdot G \cdot M_{sun}}{c^2} = 2.954 \cdot \mathrm{km}.$ .... Schwarzschild radius of Sun

$\Omega(r) = \frac{R_{H\_sun} \cdot \alpha_{sun} \cdot c}{r^3 + \alpha_{sun}{}^2 \cdot r + R_{H\_\mathrm{sun}} \cdot \alpha_{sun}{}^2}$ ....The angular velocity around the Sun

$\theta(r) = \Omega(r) \cdot T_{earth} \cdot \frac{r}{R_{earth\_sun}}$ ....The rotation angle of Earth around the Sun

Finding the length of a geodesic $L_{geodesic}$ is done by using the formula of curve length:

$$L_{geodesic} = \int_{R_{sun}}^{R_{earth\_sun}} \sqrt{\left[1 + \left(\tfrac{d}{dr} \left(r \cdot \Omega(r) \cdot T_{earth} \right) \right)^2 \right]} \cdot \mathrm{dr} = 148.8 \cdot 10^{6} \text{ km}$$ ....Length of-

-geodesic Earth to Sun

And the result is $\frac{L_{geodesic}}{R_{earth\_sun}} \cong 1$, therefore Newton’s approximation is good.

Fig. 1 shows the straight-line connecting Sun to Earth as formulated by Newton’s law.

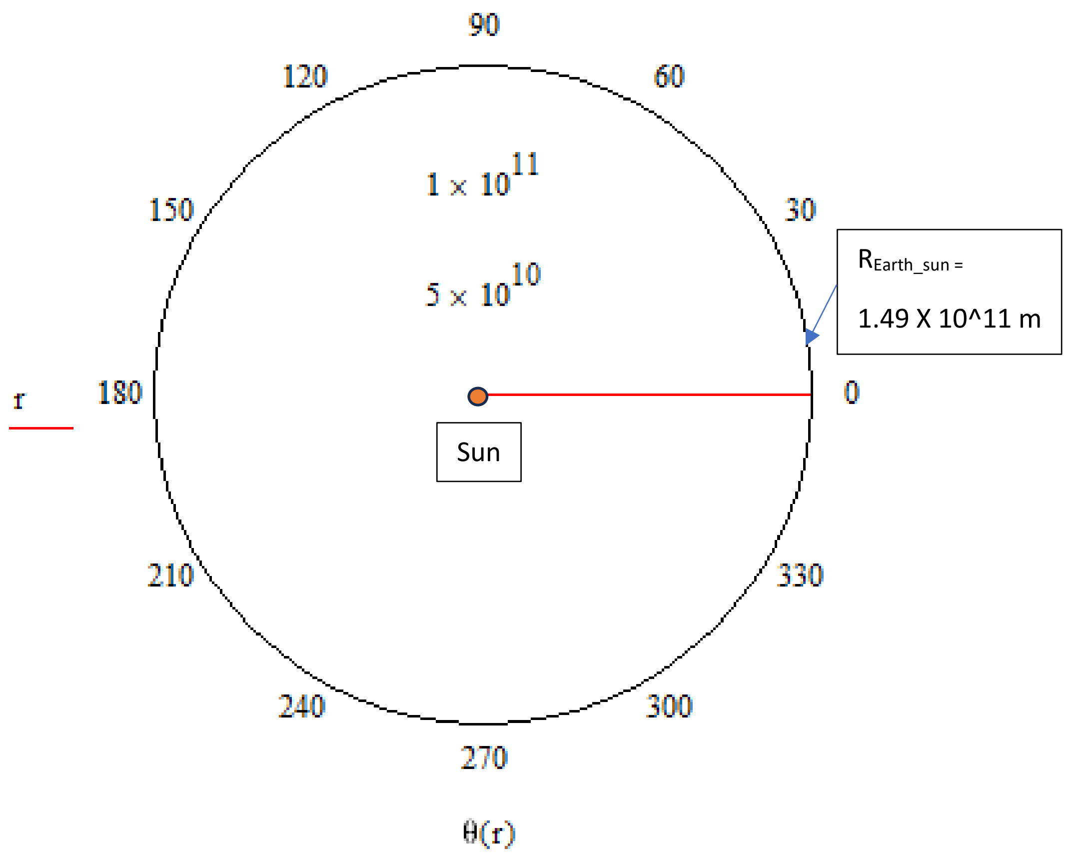

Fig. 1- The line connecting Earth- Sun – According to Newton

However, as GR predicts the dragging is minuscule. To see the geodesic, the angle $\theta(r)$ is scaled up by a factor of 10^8. See Fig. 2. The geodesic becomes evident near the Sun.

