Polynomials Generating Prime Numbers No.2

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Abstract

In the Ulam spiral, there are places where prime numbers appear continuously on line. Integers are arranged in a square spiral in the Ulam spiral. I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315, 153, 160 degrees, hexagonal arrangement etc.. Then, prime numbers appeared continuously on line.
Looking at the prime numbers vertical column of Euler's polynomial generating prime numbers, that I created in the hexagonal 90 degrees arrangement, I think how far the continuous numbers will continue if Euler prime numbers is selected to skip by 2 or skip by 3 etc., avoiding the value where the continuous prime number is interrupted, and I investigate. As a result, I found many polynomials generating 40 to 10 consecutive prime numbers.

Contents

1 Introduction

I was interested in prime numbers looking at the Ulam spiral, I analyzed it myself.
And I learned that Euler's polynomial generating prime numbers is simple and great.
I thought that other polynomials generating prime numbers may be found in other arrangements, I investigate.
Looking at the prime numbers vertical column of Euler's polynomial generating prime numbers, that I created in the hexagonal 90 degrees arrangement, I think how far the continuous numbers will continue if Euler prime numbers is selected to skip by 2 or skip by 3 etc., avoiding the value where the continuous prime number is interrupted, and I investigate. As a result, I found many polynomials generating 40 to 10 consecutive prime numbers and I collect the results.

These algebraic polynomials have the property that for $n = 0, 1, \ldots, m-1$ value of the polynomial, eventually in module, are m primes.

2 Polynomials generating prime numbers

2.1 The Ulam spiral

In the Ulam spiral, there are places where prime numbers appear continuously on line. I noticed that there are places where prime numbers appear continuously in a certain pattern in the Ulam spiral, although they do not appear continuously on line. They are two polynomials, $P(n) = 4n^2 + 2n + 41$ and $P(n) = 4n^2 + 6n + 43$, generates 20 primes, see Figure 2.1. Each value is a value obtained by skipping by 1 of Euler prime numbers. When the values of the two polynomials are inserted alternately, the values are the same as values of Euler prime numbers, see Figure 2.1.

2.2 Polynomial generating prime numbers 1

Integers are arranged in a square spiral in the Ulam spiral, but I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315, 153, 160 degrees, hexagonal arrangement etc., using a computer. Then, prime numbers appeared continuously on line.

In 180 degrees arrangement, see Figure 2.2, 29 prime numbers appear continuously. It was prime numbers of Legendre polynomial [1798], $P(n) = 2n^2 + 29$, generates 29 primes: 29, 31, 37, 47, 61, 79, 101, 127, 157, 191, 229, 271, 317, 367, 421, 479, 541, 607, 677, 751, 829, 911, 997, 1087, 1181, 1279, 1381, 1487, 1597 .

2.3 Polynomial generating prime numbers 2

In 135 degrees arrangement, see Figure 2.3, 29 prime numbers appear continuously. It was prime numbers of Brox polynomial [2006], $P(n) = 6n^2 - 342n + 4903$ (or $6n^2 + 6n + 31$), generates 29 primes: 4903, 4567, 4243, 3931, 3631, 3343, 3067, 2803, 2551, 2311, 2083, 1867, 1663, 1471, 1291, 1123, 967, 823, 691, 571, 463, 367, 283, 211, 151, 103, 67, 43, 31 .

Also, in Figure 2.3, prime numbers of polynomials, $P(n) = 6n^2 + 6n + p$, p are lucky numbers p = 5, 7, 11, 17, 31, are clearly appeared. In addition, prime numbers of polynomials, $P(n) = 6n^2 + p$, p are lucky numbers p = 5, 7, 13, 17, are clearly appeared.

2.4 Polynomial generating prime numbers 3

In 270 degrees arrangement, see Figure 2.4, 22 prime numbers appear continuously. It was prime numbers of Frame polynomial [2018], $P(n) = 3n^2 + 3n + 23$, generates 22 primes: 23, 29, 41, 59, 83, 113, 149, 191, 239, 293, 353, 419, 491, 569, 653, 743, 839, 941, 1049, 1163, 1283, 1409 .

2.5 Euler's polynomial generating prime numbers

In 90 degrees arrangement, see Figure 2.5 and the hexagonal 90 degrees arrangement, see Figure 2.6 (illustrated as a rectangle for simplification in Figure 2.6), 40 prime numbers appear continuously. It was prime numbers of Euler's polynomial [1772], $P(n) = n^2 + n + 41$, generates 40 primes: 41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601 .
Also, prime numbers of polynomial, $P(n) = n^2 + n + p$, p are Euler's lucky numbers p = 3, 5, 11, 17, 41, are clearly appeared.

3 New polynomials generating prime numbers

Looking at the vertical column of $n^2 + n + 17$ with the lucky number p = 17 of Euler's polynomial generating prime numbers in Figure 2.6 that I created, this vertical column has many prime numbers, but they are not used much in the polynomial generating prime numbers. The last value of the continuous prime number of $n^2 + n + 17$ is 257. I thought that if I selected 257, 359, 479 to skip by 2 in this column, I may have polynomials generating prime numbers with many consecutive numbers, because there are many prime numbers in this vertical column. With the spirit of "mottainai" in Japan, the intention is to make use of 257, 359, 479. Then, the polynomial of Section 3.12.10 was found.
After that, since the vertical column of Euler's polynomial generating prime numbers , $n^2 + n + 41$ has many prime numbers, I think how far the continuous numbers will continue if Euler prime numbers is selected to skip by 2 or skip by 3 etc., avoiding the value of 1681, 1763, 2021 where the continuous prime number is interrupted, and I investigate. As a result, I found many polynomials generating 40 to 10 consecutive prime numbers and I collect the results.

3.1 New polynomial generating prime numbers 1

Figure 3.1 shows the same contents as Figure 2.6, but Figure 3.1 is easy to see the values. In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I avoid 1681, 1763, 2021 of non-prime numbers and I calculate polynomial including 1601, 1847, 2111 of 2 skipped numbers. 971, 1163, 1373, 1601, 1847, 2111 are prime numbers. Polynomial generating these prime numbers is calculated. The method of calculating polynomial is as follows.

971			971	n=0
	192			
1163		18	1163=971+192	n=1
	210		=971+192x1	
1373		18	1373=(971+192)+(192+18)	n=2
	228		=971+192x2+18x1	
1601		18	1601=(971+192)+(192+18)+(192+18+18)	n=3
	246		=971+192x3+18x3	
1847		18	1847=(971+192)+(192+18)+(192+18+18)+(192+18+18+18)	n=4
	264		=971+192x4+18x6	
2111			2111=(971+192)+(192+18)+(192+18+18)+(192+18+18+18)	n=5
			+(192+18+18+18+18)=971+192x5+18x10	

$f(n)=971+192n+18xn(n-1)/2=971+192n+9n^2-9n=9n^2+183n+971$

It can be confirmed later that this polynomial is prime numbers even if n = -1 to -23, so I insert n=n-23,

$f(n)=9(n-23)^2+183(n-23)+971=9n^2-9x2x23n+9x23x23+183n-183x23+971$

$=9n^2-414n+4761+183n-4209+971=9n^2-231n+1523$ (n=0,1,...,28)

And it can be confirmed later that this polynomial is prime numbers even if n = 29 to 39.

This is polynomial generating 40 prime numbers.

