A more detailed calculation of Jupiter's influence on the measurement of the gravitational constant

Zhi Cheng

gzchengzhi@hotmail.com

Abstract: Based on the analysis of the previous article, this article analyzes in detail the influence of Jupiter on the measurement of the gravitational constant. Including a specific estimate of the change in the gravitational constant caused by the change of the earth's orbital position every half month. The influence of Jupiter's perihelion and aphelion on the gravitational constant is also estimated. On this basis, this article analyzes the data with specific time records such as BIPM-01, BIPM-14, JILA-10, UCI-14, HUST-09. The analysis results show that after considering the influence of Jupiter, the experimental results have been significantly improved.

Keywords: gravitational constant; measurement of gravitational constant; Jupiter

1 Introduction

The measurement of the gravitational constant is very difficult. Nevertheless, with the development of various related technologies since 2000, the measurement accuracy of the gravitational constant has been significantly improved. Therefore, some systematic errors that could not be considered in the past can also be taken into account, thereby effectively improving data consistency.

Most of the mass of the solar system is concentrated in the sun. But even though Jupiter is only one-thousandth of the mass of the sun, it is enough to affect gravitational activity on the earth. One of the effects is the measurement experiment of the gravitational constant on the earth.

In order to explore the influence of Jupiter on the gravitational constant, a basic assumption is needed, that is, space-time compression will change the gravitational constant. The specific analysis is explained in detail in my last paper [1]. The basic idea is to assume that space-time compression will lead to a shortening of the spatial measurement scale. However, it is observed in the distant flat space-time observation system that the mass to form the space-time should not be changed, otherwise it violates the law of conservation of energy. In this way, the Schwarzschild radius of the mass measured in the flat space-time reference frame should also remain unchanged. Now switch to the compressed space-time reference system. As the space measurement unit is reduced, the measured Schwarzschild radius should become longer. Regardless of the frame of reference, the mass cannot be changed, which is the requirement of the law of conservation of energy. The speed of light is constant and cannot be changed. Therefore, what can be changed when space-time is compressed is the gravitational constant. The degree of space-time compression is proportional to the absolute value of the potential energy generated by all masses at that point. This is the most basic assumption of this article and my last paper.

With the assumption that space-time compression will increase the gravitational constant, we can specifically calculate the influence of Jupiter's position on the gravitational constant measured on the earth. This article will give a more detailed estimation method.

2 Detailed calculation of Jupiter's influence on the gravitational constant

2.1 Jupiter and Earth are in circular orbits

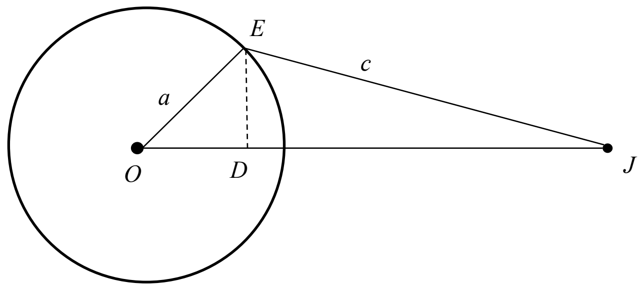

Figure 1. The positions of the earth, Jupiter and the sun

In Figure 1, *O* is the position of the sun, *E* is the position of the earth, and *J* is the position of Jupiter. Since Jupiter has an orbital period of 11.8 years, Jupiter and the sun can be regarded as stationary. For the convenience of calculation, cross point *E* to make a vertical line *ED* perpendicular to *OJ*.

OE is the orbital radius of Jupiter to the Sun. The orbit of Jupiter is approximately regarded as a circle here.

$$\angle EOD = \theta = \frac{x}{6}\pi$$

Where *x* is the time when the earth leaves Jupiter's opposition position, the unit is "month"

And the distance from Jupiter to Earth is

$$JE = c$$

Then

$$OD = acos\theta$$

$$ED = asin\theta$$

$$JD = OJ - OD$$

Therefore

$$c = \sqrt{a^2 sin^2\theta + (OJ - OD)^2}$$

Then calculate the total potential energy of the Sun and Jupiter at point *E*

$$V_E = -G_0 m_e \left(\frac{M}{a} + \frac{m}{c}\right)$$

Where G_0 is the gravitational constant, m_e is the mass of the earth, *M* is the mass of the sun, and *m* is the mass of Jupiter.

