AN INTEGRAL EQUATION FOR THE GRAMM SERIES

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ABSTRACT : In this paper we give an integral equation satisfied by the gramm series based on the use of the Borel transform

In Mathematics the Gramm series is define as the infinite series

$$G(x)=1+\sum_{n=1}^{\infty}\frac{(\log x)^{n}}{nn!\zeta(n+1)} \quad (1)$$

Let be the following integral equation

$$-\log\left(1-\frac{1}{s}\right)=s\int_{1}^{\infty}dx\frac{g(x)}{x^{s}-1}.\frac{1}{x} \quad (2)$$

Then with a simple change of variable , we have the following integral equation

$$-\log\left(1-\frac{1}{s}\right)=\sum_{n=1}^{\infty}\frac{1}{n}.\frac{1}{s^{n}}=s\int_{0}^{\infty}dt\frac{g(e^{t})}{e^{st}-1} \quad (3)$$

Using our method described in Paper [1] , which uses the Borel generalized transform to solve integral equations of this kind

$$g(s)=s\int_{0}^{\infty}dtK(st)f(t) \quad (4)$$

With a solution given by the power series $f(t)=\sum_{n=1}^{\infty}\frac{c_{n}t^{n}}{M(n+1)}$ (5)

With $c(n)=\frac{1}{2\pi i}\int_{C}g(z)z^{n-1}$ $\qquad g(s)=\sum_{n=0}^{\infty}\frac{c(n)}{s^{n}}$ (6)

Then,we can find a series solution for this integral equation as follows

$$G(x)-1=g(x)=\sum_{n=1}^{\infty}\frac{(\log x)^{n}}{nn!\zeta(n+1)} \quad (7)$$

Which is precisely the Gramm series (minus a constant 1), so the Integral given in formula (   ) is just the integral equation satisfied by the Gramm function

References:

[1] Garcia J.J "Borel resummation and the solution of integral equatio· e-print vixra.org/abs/1304.0013

[2] Ingham, A. E. Ch. 5 in "*The Distribution of Prime Numbers"*. New York: Cambridge University Press, 1990.

[3] Weisstein, Eric W. "Gram Series." From *MathWorld*--A Wolfram Web Resource. https://mathworld.wolfram.com/GramSeries.html