A CLASSIC ALGEBRAIC IDENTITY IMPLIES FERMAT'S LAST THEOREM (FLT) FOR INTEGRAL EXPONENT LARGER THAN TWO, V. 12

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Abstract. We solve the open problem of a simple proof of FLT for $n > 2$ by directly inferring, from Euclid's formula, a generalization that holds for the set of all coprime triples equal to the set of all coprime $\{(z, y, x)\}$ for which $z^n - y^n = x^n$ holds. Our nth generalization allows us to deduce a *necessary condition* for coprime $\{(z, y, x)\}$ to satisfy $z^n - y^n = x^n$, the condition being $n \not> 2$.

$$z^n - y^n = x^n. \tag{1}$$

Fermat's last theorem (FLT) states, for (1), with z, y, x, n positive integers, $z > y, x$, that the triple (z, y, x) can not be integral for $n > 2$. We propose a *simple proof of FLT* (which is still an open problem) by direct inference (not by deriving a contradiction) for any given $n > 2$.

Our argument begins with the well-known true statement, Euclid's formula (with terms rearranged and notation changed). Per the well-known demonstration of rational points on a unit circle, Euclid's formula, (2), holds for the set of all positive coprime $\{(z, y, x)\}$ if and only if t, s each is every positive coprime, with $t > s$, and t, s of opposite parity; the coprime triple for which (2), below, holds is $\{((t^2 + s^2), (t^2 - s^2), (2ts))\}$:

$$(t^2 + s^2)^2 - (t^2 - s^2)^2 = (2ts)^2. \tag{2}$$

For at least one value of $n > 1$ there is a set of all non-null coprime triples for which (2) to (6), (9) below, respectively hold, and for which t, s remain solely coprime the entire argument, per above. This truth allows respective coprime triples to imply, below, a *necessary condition* for positive coprime (z, y, x) to satisfy $z^n - y^n = x^n$, that being $n \not> 2$.

Since (2) is an identity, we can *substitute* $t^{\frac{n}{2}}$ for t, and $s^{\frac{n}{2}}$ for s so that coprime $\{((t^2 + s^2), (t^2 - s^2), (2ts))\}$ for which (2) holds implies coprime $\{((t^{\frac{n}{2}})^2 + (s^{\frac{n}{2}})^2), ((t^{\frac{n}{2}})^2 - (s^{\frac{n}{2}})^2), 2t^{\frac{n}{2}}s^{\frac{n}{2}})\}$ for which (3) holds :

$$\left((t^{\frac{n}{2}})^2 + (s^{\frac{n}{2}})^2\right)^2 - \left((t^{\frac{n}{2}})^2 - (s^{\frac{n}{2}})^2\right)^2 = (2t^{\frac{n}{2}}s^{\frac{n}{2}})^2. \tag{3}$$

Equation (3) reduces to (4), with the set of all positive coprime $\{(t,s)\}$:

$$(t^n+s^n)^2-(t^n-s^n)^2=(2t^{\frac{n}{2}}s^{\frac{n}{2}})^2. \tag{4}$$

With t,s coprime, the coprime triple for which (4) holds, $\{\big((t^n+s^n),(t^n-s^n),(2t^{\frac{n}{2}}s^{\frac{n}{2}})\big)\}$ implies the coprime triple for which (5) holds, $\{\Big((t^n+s^n)^{\frac{2}{n}},(t^n-s^n)^{\frac{2}{n}},(2t^{\frac{n}{2}}s^{\frac{n}{2}})^{\frac{2}{n}}\Big)\}$:

$$\Big((t^n+s^n)^{\frac{2}{n}}\Big)^n-\Big((t^n-s^n)^{\frac{2}{n}}\Big)^n=\Big((2t^{\frac{n}{2}}s^{\frac{n}{2}})^{\frac{2}{n}}\Big)^n. \tag{5}$$

Reduce $(2t^{\frac{n}{2}}s^{\frac{n}{2}})^{\frac{2}{n}}$ of (5) to imply (6) (taking $(t^n-s^n)^{\frac{2}{n}}$ as odd for a non-null set), treating (6) as a Fermat equation with s,t solely integral:

$$\Big((t^n+s^n)^{\frac{2}{n}}\Big)^n-\Big((t^n-s^n)^{\frac{2}{n}}\Big)^n=(2^{\frac{2}{n}}ts)^n. \tag{6}$$

Note : For $n>2$ there is no coprime Fermat triple with solely coprime $\{(t,s)\}$ that satisfies (6) since $2^{\frac{2}{n}}ts$ can not be rational.

For any given $n>1$, there must exist coprime left and middle parts of the real triple for which (6) holds. We notate such parts respectively:

$$(t^n+s^n)^{\frac{2}{n}}=w;(t^n-s^n)^{\frac{2}{n}}=v. \tag{7}$$

Raising expressions in (7) to the power of $\frac{n}{2}$ implies useful equation (8):

$$t^n-s^n=v^{\frac{n}{2}}. \tag{8}$$

We rewrite (8) as a deductively true Fermat equation (9) :

$$t^n-s^n=(v^{\frac{1}{2}})^n. \tag{9}$$

For any given $n>1$, with non-null sets : The set of all coprime Fermat triples for which (9) holds is $\{(t,s,v^{\frac{1}{2}})\}$ (taking s as even) since, with proper choices of coprime $\{(t,s)\}$, term $v^{\frac{1}{2}}$ is constrained to be integral. For any given $n>1$, equation (9) holds for a subset of all positive coprime pairs $\{(t,s)\}$, a subset that is equal to coprime $\{(z,y)\}$ for which (1) holds. Hence, coprime $\{(t,s,v^{\frac{1}{2}})\}$ is equal to coprime $\{(z,y,x)\}$.

With non-null such sets : Coprime $\{((t^n+s^n)^{\frac{2}{n}},(t^n-s^n)^{\frac{2}{n}},2^{\frac{2}{n}}ts)\}$ implies coprime $\{(t,s,(v^{\frac{1}{2}}))\}$; So, coprime $\{((t^n+s^n)^{\frac{2}{n}},(t^n-s^n)^{\frac{2}{n}},2^{\frac{2}{n}}ts)\}$ implies coprime $\{(z,y,x)\}$ for which $z^n-y^n=x^n$ holds.

So, for $n>1$, the set of all coprime triples for which (6) holds equals the set of all coprime triples $\{(z,y,x)\}$ for which $z^n-y^n=x^n$ holds.

Ergo, for $n>2$ there is a null set of all coprime $\{(z,y,x)\}$ (thus, a null set of all integral $\{(z,y,x)\}$) for which $z^n-y^n=x^n$ holds. Q.E.D.

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