Notation for Operators Based in Knuth's Up Arrow

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Abstract:

With this kind of notation we can use the method of Donald Knuth in iterations to express serial operators with them variables iterated.

Introduction:

The Knuth's up arrow notation is one of the notations which is used to explain extended operations beyond exponentiation. In this method we use an arrow to indicate an exponentiation, two arrows to indicate tetration, three arrow to indicate pentation…
The idea of this paper is use this properties to do a continuity succession of variables defined by an operator. For this reason we are going to use a general symbol "Greek capital letter xi" with a subscript: 1 for additions, 2 for products, 3 for exponents, 4 for tetrations, 5 for pentations, etc.
In the final part we are going to define the equivalences between operators.

Formulas:

Summation:

(1) $$\underset{n=a}{\overset{b}{\Xi_1}} f(x) = \sum_{n=a}^{b} f(x) = x_a + x_{a+1} + x_{a+2} + \ldots + x_{b-1} + x_b$$

Productory:

(2) $$\underset{n=a}{\overset{b}{\Xi_2}} f(x) = \prod_{n=a}^{b} f(x) = x_a \cdot x_{a+1} \cdot x_{a+2} \cdot \ldots \cdot x_{b-1} \cdot x_b$$

Exponentory:

(3) $$\underset{n=a}{\overset{b}{\Xi_3}} k^{f(x)} = \underset{n=a}{\overset{b}{\Theta}} k^{f(x)} = (((((k^a)^{a+1})^{a+2})^{\ldots})^{(b-1)})^b = k \uparrow a \uparrow (a+1) \uparrow (a+2) \uparrow \ldots \uparrow (b-1) \uparrow b$$

Tetratory:

(4) $$\underset{n=a}{\overset{b}{\Xi_4}} k^{f(x)} = k \uparrow\uparrow a \uparrow\uparrow (a+1) \uparrow\uparrow (a+2) \uparrow\uparrow \ldots \uparrow\uparrow (b-1) \uparrow\uparrow b$$

(5) $$\underset{n=a}{\overset{b}{\Xi_4}} k^{f(x)} = k \uparrow^2 a \uparrow^2 (a+1) \uparrow^2 (a+2) \uparrow^2 \ldots \uparrow^2 (b-1) \uparrow^2 b$$

Pentatory:

(6) $$\overset{b}{\underset{n=a}{\Xi_5}} k^{f(x)} = k \uparrow^3 a \uparrow^3 (a+1) \uparrow^3 (a+2) \uparrow^3 \ldots \uparrow^3 (b-1) \uparrow^3 b$$

In general for any case subscript equals to n:

(7) $$\overset{b}{\underset{n=a}{\Xi_n}} k^{f(x)} = k \uparrow^{(n-2)} a \uparrow^{(n-2)} (a+1) \uparrow^{(n-2)} (a+2) \uparrow^{(n-2)} \ldots \uparrow^{(n-2)} (b-1) \uparrow^{(n-2)} b \qquad \forall n \geq 3$$

Relation formulas:

Any constant whose power is a summation of a function is equal to the productory of these constant whose power is the function.

(8) $$c^{\sum_{n=a}^{b} f(x)} = \prod_{n=a}^{b} c^{f(x)}$$

Any constant whose power is a productory of a function is equal to the exponentory of these constant whose power is the function.

(9) $$c^{\prod_{n=a}^{b} f(x)} = \overset{b}{\underset{n=a}{\Theta}} c^{f(x)}$$

This is the same for exponentory and tetratory:

(10) $$c^{\overset{b}{\underset{n=a}{\Theta}} f(x)} = \overset{b}{\underset{n=a}{\Xi_4}} c^{f(x)}$$

In general, we can follow the next rule:

(11) $$c^{\overset{b}{\underset{n=a}{\Xi_n}} f(x)} = \overset{b}{\underset{n=a}{\Xi_{(n-1)}}} c^{f(x)}$$

Conclusion:

With the notation we can write series of very large integers. It will be interesting to see in the future if it has some applications in astrophysics, quantum physics or maybe in economics. Anyway this paper show how it is possible to understand with precision a very large numbers if you do a practical application.