A New Sieve for Twin Primes

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Abstract

In this article, we find a new sieve for twin primes to prove the twin prime theory.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

When we sieve numbers 2 and 3 to the whole natural numbers by removing all the multiplies of 2 or 3, the remaining numbers are 1 and all pairs of {6k-1, 6k+1; k=1,2,3,....}. The remaining numbers set is a group of multiply.

We call the integer k as the id of the pair (6k-1, 6k+1).

By mulitiplication, two pairs of $(6k_1 - 1, 6k_1 + 1)$, and $(6k_2 - 1, 6k_2 + 1)$, can generate four new numbers,6k-1,or 6k+1, with id k takes four new integers of $(6k_1 \pm 1)k_2 \pm k_1$ when $k_1 \neq k_2$, while four new k will reduce to three when $k_1 = k_2$.

We have a new sieve based on the above observation, we sieve the whole integer set I of id integers by the numbers of $6k_1$-1, and $6k_1 + 1$ for $k_1 = 1, 2, ..., m$, by removing all the numbers of $\{i \in I;\ i = \pm k_1,$ mod $(6k_1 \pm 1),\ k_1 \leq m\}$;

the remaining numbers are set of $\{i \in I;\ i \neq \pm k_1,$ mod $(6k_1 \pm 1),\ k_1 \leq m\}$;

If we limit our sieve upto a finite large number N, we have m = $[\sqrt{\frac{N}{6}}]$, here [a] means the largest integer less than a.

Theorem;

By using the above sieve when sieve all $(6k_1 \pm 1)$, $k_1 \leq m$ for the first N integers (0, N), the total number of the remaining numbers inside (0, N)is larger than $N \times \prod_{5 \leq p \leq (6m+1)} (1 - 2/p)$,

The remaining numbers less than N is the set, $\{i \leq N;\ i \neq \pm k_1$, mod $(6k_1 \pm 1),\ k_1 \leq m\}$;

Each remaining number i is an id for a twin primes of $(6i \pm 1)$,

For example, if N= 100, we get m=4. Here we only need to sieve by primes of, 5,7,11,13,17,19,23,which are less than (6m+1);

The total remaining number is larger than 100(1-2/5)(1-2/7)(1-2/11)(1-2/13)(1-2/17)(1-2/19)(1-2/23), which is about 21;

with i equals to 5,7, 10,12,17,18,23,25,30,32,33,38,40,45,47,52,58,70,72,77,87,95;

here the actual total remaining id's is 22. For example i=95, the twin primes is (569,571).

This also proves the twin prime conjecture.