$P(n) = 9n^2 - 231n + 1523$, generates 40 primes: 1523, 1301, 1097, 911, 743, 593, 461, 347, 251, 173, 113, 71, 47, 41, 53, 83, 131, 197, 281, 383, 503, 641, 797, 971, 1163, 1373, 1601, 1847, 2111, 2393, 2693, 3011, 3347, 3701, 4073, 4463, 4871, 5297, 5741, 6203 .

The values of 1523 to 47 in the first half of the above value and 41 to 1601 in the middle part are the values of Euler prime numbers skipped by 2, and the values in the latter half are prime numbers larger than the maximum Euler prime number 1601.

3.2 New polynomial generating prime numbers 2

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1163, 1523, 1933 of 4 skipped numbers. 383, 593, 853, 1163, 1523, 1933 are prime numbers. Polynomial generating these prime numbers is calculated. The method of calculating polynomial is as follows.

383			383	n=0
	210			
593		50	593=383+210	n=1
	260		=383+210x1	
853		50	853=(383+210)+(210+50)	n=2
	310		=383+210x2+50x1	
1163		50	1163=(383+210)+(210+50)+(210+50+50)	n=3
	360		=383+210x3+50x3	
1523		50	1523=(383+210)+(210+50)+(210+50+50)+(210+50+50+50)	n=4
	410		=383+210x4+50x6	
1933			1933=(383+210)+(210+50)+(210+50+50)+(210+50+50+50)	n=5
			+(210+50+50+50+50)=383+210x5+50x10	

$f(n)=383+210n+50xn(n-1)/2=383+210n+25n^2-25n$

$=25n^2+185n+383$

It can be confirmed later that this polynomial is prime numbers even if n = -1 to -11, so I insert n=n-11,

$f(n)=25(n-11)^2+185(n-11)+383=25n^2-25x2x11n+25x11x11+185n-185x11+383$

$=25n^2-550n+3025+185n-2035+383=25n^2-365n+1373$ (n=0,1,...,16)

And it can be confirmed later that this polynomial is prime numbers even if n = 17 to 31.

This is polynomial generating 32 prime numbers.

$P(n) = 25n^2 - 365n + 1373$, generates 32 primes: 1373, 1033, 743, 503, 313, 173, 83, 43, 53, 113, 223, 383, 593, 853, 1163, 1523, 1933, 2393, 2903, 3463, 4073, 4733, 5443, 6203, 7013, 7873, 8783, 9743, 10753, 11813, 12923, 14083 .

3.3 New polynomial generating prime numbers 3

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1301, 1601, 1933 of 3 skipped numbers. 593, 797, 1033, 1301, 1601, 1933 are prime numbers. Polynomial generating these prime numbers is calculated. The method of calculating polynomial is as follows.

593			593	n=0
	204			
797		32	797=593+204	n=1
	236		=593+204x1	
1033		32	1033=(593+204)+(204+32)	n=2
	268		=593+204x2+32x1	
1301		32	1301=(593+204)+(204+32)+(204+32+32)	n=3
	300		=593+204x3+32x3	
1601		32	1601=(593+204)+(204+32)+(204+32+32)+(204+32+32+32)	n=4
	332		=593+204x4+32x6	
1933			1933=(593+204)+(204+32)+(204+32+32)+(204+32+32+32)	n=5
			+(204+32+32+32+32)=593+204x5+32x10	

$f(n)=593+204n+32xn(n-1)/2=593+204n+16n^2-16n$

$=16n^2+188n+593$

It can be confirmed later that this polynomial is prime numbers even if n = -1 to -15, so I insert n=n-15,

$f(n)=16(n-15)^2+188(n-15)+593=16n^2-16x2x15n+16x15x15+188n-188x15+593$

$=16n^2-480n+3600+188n-2820+593=16n^2-292n+1373$ (n=0,1,...,20)

And it can be confirmed later that this polynomial is prime numbers even if n = 21 to 30.

This is polynomial generating 31 prime numbers.

$P(n) = 16n^2 - 292n + 1373$, generates 31 primes: 1373, 1097, 853, 641, 461, 313, 197, 113, 61, 41, 53, 97, 173, 281, 421, 593, 797, 1033, 1301, 1601, 1933, 2297, 2693, 3121, 3581, 4073, 4597, 5153, 5741, 6361, 7013 .

3.4 New polynomial generating prime numbers 4

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1231, 1523, 1847 of 3 skipped numbers. 547, 743, 971, 1231, 1523, 1847 are prime numbers. Polynomial generating these prime numbers is calculated. The method of calculating polynomial is as follows.

547			547	n=0
	196			
743		32	743=547+196	n=1
	228		=547+196x1	
971		32	971=(547+196)+(196+32)	n=2
	260		=547+196x2+32x1	
1231		32	1231=(547+196)+(196+32)+(196+32+32)	n=3
	292		=547+196x3+32x3	
1523		32	1523=(547+196)+(196+32)+(196+32+32)+(196+32+32+32)	n=4
	324		=547+196x4+32x6	
1847			1847=(547+196)+(196+32)+(196+32+32)+(196+32+32+32)	n=5
			+(196+32+32+32+32)=547+196x5+32x10	

$$f(n)=547+196n+32 \times n(n-1)/2=547+196n+16n^2-16n$$
$$=16n^2+180n+547$$

It can be confirmed later that this polynomial is prime numbers even if n = -1 to -15, so I insert n=n-15,

$f(n)=16(n-15)^2+180(n-15)+547=16n^2-16 \times 2 \times 15n+16 \times 15 \times 15+180n-180 \times 15+547$

$=16n^2-480n+3600+180n-2700+547=16n^2-300n+1447$ (n=0,1,…,20)

And it can be confirmed later that this polynomial is prime numbers even if n = 21 to 29.

This is polynomial generating 30 prime numbers.

$P(n) = 16n^2 - 300n + 1447$, generates 30 primes: 1447, 1163, 911, 691, 503, 347, 223, 131, 71, 43, 47, 83, 151, 251, 383, 547, 743, 971, 1231, 1523, 1847, 2203, 2591, 3011, 3463, 3947, 4463, 5011, 5591, 6203 .

3.5 New polynomial generating prime numbers 5

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 691, 1231, 1933 of 8 skipped numbers. 43, 97, 313, 691, 1231, 1933 are prime numbers. Since the method of obtaining the polynomial is the same as the method described above, so I will omit below.

This is polynomial generating 28 prime numbers.

$P(n) = 81n^2 - 1323n + 5443$, generates 28 primes: 5443, 4201, 3121, 2203, 1447, 853, 421, 151, 43, 97, 313, 691, 1231, 1933, 2797, 3823, 5011, 6361, 7873, 9547, 11383, 13381, 15541, 17863, 20347, 22993, 25801, 28771 .

3.6 New polynomial generating prime numbers 6

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 853, 1447, 2203 of 8 skipped numbers.

This is polynomial generating 28 prime numbers.
$P(n) = 81n^2 - 3051n + 28771$, generates 28 primes: 28771, 25801, 22993, 20347, 17863, 15541, 13381, 11383, 9547, 7873, 6361, 5011, 3823, 2797, 1933, 1231, 691, 313, 97, 43, 151, 421, 853, 1447, 2203, 3121, 4201, 5443 .

3.7 New polynomial generating prime numbers 7

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 911, 1373, 1933 of 6 skipped numbers.

This is polynomial generating 23 prime numbers.
$P(n) = 49n^2 - 469n + 1163$, generates 23 primes: 1163, 743, 421, 197, 71, 43, 113, 281, 547, 911, 1373, 1933, 2591, 3347, 4201, 5153, 6203, 7351, 8597, 9941, 11383, 12923, 14561 .