Suppose Jupiter's opposition to the sun is the base point and its potential energy is V_0, then

$$V_0 = -G_0 m_e \left(\frac{M}{a} + \frac{m}{OJ - a}\right)$$

Note that the above potential energy is the result of observation in the distant flat space-time reference frame. Therefore, the obtained gravitational constant is set as G_0. However, the gravitational constant can be cancelled out.

$$\frac{G'}{G} = \frac{V_0}{-G_0 m_e \left(\frac{M}{a} + \frac{m}{c}\right)}$$

Or

$$G' = \frac{1 + \frac{m}{M}\frac{a}{OJ - a}}{1 + \frac{m}{M}\frac{a}{c}} G$$

Among the above parameters

$$M = 2 \times 10^{30} kg$$

$$m = 2 \times 10^{27} kg$$

$$a = 1.5 \times 10^{11} m$$

$$OJ = 7.8 \times 10^{11} m$$

In this way, the change in the gravitational constant caused by the deviation from Jupiter's opposition point every half month can be calculated, namely

$$G' = (1 + \alpha)G$$

Where G is the time of Jupiter's opposition to the sun, the value of the gravitational constant measured at the position of the earth. G' is the value of the gravitational constant measured at other locations.

The results are shown in Table 1

Table 1 The error of the gravitational constant caused by the time deviation from Jupiter opposition

Time (month)	1+α	Errors ($\times 10^{-11} m^3 kg^{-1} s^{-2}$)
0.5	1.000002	1.57215E-05
1	1.000009	5.92566E-05
1.5	1.000018	0.00012163
2	1.000029	0.000192522
2.5	1.000039	0.000263405
3	1.000049	0.000328628
3.5	1.000058	0.000385083
4	1.000065	0.000431393
4.5	1.000070	0.000467166
5	1.000074	0.000492474
5.5	1.000076	0.000507533
6	1.000077	0.000512529

2.2 The influence of Jupiter's perihelion and aphelion on the gravitational constant

Since the Earth's eccentricity is relatively low compared to Jupiter, the Earth's orbit can be regarded as a circle. Let us consider the influence of Jupiter's own perihelion and aphelion on the gravitational constant. The distance of Jupiter's perihelion is $7.41 \times 10^{11} m$, and the aphelion is $8.14 \times 10^{11} m$

By formula

$$G' = \frac{1 + \frac{m}{M}\frac{a}{d}}{1 + \frac{m}{M}\frac{a}{2a + d}} G$$

The distance between the earth and the Sun is a; when Jupiter opposes the sun, the distance between the earth and Jupiter is d.

If Jupiter is at the aphelion position, the difference in the gravitational constant measured during Jupiter’s opposition is

$$G' = (1 + 7 \times 10^{-5})G$$

If Jupiter is at the perihelion, then

$$G' = (1 + 8.5 \times 10^{-5})G$$

In the above formula, G represents the gravitational constant measured on the surface of the earth when the earth is on the other side of Jupiter and the sun.

It can be seen that Jupiter's perihelion and aphelion have little effect on the gravitational constant measured on the earth. The impact is approximately $\pm 7ppm$.

However, when calculating, if you consider whether Jupiter is at the perihelion or the aphelion position, the calculation results should be improved. At that time, this improvement should be related to the accuracy of the instrument used. For example, theoretical calculations show that the measurement interval deviates from Jupiter's perihelion or aphelion by about 6 years, and the position of Jupiter's perihelion or aphelion has a greater influence on the result. However, if the time span is so large, factors such as the aging of equipment and materials may cause greater system errors.

3 Analysis of some time-recorded measurement values of gravitational constants

Since Cavendish measured the gravitational constant, humans have measured the gravitational constant many times. However, considering that the relative error of the measurement of the gravitational constant before 2000 is close to 150ppm. Such a relative error exceeds the estimation results of this article and my previous days, so these data are not suitable for analyzing the data of Jupiter's orbit on the earth's gravity measurement. For example, for the measurement result of TR&D-96 [2], the accuracy is too low, about 6.6730 (90). It can be seen that the gravitational constant measured before 2000 has no obvious correlation with the specific month. Therefore, below this accuracy, the changes in the positions of Jupiter and the Earth have little effect on the results.