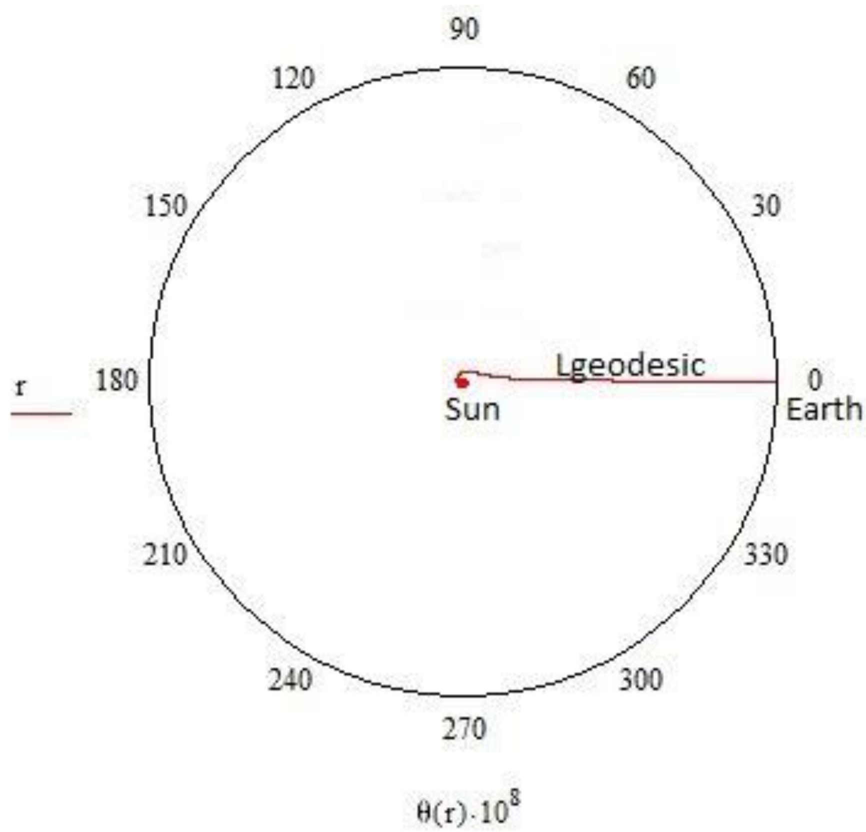

Fig. 2- The geodesic around the Sun – According to GR

2) Frame – dragging around Neutron star PSR J1748 – 2446ad

Given

$M_{ns} = 2 \cdot 1.99 \cdot 10^{30} \cdot kg.$ ....The Neutron star mass: 2x Sun mass

$R_{ns} = R_{neutron} \cdot (\frac{M_{ns}}{m_{neutron}})^{1/3} = 10.67 \cdot \text{km}$.....Calculated radius of the Neutron star

$f_{ns} = 716 \frac{1}{s}.$ ....Measured spin of neutron star

$T_{ns} = \frac{1}{f_{ns}} = 1.4 \cdot 10^{-3} \cdot s$

$J_{ns} = \hbar \cdot \left(\frac{M_{ns}}{m_{neutron}} \right)^{\frac{4}{3}} = 3.34 \cdot 10^{42}\, J \cdot s.$ .... The angular momentum of Neutron star

$\alpha_{ns} = \frac{J_{ns}}{M_{ns} \cdot c} = 2.8 \cdot km$ ....Spin parameter

$R_{H\_\text{ns}} = \frac{2 \cdot G \cdot M_{ns}}{c^2} = 5.9 \cdot \text{km}.$ ... The Schwarzschild radius of the Neutron star

$\Omega(r) = \frac{R_{H\_\text{ns}} \cdot \alpha_{ns} \cdot c}{r^3 + \alpha_{ns}{}^2 \cdot r + R_{H\_\text{ns}} \cdot \alpha_{ns}{}^2}$ ....The ngular velocity around the Neutron star

$R_{out} = 20 \cdot km.$ ... Assumption: distance of a particle from Neutron star

$L_{geodesic\_ns} = \int_{R_{ns}}^{R_{out}} \sqrt{\left[1 + \left(\frac{d}{dr}\left(r \cdot \Omega(r) \cdot T_{ns}\right)\right)^2\right]} \cdot \text{dr} = 39.4 \cdot \text{km}$ ....Length of geodesic-

-particle to Neutron star

The ratio is: $\frac{L_{geodesic\_ns}}{R} = 1.97$

$\theta(r) = \Omega(r) \cdot T_{ns} \cdot \frac{r}{R_{ns}}$ ....Polar plot around the neutron star

Fig. 3 shows frame-dragging around a Neutron star. It shows $L_{geodesic\_ns}$ Vs. Newton's distance R. The gravity force between the Neutron star and a particle orbiting it differs significantly from Newton's law.

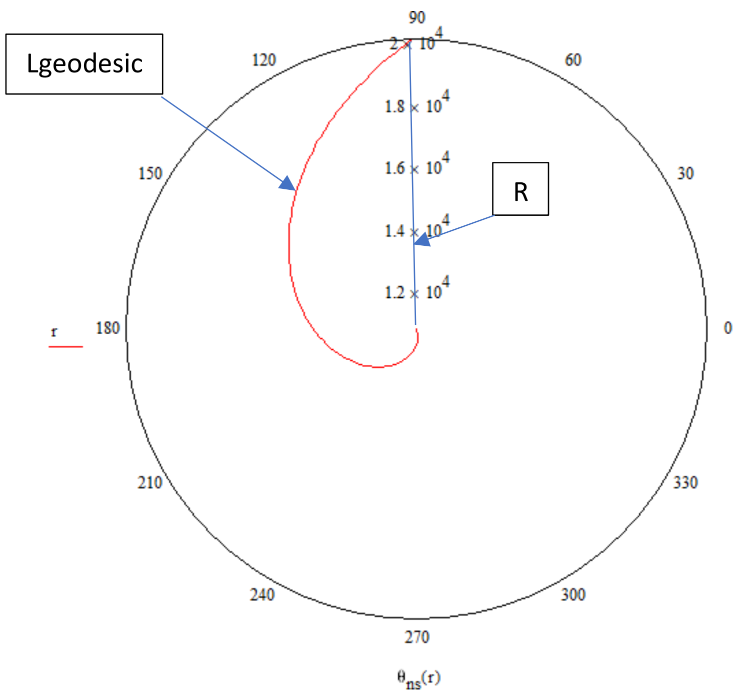

Fig. 3 - The geodesic around the Neutron star per GR.