3.8 New polynomial generating prime numbers 8

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 911, 1447, 2111 of 7 skipped numbers.

This is polynomial generating 23 prime numbers.
$P(n) = 64n^2 - 1192n + 5591$, generates 23 primes: 5591, 4463, 3463, 2591, 1847, 1231, 743, 383, 151, 47, 71, 223, 503, 911, 1447, 2111, 2903, 3823, 4871, 6047, 7351, 8783, 10343 .

3.9 New polynomial generating prime numbers 9

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 743, 1231, 1847 of 7 skipped numbers.

This is polynomial generating 23 prime numbers.

$P(n) = 64n^2 - 1624n + 10343$, generates 23 primes: 10343, 8783, 7351, 6047, 4871, 3823, 2903, 2111, 1447, 911, 503, 223, 71, 47, 151, 383, 743, 1231, 1847, 2591, 3463, 4463, 5591 .

Note that the above values are the same as in Section 3.8 and they will appear in reverse order, like mirror.

3.10 New polynomial generating prime numbers 10

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 971, 1373, 1847 of 5 skipped numbers.

This is polynomial generating 20 prime numbers.

$P(n) = 36n^2 - 426n + 1301$, generates 20 primes: 1301, 911, 593, 347, 173, 71, 41, 83, 197, 383, 641, 971, 1373, 1847, 2393, 3011, 3701, 4463, 5297, 6203 .

3.11 New polynomial generating prime numbers 11

In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1163, 1601, 2111 of 5 skipped numbers.

This is polynomial generating 20 prime numbers.

$P(n) = 36n^2 - 462n + 1523$, generates 20 primes: 1523, 1097, 743, 461, 251, 113, 47, 53, 131, 281, 503, 797, 1163, 1601, 2111, 2693, 3347, 4073, 4871, 5741 .

3.12 Other new polynomial generating prime numbers

3.12.1 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 347, 853, 1601 of 10 skipped numbers.
This is polynomial generating 18 prime numbers.

$P(n) = 121n^2 - 1551n + 5011$, generates 18 primes: 5011, 3581, 2393, 1447, 743, 281, 61, 83, 347, 853, 1601, 2591, 3823, 5297, 7013, 8971, 11171, 13613 .

3.12.2 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 281, 743, 1447 of 10 skipped numbers.
This is polynomial generating 18 prime numbers.

$P(n) = 121n^2 - 2563n + 13613$, generates 18 primes: 13613, 11171, 8971, 7013, 5297, 3823, 2591, 1601, 853, 347, 83, 61, 281, 743, 1447, 2393, 3581, 5011 .

3.12.3 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 593, 1301, 2297 of 11 skipped numbers.
This is polynomial generating 18 prime numbers.
$P(n) = 144n^2 - 1740n + 5297$, generates 18 primes: 5297, 3701, 2393, 1373, 641, 197, 41, 173, 593, 1301, 2297, 3581, 5153, 7013, 9161, 11597, 14321, 17333 .

3.12.4 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 197, 641, 1373 of 11 skipped numbers.
This is polynomial generating 18 prime numbers.
$P(n) = 144n^2 - 3156n + 17333$, generates 18 primes: 17333, 14321, 11597, 9161, 7013, 5153, 3581, 2297, 1301, 593, 173, 41, 197, 641, 1373, 2393, 3701, 5297 .

3.12.5 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1097, 1601, 2203 of 6 skipped numbers.
This is polynomial generating 17 prime numbers.
$P(n) = 49n^2 - 525n + 1447$, generates 17 primes: 1447, 971, 593, 313, 131, 47, 61, 173, 383, 691, 1097, 1601, 2203, 2903, 3701, 4597, 5591 .

3.12.6 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 971, 1523, 2203 of 7 skipped numbers.
This is polynomial generating 17 prime numbers.
$P(n) = 64n^2 - 536n + 1163$, generates 17 primes: 1163, 691, 347, 131, 43, 83, 251, 547, 971, 1523, 2203, 3011, 3947, 5011, 6203, 7523, 8971 .

3.12.7 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 383, 853, 1523 of 9 skipped numbers.
This is polynomial generating 17 prime numbers.
$P(n) = 100n^2 - 630n + 1033$, generates 17 primes: 1033, 503, 173, 43, 113, 383, 853, 1523, 2393, 3463, 4733, 6203, 7873, 9743, 11813, 14083, 16553 .

3.12.8 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 593, 1163, 1933 of 9 skipped numbers.
This is polynomial generating 17 prime numbers.
$P(n) = 100n^2 - 930n + 2203$, generates 17 primes: 2203, 1373, 743, 313, 83, 53, 223, 593,

1163, 1933, 2903, 4073, 5443, 7013, 8783, 10753, 12923 .

3.12.9 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 743, 1373, 2203 of 9 skipped numbers.
This is polynomial generating 17 prime numbers.
$P(n) = 100n^2 - 2270n + 12923$, generates 17 primes: 12923, 10753, 8783, 7013, 5443, 4073, 2903, 1933, 1163, 593, 223, 53, 83, 313, 743, 1373, 2203 .

3.12.10 In Figure 3.1, in the $n^2 + n + 17$ vertical column of Euler's polynomial generating prime with lucky number p = 17, I calculate polynomial including 257, 359, 479 of 2 skipped numbers.
This is polynomial generating 16 prime numbers.
$P(n) = 9n^2 - 87n + 227$, generates 16 primes: 227, 149, 89, 47, 23, 17, 29, 59, 107, 173, 257, 359, 479, 617, 773, 947 .

3.12.11 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1097, 1447, 1847 of 4 skipped numbers.
This is polynomial generating 16 prime numbers.
$P(n) = 25n^2 + 25n + 47$, generates 16 primes: 47, 97, 197, 347, 547, 797, 1097, 1447, 1847, 2297, 2797, 3347, 3947, 4597, 5297, 6047 .
This polynomial generating prime number produces the same prime number even if $n = -1$ to -16.

3.12.12 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 797, 1301, 1933 of 7 skipped numbers.
This is polynomial generating 16 prime numbers.
$P(n) = 64n^2 - 584n + 1373$, generates 16 primes: 1373, 853, 461, 197, 61, 53, 173, 421, 797, 1301, 1933, 2693, 3581, 4597, 5741, 7013 .

3.12.13 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 797, 1447, 2297 of 9 skipped numbers.
This is polynomial generating 16 prime numbers.
$P(n) = 100n^2 - 1450n + 5297$, generates 16 primes: 5297, 3947, 2797, 1847, 1097, 547, 197, 47, 97, 347, 797, 1447, 2297, 3347, 4597, 6047 .

3.12.14 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 461, 1033, 1847 of 10 skipped numbers. This is polynomial generating 16 prime numbers.
$P(n) = 121n^2 - 1001n + 2111$, generates 16 primes: 2111, 1231, 593, 197, 43, 131, 461, 1033, 1847, 2903, 4201, 5741, 7523, 9547, 11813, 14321 .

3.12.15 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 593, 1231, 2111 of 10 skipped numbers. This is polynomial generating 16 prime numbers.
$P(n) = 121n^2 - 2629n + 14321$, generates 16 primes: 14321, 11813, 9547, 7523, 5741, 4201, 2903, 1847, 1033, 461, 131, 43, 197, 593, 1231, 2111 .