Since 2000, the accuracy of the measurement of the gravitational constant has been improved to a certain extent. By 2018, the accuracy of the measurement of the gravitational constant has been improved to 12ppm [3]. Such a relative error exceeds the influence of Jupiter on the measurement of the gravitational constant. Therefore, some common changing laws should be found from these data and considered as systematic errors.

In addition, at present, the systematic error caused by different measuring devices is indeed relatively large. For example, the measurement accuracy of BIPM-01[4] and BIPM-14[5,6] are very high, but their results are very different from the measurement results of the HUST series, and it is difficult to explain the influence of Jupiter. Therefore, it can be determined that the difference of the measuring device will cause a large systematic error. Therefore, to compare the data and understand the influence of Jupiter on the measurement of the gravitational constant on the earth, the same device should be mainly used.

3.1 BIPM-01 and BIPM-14

The first measurement by Quinn et al. was from October 1 to October 30, 2000 [4]. The result of the measurement is $6.67559 \times 10^{-11} m^3 kg^{-1} s^{-2}$. In 2000, Jupiter's opposition time was November 28, 2000, and there was a **one-month** difference between the two. It shows that the earth and Jupiter are basically on the same side.

The second measurement was from August 7th to September 7th, 2007 [5,6]. The result of the measurement is $6.67545 \times 10^{-11} m^3 kg^{-1} s^{-2}$. Jupiter will oppose the sun in 2007 on June 5, 2007. There is a difference of **two months** between the two, and the earth has already begun to leave Jupiter. The measured result is slightly smaller.

The relative position between the Earth and Jupiter during these two measurements can be shown in Figure 2. Since the *A* position measured for the first time is closer to Jupiter, the measured gravitational constant will be larger.。

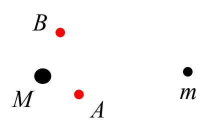

Figure 2. A: BIPM-01; B: BIPM-14

According to Table 1, the first measurement is 0.000009G smaller than Jupiter's opposition point. Now add 0.000009G to the first measurement result to get

$$6.67565 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

The second measurement is 0.000029G smaller than Jupiter's opposition. Now add 0.000029G to the result of the second measurement, we can get:

$$6.67564 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

It can be seen that after considering the influence of Jupiter, the results of the two experiments have been significantly improved.

3.2 JILA-10

The experiment of JILA-10 was mainly completed from May to June 2004 [7], and their measurement results are $(6.67234 \pm 0.00014) \times 10^{-11} m^3 kg^{-1} s^{-2}$. Considering that Jupiter's opposition in 2004 is March 4th. Therefore, during the measurement process from May to June, the earth is gradually moving away from Jupiter, so the measured value will gradually decrease. In the paper by Parks et al., Figure 2 shows the series of data measured during this time [7]. It can be seen that the value of the gravitational constant measured in June has dropped significantly.

Although there is no specific data, it can be roughly seen from the figure that the average difference between the data in May and the data measured in June is about $0.00001G$.

3.3 UCI-14

The measurement time of UCI-14 is 9-11/2000, 12/2000, 3-5/2002, 3-5/2006[8]. Among them, the measurement in 2004 was discarded due to too much noise signal. Newman et al. used three types of fibres. Among them, 9-11/2000 used the first fibre, 12/2000, 3-5/2002 used the second fibre, and 3-5/2006 used the third fibre.

Although the second fibre was used in 12/2000, the number of experiments was only more than one hundred, so the second fibre was mainly used in 3-5/2002.

In 2000, Jupiter opposed the sun on November 28. Therefore, the 9-11/2000 experiment differs from Jupiter's opposition time by about **2 months**. The 12/2000 experiment coincided with Jupiter's opposition to the sun. The opposition of Jupiter in 2002 was January 1st. Thus, the time difference between the 3-5/2002 experiment and Jupiter's opposition is about **4 months**. Jupiter's opposition in 2006 was May 4th. It can be seen that in 3-5/2006, the Earth was closest to Jupiter, a difference of about **half a month**. At this time, the maximum gravitational constant value should be measured. Although the earth was very close to Jupiter at the time of 12/2000, considering that the data measured in this month was relatively small and averaged by the data of 3-5/2002, the measurement result of fibre 2 was mainly affected by the measurement data of 3-5/2002. Considering that the earth began to move away from Jupiter in 3-5/2002, the value of the gravitational constant measured during this period should be relatively small.

Through the above analysis, we can conclude that the order of the experimental measurement results is:

$$\text{Fibre 3} > \text{Fibre 1} > \text{Fibre 2}$$

The actual measurement result is

Fibre 1: $6.67435(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Fibre 2: $6.67408(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$

Fibre 3: $6.67455(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$

It can be seen from the results of UCI-14 that the first result in 2000 was greater than that in 2002, and 2006 was closer to Jupiter, and the result was the largest.

Therefore, according to Table 1, adding the data of Fibre1 to the effect of **two months**, that is, the difference of 0.000029G, you can get

$$6.67454(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Add the data of Fibre2 to the impact of **four months**, that is, the difference of 0.000065G, you can get

$$6.67451(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Adding the Fibre3 data to the impact of **half a month**, a difference of about 0.000002G, you can get

$$6.67457(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

It can be seen that the three sets of data have been significantly improved.

3.4 HUST-09

The first experiment of HUST-09 was from March 21, 2007 to May 20, 2007, and from April 19, 2008 to May 10, 2008 [9, 10]. The time of Jupiter's opposition in 2007 and 2008 are June 5, 2007 and July 9, 2008.

The second experiment was from August 25, 2008 to September 28, 2008, and from October 8, 2008 to November 16, 2008.

It can be seen that the first experiment is closer to the time of Jupiter opposition. The time interval is about **2 months**. The second experiment was a little farther away from Jupiter's opposition, about **two and a half months** on average. Therefore, theoretically, the result of the first experiment measurement will be larger than the result of the second experiment.

The reality is that the gravitational constant measured by Luo's team in the first experiment is：

$$(6.67352 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

The gravitational constant measured in the second experiment is:

$$(6.67346 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

According to the calculation in Table 1, if the data of the first experiment plus the influence of $0.000029G$ Jupiter, the result of the first experiment is

$$(6.67371 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

The second experiment took **two and a half months** longer than Jupiter's opposition to the sun. According to Table 1, it can be seen that the result of the second experiment is $0.000039G$ less than the gravitational constant of Jupiter's opposition point. So, the second experiment data plus the Jupiter influence of $0.000039G$, the second data will become:

$$(6.67372 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

This is exactly the same as the result after the first experimental correction. The experimental data has improved significantly.

4 Conclusion

For the measurement data of the gravitational constant before 2000, it is difficult to consider the influence of Jupiter on the measurement results of the gravitational constant on the earth due to insufficient accuracy. But after 2000, due to the improvement of various technologies, the accuracy of the measurement of the gravitational constant has also been improved to a certain extent. The current best accuracy has reached 12ppm. This has exceeded the estimated influence of Jupiter on the earth's gravitational constant. The estimated result of this paper is that if we consider the two extreme cases of Jupiter's opposition and the earth's distance from Jupiter, the relative systematic error of measuring the gravitational constant on the earth will reach 77ppm. Based on this consideration, this paper retrieves the experiment data of the measurement of the gravitational constant since 2000, and specifically analyzed the results with more detailed time records. The analysis results show that if Jupiter's influence on the measurement of the gravitational constant is taken into account, the accuracy of the revised experimental data will be significantly improved.

References

[1] Cheng, Z. The Influence of the Spacetime Compression on the Gravitational Constant. https://vixra.org/author/zhi_cheng (2020).
[2] Karagioz, O. & Izmailov, V. Measurement of the gravitational constant with a torsion balance. Meas. Tech. 39, 979–987 (1996).
[3] Li, Q., Xue, C., Liu, J. *et al.* Measurements of the gravitational constant using two independent methods. *Nature* **560,** 582–588 (2018).

[4] Quinn, T. J., Speake, C. C., Richman, S. J., Davis, R. S. & Picard, A. A new determination of G using two methods. Phys. Rev. Lett. 87, 111101 (2001).
[5] Quinn, T. J., Parks, H. V., Speake, C. C. & Davis, R. S. Improved determination of G using two methods. Phys. Rev. Lett. 111, 101102 (2013).
[6] Quinn, T., Speake, C., Parks, H. & Davis, R. The BIPM measurements of the Newtonian constant of gravitation, G. Phil. Trans. R. Soc. A 372, 20140032 (2014).
[7] Parks, H. V. & Faller, J. E. Simple pendulum determination of the gravitational constant. Phys. Rev. Lett. 105, 110801 (2010).
[8] Newman, R., Bantel, M., Berg, E. & Cross, W. A measurement of G with a cryogenic torsion pendulum. Phil. Trans. R. Soc. A 372, 20140025 (2014).
[9] Luo, J. et al. Determination of the Newtonian gravitational constant G with time-of-swing method. Phys. Rev. Lett. 102, 240801 (2009).
[10] Tu, L. C. et al. New determination of the gravitational constant G with time-of-swing method. Phys. Rev. D 82, 022001 (2010).

木星对引力常数测量的影响的更详细计算

Zhi Cheng

gzchengzhi@hotmail.com

摘要：本文在上一篇分析的基础上，详细分析了木星对引力常数测量的影响。包括具体估算了每半个月地球轨道位置的变化引起的引力常数的变化。也估算了木星近日点和远日点对引力常数的影响。在此基础上，本文分别对 BIPM-01, BIPM-14, JILA-10, UCI-14, HUST-09 等有具体时间记录的数据进行了分析。分析结果表明，考虑了木星的影响之后，实验结果都有较明显改善。

关键词：引力常数；引力常数测量；木星

1 引言

引力常数的测量是一件很困难的事情。尽管如此，自 2000 年以来随着各种相关技术的发展，引力常数的测量精度还是有很明显的提高。因此一些过去无法考虑的系统误差也可以被考虑进来，从而有效提升数据的一致性。

太阳系的绝大部分质量都集中在太阳。但是尽管木星质量只有太阳的千分之一，却也足以对地球上的引力活动产生影响。其中的一个影响就是在地球上进行的引力常数测量实验。

为了探讨木星对引力常数的影响，需要一个基本假设，就是时空压缩会改变引力常数。具体的分析在我上一篇论文中有详细的解释[1]。其基本思路就是假设时空压缩将导致空间的度量尺度缩短。然而在远处平坦时空观察系观察到该时空中的质量不应该改变，否则违背能量守恒定律。这样在平坦时空参照系测量到的该质量的史瓦西半径也应该保持不变。现在切换到被压缩的时空参照系中，由于空间的度量单位缩小了，所测量到的史瓦西半径应该变长。而无论在哪个参照系，质量不能改变，这是能量守恒定律的要求。光速则是常数，也不能够改变。因此时空被压缩能够被改变的就是引力常数。时空被压缩的程度和所有质量在该点产生的势能的绝对值成正比。这是本文和我的上一篇论文的最基本假设。

有了时空压缩将使引力常数增大的假设之后，我们就可以具体计算木星位置不同对地球上测量到的引力常数的影响。本文将给出比较详细的估算方法。

2 木星对引力常数影响的详细计算

2.1 木星和地球都是圆轨道

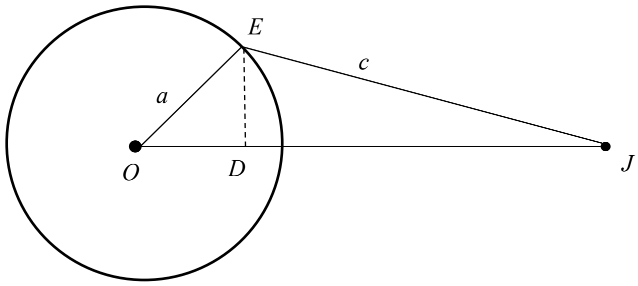

Figure 1. The position between the earth, Jupiter and the sun

在图 1 中，O 为太阳位置，E 为地球位置，J 为木星位置。由于木星的轨道周期达到 11.8 年，因此可以近似将木星和太阳看做静止不动。为了计算方便，过 E 点做垂直于 OJ 的垂线 ED.

OE 为木星到太阳的轨道半径。这里近似将木星的轨道看作是圆形。

$$\angle EOD = \theta = \frac{x}{6}\pi$$

其中 x 为地球离开木星冲日位置的时间，单位为“月”

而木星到地球的距离

$$JE = c$$

这样

$$OD = acos\theta$$

$$ED = asin\theta$$

$$JD = OJ - OD$$

因此

$$c = \sqrt{a^2sin^2\theta + (OJ - OD)^2}$$

然后计算出在 E 点太阳和木星的总势能：

$$V_E = -G_0 m_e \left(\frac{M}{a} + \frac{m}{c}\right)$$

其中 G_0 为引力常数，m_e 为地球质量，M 为太阳质量，m 为木星质量。

设木星冲日点为基点，其势能为 V_0, 则

$$V_0 = -G_0 m_e \left(\frac{M}{a} + \frac{m}{OJ - a}\right)$$

注意到上述势能都是在远处平坦时空参照系观察到的结果。因此所获得的引力常数定为 G_0. 不过该引力常数可以被相互抵消掉。

$$\frac{G'}{G} = \frac{V_0}{-G_0 m_e \left(\frac{M}{a} + \frac{m}{c}\right)}$$

或者：

$$G' = \frac{1 + \frac{m}{M}\frac{a}{OJ - a}}{1 + \frac{m}{M}\frac{a}{c}} G$$

上述参数中

$$M = 2 \times 10^{30} kg$$

$$m = 2 \times 10^{27} kg$$

$$a = 1.5 \times 10^{11} m$$

$$OJ = 7.8 \times 10^{11} m$$

这样可以计算出偏离木星冲日点之后每半个月造成的引力常数的变化，即：

$$G' = (1 + \alpha) G$$

其中 G 为木星冲日时间，在地球位置测量到的引力常数数值。G’为其他位置测量到的引力常数数值。

结果如表 1 所示：

Table 1 The error of the gravitational constant caused by the time deviation from Jupiter opposition

Time (month)	1+α	Errors ($\times10^{-11}m^3kg^{-1}s^{-2}$)
0.5	1.000002	1.57215E-05
1	1.000009	5.92566E-05
1.5	1.000018	0.00012163
2	1.000029	0.000192522
2.5	1.000039	0.000263405
3	1.000049	0.000328628
3.5	1.000058	0.000385083
4	1.000065	0.000431393
4.5	1.000070	0.000467166
5	1.000074	0.000492474
5.5	1.000076	0.000507533
6	1.000077	0.000512529

2.2 木星近日点和远日点对引力常数的影响

由于相对木星而言，地球的偏心率比较低，因此可以将地球的轨道看作是圆形。我们再来考虑木星本身的近日点和远日点对引力常数的影响。木星近日点距离为$7.41 \times 10^{11}m$，远日点为$8.14 \times 10^{11}m$

通过公式：

$$G' = \frac{1 + \frac{m}{M}\frac{a}{d}}{1 + \frac{m}{M}\frac{a}{2a + d}} G$$

其中地球到 Sun 之间的距离为 a；木星冲日的时候，地球到 Jupiter 之间的距离为 d.

如果木星位于远日点位置，则木星冲日时所测量出来的引力常数差值为：

$$G' = (1 + 7 \times 10^{-5})G$$

如果木星位于近日点位置，则

$$G' = (1 + 8.5 \times 10^{-5})G$$

上述公式中 G 都表示地球位于木星和太阳另一侧的时候，地球表面测量到的引力常数。

可以看出木星近日点和远日点对于在地球上测量到的引力常数的影响不大。影响大约为$\pm 7ppm$.

不过在进行计算的时候，如果考虑到木星是处于近日点还是远日点的位置，应该可以改善计算的结果。当时这种改善的情况应该和所使用的仪器的精度有关系。比如理论上计算表明，测量时间间隔偏离木星近日点或远日点 5 年左右，木星近日点或远日点位置对结果影响较

大。然而如果时间跨度这么大，则仪器设备材料的老化等因素将有可能造成更大的系统误差。

3 一些有时间记录的引力常数测量数值的分析

自卡文迪许测量引力常数以来，人类已经对引力常数进行了非常多次数的测量。不过考虑到2000 年之前对引力常数的测量其相对误差接近 150ppm. 这样的相对误差超过了本文以及我前几天的估算结果，因此这些数据不适合用来分析木星轨道对地球上引力测量数据的影响。比如对于 TR&D-96 的测量结果[2]，由于精度太低，大约为 6.6730(90)，可以看出 2000 年之前测量出来的引力常数数值跟具体的月份没有什么明显的关联性。因此在这样的精度下面，木星和地球位置的变化对结果的影响不大。

而自 2000 年以来，引力常数测量的精度得到一定程度的提高，到了 2018 年引力常数测量的精度已经提高到了 12ppm[3]. 这样的相对误差超过了木星对引力常数测量数值的影响。因此从这些数据中应该可以发现一些共同的变化规律，并将其作为系统误差来进行考虑。

另外目前来看，因为测量装置不同而造成的系统误差确实比较大。比如 BIPM-01[4]和 BIPM-14[5,6]的测量精度都非常高，但是他们的结果同 HUST 系列的测量结果差距却非常大，难以用木星的影响来解释。因此可以确定测量装置的不同将会造成很大的系统误差。因此要进行数据的对比，了解木星对地球上测量引力常数的影响，应该主要使用同一台装置来进行。

3.1 BIPM-01 和 BIPM-14

Quinn 等人的第一次测量是在 2000 年 10 月 1 日至 10 月 30 日[4]。测量的结果是 $6.67559 \times 10^{-11} m^3 kg^{-1} s^{-2}$. 而在 2000 年木星冲日时间为 2000 年 11 月 28 日，二者相差一个月。说明地球和木星基本位于同一侧位置。

第二次测量是 2007 年 8 月 7 日至 9 月 7 日[5,6]。测量的结果是 $6.67545 \times 10^{-11} m^3 kg^{-1} s^{-2}$. 2007 年木星冲日时间为 2007 年 06 月 05 日。二者相差两个月，地球已经开始离开木星。测得的结果略小一些。

这两次测量的时候地球和木星之间的相对位置可以用图 2 来表示。由于第一次测量的 A 位置更靠近木星，因此测量出来的引力常数会更大一些。

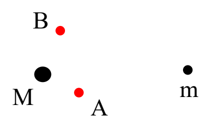

Figure 2. A: BIPM-01; B: BIPM-14

对照表 1，第一次测量比木星冲日点小 0.000009G，现在将第一次测量结果加上 0.000009G, 可以得到:

$$6.67565 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

第二次测量比木星冲日点小 0.000029G。现在将第二次测量的结果加上 0.000029G，可以得到：

$$6.67564 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

可以看出，考虑了木星的影响之后，两次实验的结果有非常明显改善。

3.2 JILA-10

JILA-10 的实验主要在 2004 年五月至六月完成[7]，他们的测量结果为$(6.67234 \pm 0.00014) \times 10^{-11} m^3 kg^{-1} s^{-2}$. 考虑到 2004 年木星冲日时间为 03 月 04 日。因此在五月到六月的测量过程中，地球是逐渐远离木星的，这样测量出来的数值将逐渐减少。在 Parks 等人的论文中，其中图 2 显示了在这段时间测量的系列数据[7]。可以看出到了六月份所测量出来的引力常数数值明显下降。

虽然没有具体的数据，从图中大致可以看出，五月份的数据和六月份测量的数据平均值大约相差 0.00001G.

3.3 UCI-14

UCI-14 的测量时间分别是 9-11/2000，12/2000,3-5/2002,3-5/2006[8]. 其中 2004 年的测量由于噪音信号太大被舍弃掉了。Newman 等人用了三种扭秤纤维。其中 9-11/2000 用了第一种纤维，12/2000, 3-5/2002 用了第二种纤维，3-5/2006 用了第三种纤维。

12/2000 虽然使用了第二种纤维，但是实验次数只有一百多次，因此第二种纤维主要集中在 3-5/2002 使用。

而 2000 年木星冲日时间为 11 月 28 日。因此 9-11/2000 的实验与木星冲日时间大约相差 2 个月。12/2000 的实验正好与木星冲日时间吻合。2002 年木星冲日时间为 01 月 01 日。这样 3-5/2002 的实验与木星冲日时间相差大约 4 个月。2006 年木星冲日时间为 05 月 04 日。可以看出 3-5/2006 的时候，地球最接近木星，相差大约半个月。这时候应该测量出最大的引力常数数值。虽然 12/2000 的时候地球非常接近木星，但考虑到在这一个月测量的数据比较少，被 3-5/2002 的数据平均，因此纤维 2 的测量结果主要受到 3-5/2002 测量数据的影响。考虑到 3-5/2002 地球开始远离木星，因此这段时间测量的引力常数数值应该比较小。

通过上述分析，我们可以得出实验测量的结果大小顺序为：

$$\text{Fibre } 3 > \text{Fibre } 1 > \text{Fibre } 2$$

实际的测量结果为：

Fibre 1: $6.67435(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Fibre 2: $6.67408(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$

Fibre 3: $6.67455(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$

从 UCI-14 的结果可以看出，2000 年第一次结果大于 2002 年，2006 年更接近木星，结果最大。

因此对照表 1，将 Fibre1 的数据加上两个月的影响，即 0.000029G 的差异，可以得到

$$6.67454(10) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

将 Fibre2 的数据加上四个月的影响，即 0.000065G 的差异，可以得到

$$6.67451(15) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

将 Fibre3 的数据加上半个月的影响，大约 0.000002G 的差异，可以得到

$$6.67457(13) \times 10^{-11} m^3 kg^{-1} s^{-2}$$

可以看出三组数据有非常明显改善。

3.4 HUST-09

HUST-09 的第一次实验是在 2007 年 3 月 21 日到 2007 年 5 月 20 日，2008 年 4 月 19 日到 2008 年 5 月 10 日[9, 10]。可以看出，这段时间比较接近地球和木星在同一侧轨道的情况。而 2007 年和 2008 年木星冲日的时间分别是：2007 年 06 月 05 日，2008 年 07 月 09 日

第二次实验是在 2008 年 8 月 25 日到 2008 年 9 月 28 日，2008 年 10 月 8 日到 2008 年 11 月 16 日。

可以看出第一次实验更接近与木星冲日时间。时间大约为 2 个月时间。而第二次实验离木星冲日时间要远一点，平均大约两个半月。两个实验距离木星冲日的时间差距大约为半个月。因此理论上第一次实验测量的结果会比第二次实验的结果要大一些。

实际情况是，第一次实验罗俊小组测量的引力常数数值为：

$$(6.67352 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

第二次实验测得的引力常数为：

$$(6.67346 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

按照表 1 的计算，如果第 1 次实验的数据加上 0.000029G 木星的影响，则第一次实验结果为

$$(6.67371 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

第 2 次实验比木星冲日时间要多两个半月。根据表 1，可以看出第二次实验结果比木星冲日点的引力常数要少 0.000039G. 这样第二次实验数据加上 0.000039G 的木星影响，第二次数据将变成：

$$(6.67372 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

这正好与第一次实验矫正后的结果一致。实验数据有明显改善。

4 结论

对于 2000 年之前的引力常数测量数据，由于精度不够，因此很难考虑到木星对地球上引力常数测量结果的影响。但是在 2000 年之后，由于各方面技术的提升，引力常数测量的精度也有一定程度的提升。目前最好的精度已经达到了 12ppm. 这已经超过了本文估算的木星对地球上测量引力常数的影响。本文估算的结果是，如果考虑木星冲日和地球远离木星的两种极端情况，在地球上测量引力常数产生的相对系统误差将达到 77ppm. 基于这样的考虑，本文检索了 2000 年以来测量引力常数的高精度实验结果，并专门对其中有比较详细时间记录的结果进行了分析。分析结果表明，如果考虑到木星对引力常数测量的影响，则修正以后的实验数据在精度方面有非常明显的改善。

参考文献

[1] Cheng, Z. The Influence of the Spacetime Compression on the Gravitational Constant. https://vixra.org/author/zhi_cheng (2020).
[2] Karagioz, O. & Izmailov, V. Measurement of the gravitational constant with a torsion balance. Meas. Tech. 39, 979–987 (1996).
[3] