A Sieve for Goldbach Conjecture

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Abstract

In this article, we find a new sieve for pair of primes whose summation equals to a given Even Number.

When we sieve numbers 2 and 3 to the whole natural numbers by removing all the multiplies of 2 or 3, the remaining numbers are 1 and all pairs of {6k-1, 6k+1; k=1,2,3,....}. The remaining numbers set is a group of multiply.

We call the integer k as the id of the pair (6k-1, 6k+1).

By mulitiplication, two pairs of $(6k_1 - 1, 6k_1 + 1)$, and $(6k_2 - 1, 6k_2 + 1)$, can generate four new numbers,6k-1,or 6k+1, with id k takes four new integers of $(6k_1 \pm 1)k_2 \pm k_1$ when $k_1 \neq k_2$, while four new k will reduce to three when $k_1 = k_2$.

We have a new sieve for Goldbach Conjecture based on the above observation,

For any large Even Number of forms as, 6N, or 6N+2, or 6N-2. we sieve the whole integer set I of id integers by the numbers of $6k_1$-1, and $6k_1 + 1$ for $k_1 = 1, 2, ..., m$, by removing all the numbers of $\{i \in I$; $i = k_1$,or N-k_1, mod $(6k_1 + 1)$, or $i = -k_1$,or N+k_1, mod $(6k_1 - 1)$, $k_1 \leq m\}$;

the remaining numbers are set of $\{i \in I; i \neq k_1$,or N-k_1, mod $(6k_1+1)$, and $i \neq -k_1$,or N+k_1, mod $(6k_1-1)$, $k_1 \leq m\}$;

If we limit our sieve upto this large number N, we have m = $[\sqrt{\frac{N}{6}}]$, here [a] means the largest integer less than a.

Theorem;

By using the above sieve when sieve all $(6k_1 \pm 1)$, $k_1 \leq m$ for the first N integers, (0, N), the total number of the remaining numbers inside (0, N)is larger than $N \times \prod_{5 \leq p \leq (6m+1)}(1-2/p)$,

The remaining numbers less than N is the set, $\{i < N; i \neq k_1$,or N-k_1, mod $(6k_1+1)$, and $i \neq -k_1$,or N+k_1, mod $(6k_1-1)$, $k_1 \leq m\}$;

Each remaining number i and N-i are id's for possible primes of $(6i \pm 1)$, and $(6(N-i) \pm 1)$

When the even Number is 6N+2, take the possible pairs of, $(6i+1)$, and $(6(N-i)+1)$;

When the even Number is 6N, take the possible pairs of, $(6i+1)$, and $(6(N-i)-1)$; or $(6i-1)$, and $(6(N-i)+1)$;

When the even Number is 6N-2, take the possible pairs of, $(6i-1)$, and $(6(N-i)-1)$;

For example, if N= 100, we get m=4. Here we only need to sieve by primes of, 5,7,11,13,17,19,23,which are less than (6m+1);

The total remaining number is larger than 100(1-2/5)(1-2/7)(1-2/11)(1-2/13)(1-2/17)(1-2/19)(1-2/23), which is about 21;

with i equals to 5, 10,12,13,17,23,27,30,32,37,38,45,55,62,63,68,70,73,77,87,90,95;

here the actual total remaining id's is 22. For example, when i =5, N-i=95, the pair primes is (31,571) which add up to 602; or pair primes of (29, 569) which add up to 698; or pair primes of (29,571),or (31, 569) which add up to 600.

This proves the Goldbach conjecture.