3.12.16 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 503, 1163, 2111 of 11 skipped numbers. This is polynomial generating 16 prime numbers.
$P(n) = 144n^2 - 2652n + 12251$, generates 16 primes: 12251, 9743, 7523, 5591, 3947, 2591, 1523, 743, 251, 47, 131, 503, 1163, 2111, 3347, 4871 .

3.12.17 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 197, 691, 1523 of 12 skipped numbers. This is polynomial generating 16 prime numbers.
$P(n) = 169n^2 - 1365n + 2797$, generates 16 primes: 2797, 1601, 743, 223, 41, 197, 691, 1523, 2693, 4201, 6047, 8231, 10753, 13613, 16811, 20347 .

3.12.18 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 223, 743, 1601 of 12 skipped numbers. This is polynomial generating 16 prime numbers.
$P(n) = 169n^2 - 3705n + 20347$, generates 16 primes: 20347, 16811, 13613, 10753, 8231, 6047, 4201, 2693, 1523, 691, 197, 41, 223, 743, 1601, 2797 .

3.12.19 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 593, 1033, 1601 of 7 skipped numbers. This is polynomial generating 15 prime numbers.
$P(n) = 64n^2 - 520n + 1097$, generates 15 primes: 1097, 641, 313, 113, 41, 97, 281, 593, 1033, 1601, 2297, 3121, 4073, 5153, 6361 .

3.12.20 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 911, 1523, 2297 of 8 skipped numbers. This is polynomial generating 15 prime numbers.

$P(n) = 81n^2 - 855n + 2297$, generates 15 primes: 2297, 1523, 911, 461, 173, 47, 83, 281, 641, 1163, 1847, 2693, 3701, 4871, 6203 .

3.12.21 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 911, 1523, 2297 of 8 skipped numbers. This is polynomial generating 15 prime numbers.

$P(n) = 81n^2 - 1413n + 6203$, generates 15 primes: 6203, 4871, 3701, 2693, 1847, 1163, 641, 281, 83, 47, 173, 461, 911, 1523, 2297 .

3.12.22 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 547, 1097, 1847 of 9 skipped numbers. This is polynomial generating 15 prime numbers.

$P(n) = 100n^2 - 1550n + 6047$, generates 15 primes: 6047, 4597, 3347, 2297, 1447, 797, 347, 97, 47, 197, 547, 1097, 1847, 2797, 3947 .

3.12.23 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 461, 1163, 2203 of 12 skipped numbers. This is polynomial generating 15 prime numbers.

$P(n) = 169n^2 - 819n + 1033$, generates 15 primes: 1033, 383, 71, 97, 461, 1163, 2203, 3581, 5297, 7351, 9743, 12473, 15541, 18947,22691 .

3.12.24 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1033, 1447, 1933 of 5 skipped numbers. This is polynomial generating 14 prime numbers.

$P(n) = 36n^2 - 414n + 1231$, generates 14 primes: 1231, 853, 547, 313, 151, 61, 43, 97, 223, 421, 691, 1033, 1447, 1933 .

3.12.25 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 641, 1301, 2203 of 10 skipped numbers. This is polynomial generating 14 prime numbers.

$P(n) = 121n^2 - 671n + 971$, generates 14 primes: 971, 421, 113, 47, 223, 641, 1301, 2203, 3347, 4733, 6361, 8231, 10343, 12697 .

3.12.26 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 503, 1231, 2297 of 12 skipped numbers. This is polynomial generating 14 prime numbers.

$P(n) = 169n^2 - 1131n + 1933$, generates 14 primes: 1933, 971, 347, 61, 113, 503, 1231, 2297, 3701, 5443, 7523, 9941, 12697, 15791 .

3.12.27 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 347, 971, 1933 of 12 skipped numbers. This is polynomial generating 14 prime numbers.

$P(n) = 169n^2 - 3263n + 15791$, generates 14 primes: 15791, 12697, 9941, 7523, 5443, 3701, 2297, 1231, 503, 113, 61, 347, 971, 1933 .

3.12.28 In Figure 3.1, in the $n^2 + n + 17$ vertical column of Euler's polynomial generating prime with lucky number $p = 17$, I calculate polynomial including 773, 1009, 1277 of 3 skipped numbers.

This is polynomial generating 13 prime numbers.

$P(n) = 16n^2 - 100n + 173$, generates 13 primes: 173, 89, 37, 17, 29, 73, 149, 251, 397, 569, 773, 1009, 1277 .

3.12.29 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 503, 971, 1601 of 8 skipped numbers. This is polynomial generating 13 prime numbers.

$P(n) = 81n^2 - 585n + 1097$, generates 13 primes: 1097, 593, 251, 71, 53, 197, 503, 971, 1601, 2393, 3347, 4463, 5741 .

3.12.30 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 797, 1373, 2111 of 8 skipped numbers. This is polynomial generating 13 prime numbers.

$P(n) = 81n^2 - 639n + 1301$, generates 13 primes: 1301, 743, 347, 113, 41, 131, 383, 797, 1373, 2111, 3011, 4073, 5297 .

3.12.31 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 1033, 1523, 2111 of 6 skipped numbers. This is polynomial generating 12 prime numbers.

$P(n) = 49n^2 + 49n + 53$, generates 126 primes: 53, 151, 347, 641, 1033, 1523, 2111, 2797, 3581, 4463, 5443, 6521 .

This polynomial generating prime number produces the same prime number even if n = -1 to -12.

3.12.32 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 853, 1301, 1847 of 6 skipped numbers. This is polynomial generating 12 prime numbers.

$P(n) = 49n^2 - 483n + 1231$, generates 12 primes: 1231, 797, 461, 223, 83, 41, 97, 251, 503, 853, 1301, 1847 .

3.12.33 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 131, 547, 1301 of 12 skipped numbers. This is polynomial generating 12 prime numbers.

$P(n) = 169n^2 - 1781n + 4733$, generates 12 primes: 4733, 3121, 1847, 911, 313, 53, 131, 547, 1301, 2393, 3823, 5591 .

3.12.34 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 313, 911, 1847 of 12 skipped numbers. This is polynomial generating 12 prime numbers.

$P(n) = 169n^2 - 1937n + 5591$, generates 12 primes: 5591, 3823, 2393, 1301, 547, 131, 53, 313, 911, 1847, 3121, 4733 .

3.12.35 In Figure 3.1, in the $n^2 + n + 17$ vertical column of Euler's polynomial generating prime with lucky number p = 17, I calculate polynomial including 479, 887, 1423 of 7 skipped numbers.

This is polynomial generating 11 prime numbers.

$P(n) = 64n^2 - 424n + 719$, generates 11 primes: 719, 359, 127, 23, 47, 199, 479, 887, 1423, 2087, 2879 .

3.12.36 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 691, 1373, 2297 of 10 skipped numbers. This is polynomial generating 11 prime numbers.

$P(n) = 121n^2 - 649n + 911$, generates 11 primes: 911, 383, 97, 53, 251, 691, 1373, 2297, 3463, 4871, 6521 .

3.12.37 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 383, 971, 1847 of 11 skipped numbers.

This is polynomial generating 11 prime numbers.

$P(n) = 144n^2 - 708n + 911$, generates 11 primes: 911, 347, 71, 83, 383, 971, 1847, 3011, 4463, 6203, 8231 .

3.12.38 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 691, 1301, 2111 of 9 skipped numbers. This is polynomial generating 10 prime numbers.

$P(n) = 100n^2 - 690n + 1231$, generates 10 primes: 1231, 641, 251, 61, 71, 281, 691, 1301, 2111, 3121 .

3.12.39 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 421, 1033, 1933 of 11 skipped numbers. This is polynomial generating 10 prime numbers.

$P(n) = 144n^2 - 684n + 853$, generates 10 primes: 853, 313, 61, 97, 421, 1033, 1933, 3121, 4597, 6361 .

3.12.40 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 281, 797, 1601 of 11 skipped numbers. This is polynomial generating 10 prime numbers.

$P(n) = 144n^2 - 780n + 1097$, generates 10 primes: 1097, 461, 113, 53, 281, 797, 1601, 2693, 4073, 5741 .

3.12.41 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 547, 1231, 2203 of 11 skipped numbers. This is polynomial generating 10 prime numbers.

$P(n) = 144n^2 - 900n + 1447$, generates 10 primes: 1447, 691, 223, 43, 151, 547, 1231, 2203, 3463, 5011 .

3.12.42 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 251, 743, 1523 of 11 skipped numbers. This is polynomial generating 10 prime numbers.

$P(n) = 144n^2 - 1668n + 4871$, generates 10 primes: 4871, 3347, 2111, 1163, 503, 131, 47, 251, 743, 1523 .

3.12.43 In Figure 3.1, in the $n^2 + n + 41$ vertical columns of Euler's polynomial generating prime, I calculate polynomial including 223, 691, 1447 of 11 skipped numbers.

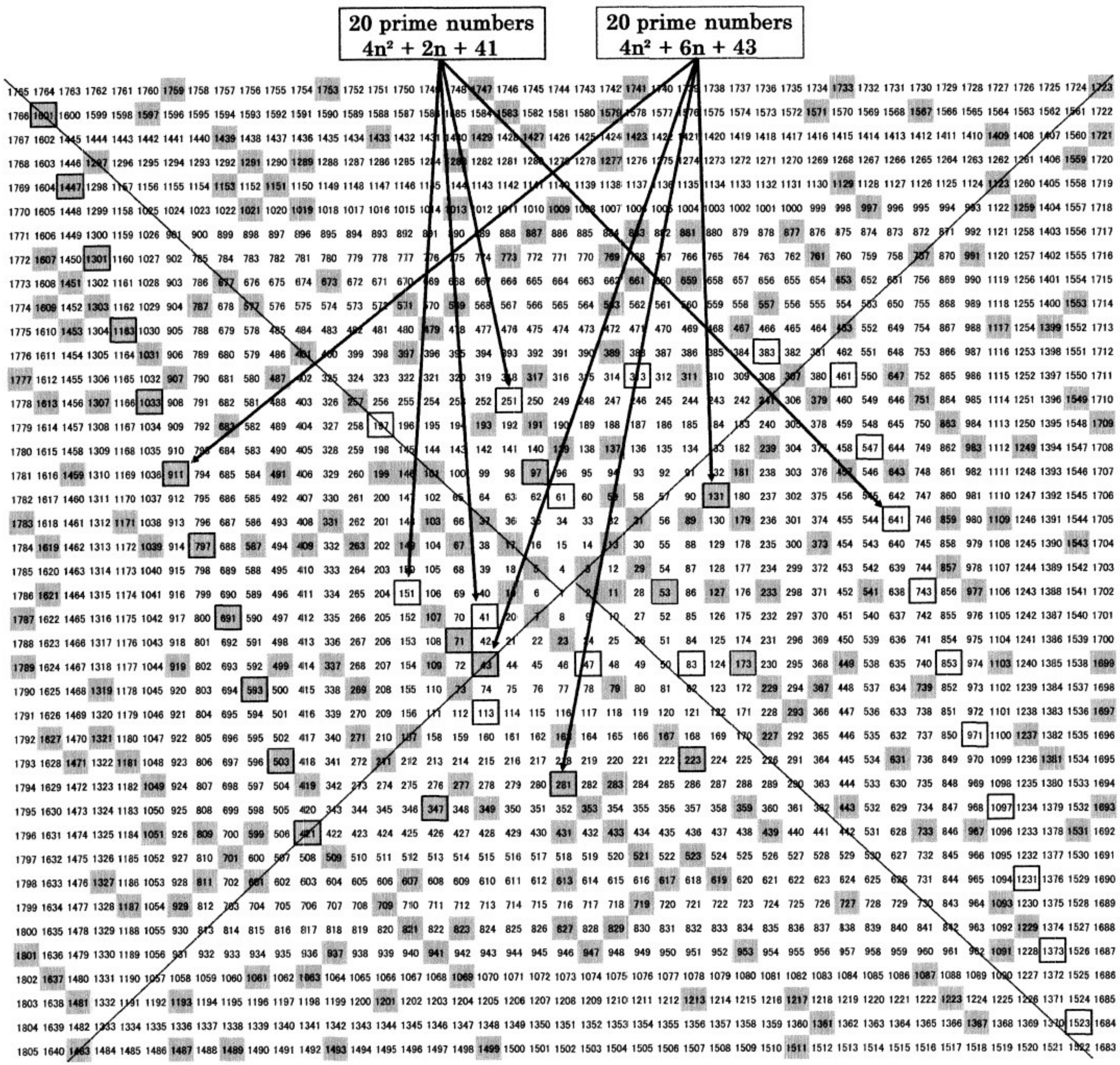
This is polynomial generating 10 prime numbers.
$P(n) = 144n^2 - 1692n + 5011$, generates 10 primes: 5011, 3463, 2203, 1231, 547, 151, 43, 223, 691, 1447 .

3.13 Skipping numbers and continuous prime numbers

I summarize continuous prime numbers for each skipping numbers in the vertical column of Euler's polynomial generating prime numbers $n^2 + n + 41$ in Figure 3.1.
If skipping numbers are small, continuous primes numbers are large, but even if skipping number are large, continuous primes numbers are large unexpectedly.

Skipping numbers	Continuous prime numbers
2	40
3	31, 30
4	32, 16
5	20, 20, 14
6	23, 17, 12, 12
7	23, 23, 17, 16, 15
8	28, 28, 15, 15, 13, 13
9	17, 17, 17, 16, 15, 10, 3
10	18, 18, 16, 16, 14, 11, 9, 4
11	18, 18, 16, 11, 10, 10, 10, 10, 10
12	16, 16, 15. 14, 14, 12, 12, 8, 6, 6,

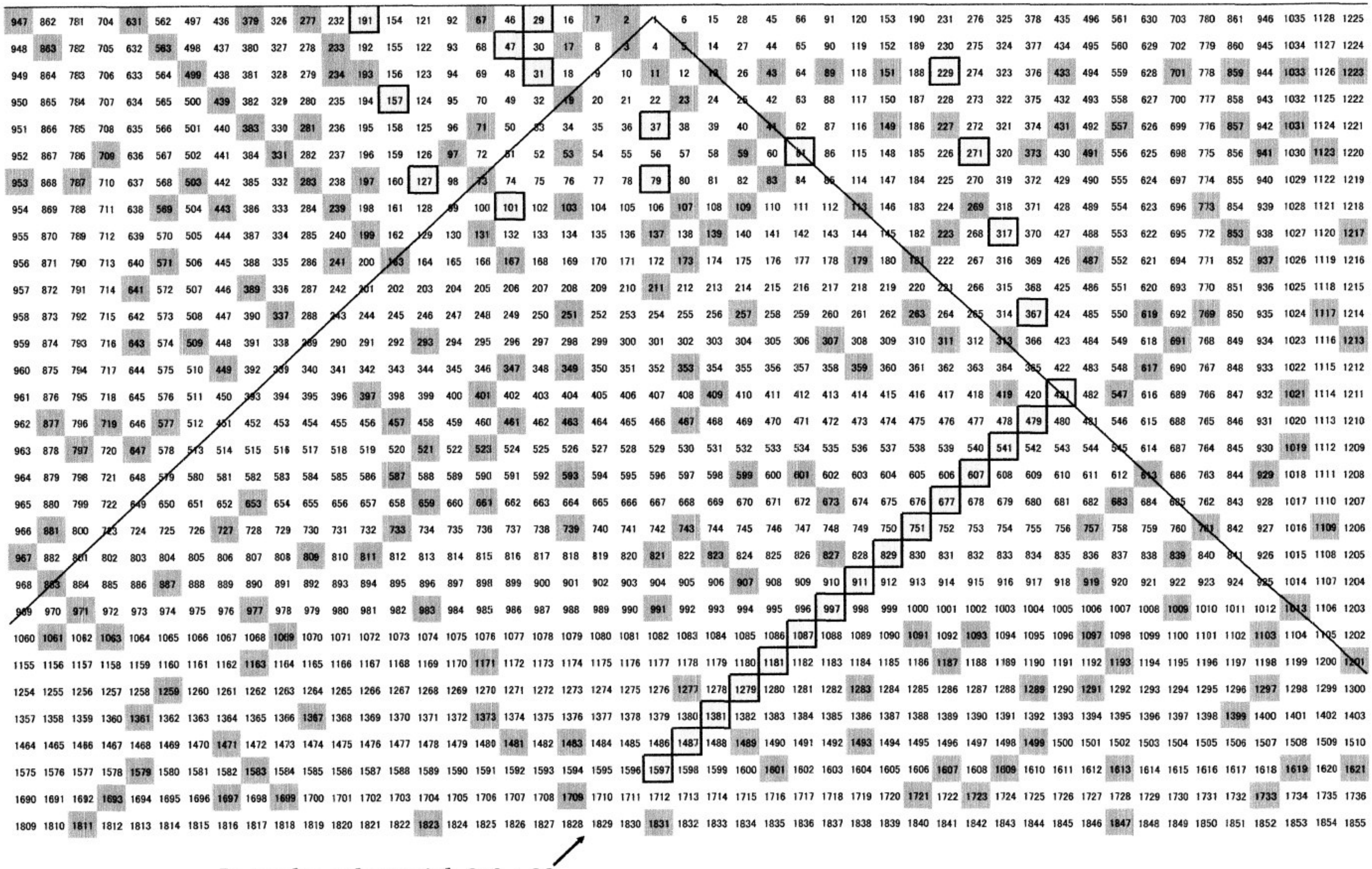
Figure 2.1: The Ulam Spiral



Euler's prime numbers

$4n^2 + 2n + 41$	$4n^2 + 6n + 43$	$n^2 + n + 41$
41		41
	43	43
47		47
	53	53
61		61
	71	71
83		83
	97	97
113		113
	131	131
151		151
	173	173
197		197
	223	223
251		251
	281	281
313		313
	347	347
383		383
	421	421
461		461
	503	503
547		547
	593	593
641		641
	691	691
743		743
	797	797
853		853
	911	911
971		971
	1033	1033
1097		1097
	1163	1163
1231		1231
	1301	1301
1373		1373
	1447	1447
1523		1523
	1601	1601

Figure 2.2: 180 degrees Arrangement Legendre Polynomial



Legendre polynomial $2n^2 + 29$

Figure 2.3: 135 degrees Arrangement Brox Polynomial

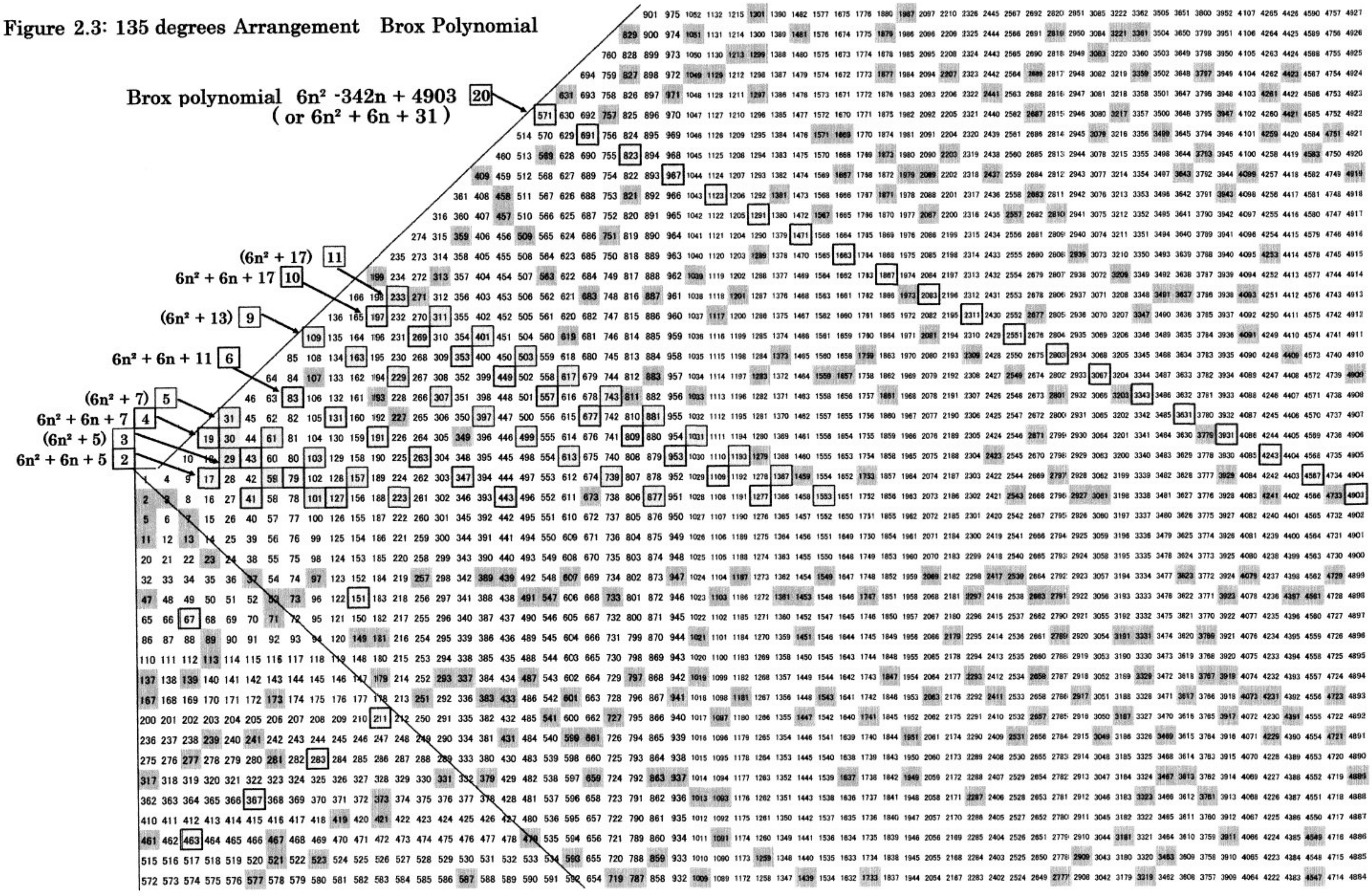


Figure 2.4: 270 degrees Arrangement　Frame Polynomial

Frame polynomial $3n^2 + 3n + 23$

																								280	279	278	277	276	275	274	273	272	271
																								225	224	223	222	221	220	219	218	217	270
																								176	175	174	173	172	171	170	169	216	269
																								133	132	131	130	129	128	127	168	215	268
																								96	95	94	93	92	91	126	167	214	267
																								65	64	63	62	61	90	125	166	213	266
																								40	39	38	37	60	89	124	165	212	265
																								21	20	19	36	59	88	123	164	211	264
																								8	7	18	35	58	87	122	163	210	263
1681	1542	1409	1282	1161	1046	937	834	737	646	561	482	409	342	281	226	177	134	97	66	41	22	9	2	1	6	17	34	57	86	121	162	209	262
1682	1543	1410	1283	1162	1047	938	835	738	647	562	483	410	343	282	227	178	135	98	67	42	23	10	3	4	5	16	33	56	85	120	161	208	261
1683	1544	1411	1284	1163	1048	939	836	739	648	563	484	411	344	283	228	179	136	99	68	43	24	11	12	13	14	15	32	55	84	119	160	207	260
1684	1545	1412	1285	1164	1049	940	837	740	649	564	485	412	345	284	229	180	137	100	69	44	25	26	27	28	29	30	31	54	83	118	159	206	259
1685	1546	1413	1286	1165	1050	941	838	741	650	565	486	413	346	285	230	181	138	101	70	45	46	47	48	49	50	51	52	53	82	117	158	205	258
1686	1547	1414	1287	1166	1051	942	839	742	651	566	487	414	347	286	231	182	139	102	71	72	73	74	75	76	77	78	79	80	81	116	157	204	257
1687	1548	1415	1288	1167	1052	943	840	743	652	567	488	415	348	287	232	183	140	103	104	105	106	107	108	109	110	111	112	113	114	115	156	203	256
1688	1549	1416	1289	1168	1053	944	841	744	653	568	489	416	349	288	233	184	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	202	255
1689	1550	1417	1290	1169	1054	945	842	745	654	569	490	417	350	289	234	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	254
1690	1551	1418	1291	1170	1055	946	843	746	655	570	491	418	351	290	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253
1691	1552	1419	1292	1171	1056	947	844	747	656	571	492	419	352	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310
1692	1553	1420	1293	1172	1057	948	845	748	657	572	493	420	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373
1693	1554	1421	1294	1173	1058	949	846	749	658	573	494	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442
1694	1555	1422	1295	1174	1059	950	847	750	659	574	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517
1695	1556	1423	1296	1175	1060	951	848	751	660	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598

Figure 2.5: 90 degrees Arrangement Euler's Polynomial

Euler prime numbers $n^2 + n + 41$

$n^2 + n + 3$

$n^2 + n + 5$

$n^2 + n + 11$

$n^2 + n + 17$

Euler prime numbers $n^2 + n + 41$

1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	441	484	529	576	625	676
2	3	8	15	24	35	48	63	80	99	120	143	168	195	224	255	288	323	360	399	440	483	528	575	624	675
5	6	7	14	23	34	47	62	79	98	119	142	167	194	223	254	287	322	359	398	439	482	527	574	623	674
10	11	12	13	22	33	46	61	78	97	118	141	166	193	222	253	286	321	358	397	438	481	526	573	622	673
17	18	19	20	21	32	45	60	77	96	117	140	165	192	221	252	285	320	357	396	437	480	525	572	621	672
26	27	28	29	30	31	44	59	76	95	116	139	164	191	220	251	284	319	356	395	436	479	524	571	620	671
37	38	39	40	41	42	43	58	75	94	115	138	163	190	219	250	283	318	355	394	435	478	523	570	619	670
50	51	52	53	54	55	56	57	74	93	114	137	162	189	218	249	282	317	354	393	434	477	522	569	618	669
65	66	67	68	69	70	71	72	73	92	113	136	161	188	217	248	281	316	353	392	433	476	521	568	617	668
82	83	84	85	86	87	88	89	90	91	112	135	160	187	216	247	280	315	352	391	432	475	520	567	616	667
101	102	103	104	105	106	107	108	109	110	111	134	159	186	215	246	279	314	351	390	431	474	519	566	615	666
122	123	124	125	126	127	128	129	130	131	132	133	158	185	214	245	278	313	350	389	430	473	518	565	614	665
145	146	147	148	149	150	151	152	153	154	155	156	157	184	213	244	277	312	349	388	429	472	517	564	613	664
170	171	172	173	174	175	176	177	178	179	180	181	182	183	212	243	276	311	348	387	428	471	516	563	612	663
197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	242	275	310	347	386	427	470	515	562	611	662
226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	274	309	346	385	426	469	514	561	610	661
257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	308	345	384	425	468	513	560	609	660
290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	344	383	424	467	512	559	608	659
325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	382	423	466	511	558	607	658
362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	422	465	510	557	606	657
401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	464	509	556	605	656
442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	508	555	604	655
485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	554	603	654
530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	602	653
577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	652
626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651
677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702
730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755
785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810
842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867
901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926
962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987
1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050
1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115
1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182
1226	1227	1228	1229	1230	1231	1232	1233	1234	1235	1236	1237	1238	1239	1240	1241	1242	1243	1244	1245	1246	1247	1248	1249	1250	1251
1297	1298	1299	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310	1311	1312	1313	1314	1315	1316	1317	1318	1319	1320	1321	1322
1370	1371	1372	1373	1374	1375	1376	1377	1378	1379	1380	1381	1382	1383	1384	1385	1386	1387	1388	1389	1390	1391	1392	1393	1394	1395
1445	1446	1447	1448	1449	1450	1451	1452	1453	1454	1455	1456	1457	1458	1459	1460	1461	1462	1463	1464	1465	1466	1467	1468	1469	1470
1522	1523	1524	1525	1526	1527	1528	1529	1530	1531	1532	1533	1534	1535	1536	1537	1538	1539	1540	1541	1542	1543	1544	1545	1546	1547
1601	1602	1603	1604	1605	1606	1607	1608	1609	1610	1611	1612	1613	1614	1615	1616	1617	1618	1619	1620	1621	1622	1623	1624	1625	1626
1682	1683	1684	1685	1686	1687	1688	1689	1690	1691	1692	1693	1694	1695	1696	1697	1698	1699	1700	1701	1702	1703	1704	1705	1706	1707
1765	1766	1767	1768	1769	1770	1771	1772	1773	1774	1775	1776	1777	1778	1779	1780	1781	1782	1783	1784	1785	1786	1787	1788	1789	1790
1850	1851	1852	1853	1854	1855	1856	1857	1858	1859	1860	1861	1862	1863	1864	1865	1866	1867	1868	1869	1870	1871	1872	1873	1874	1875

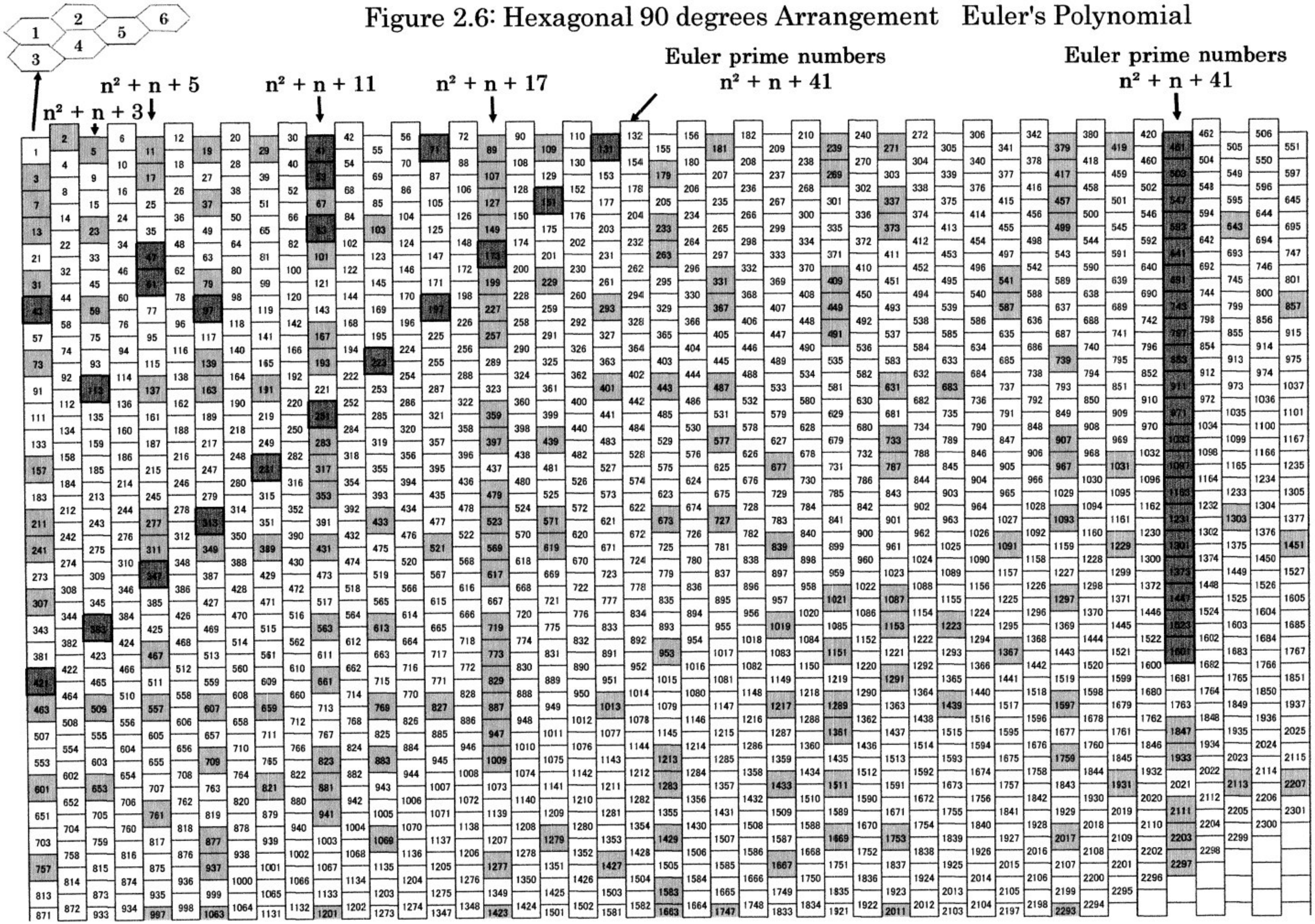
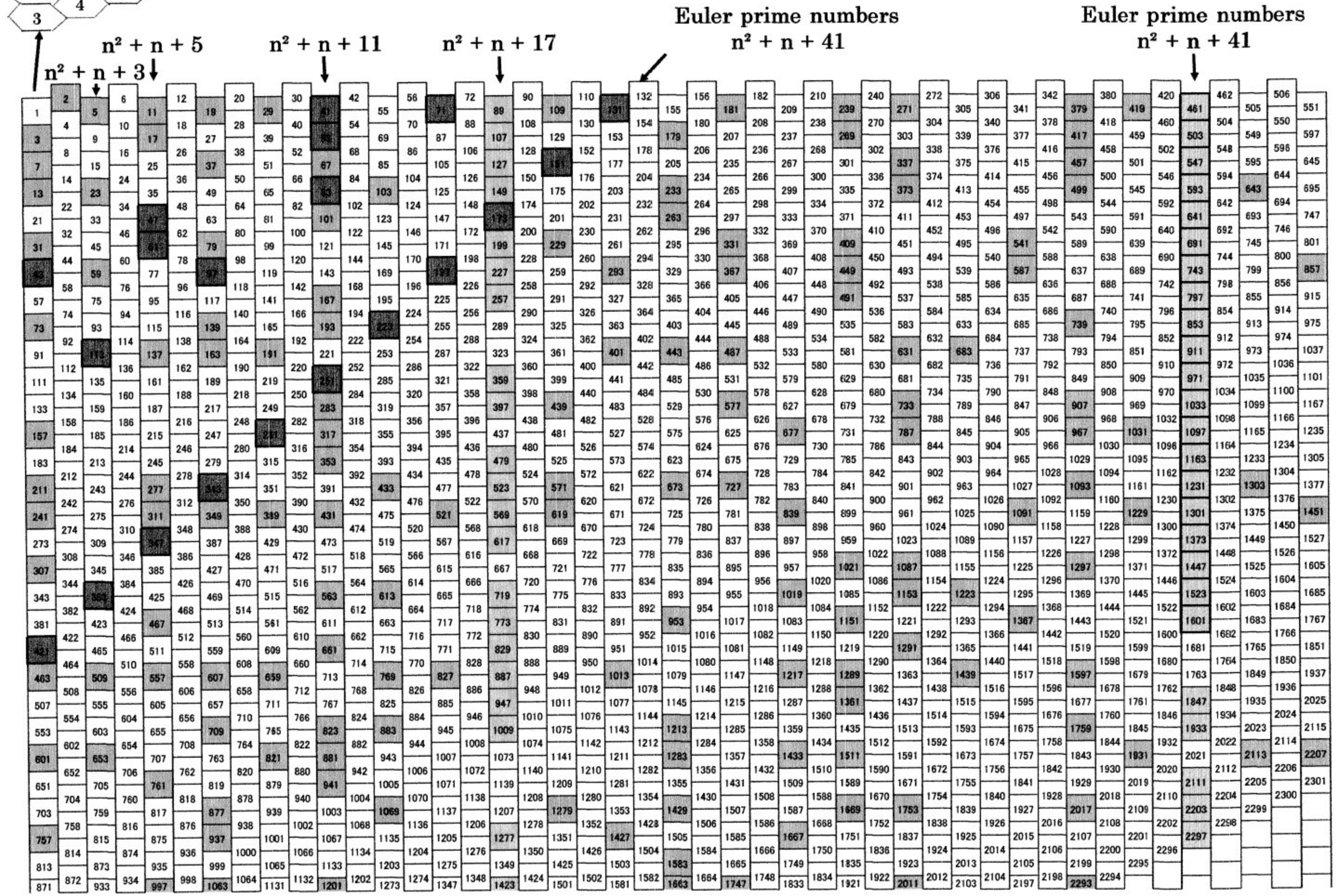


Figure 2.6: Hexagonal 90 degrees Arrangement Euler's Polynomial

Figure 3.1: Hexagonal 90 degrees Arrangement Euler's Polynomial



5 Consideration

I am very lucky that I found many polynomials generating prime numbers only to skip the values of column of Euler prime numbers.
It is expected that polynomial of many continuous prime numbers can be found by skipping successfully the values of polynomials containing many prime numbers.

6. Acknowledgment

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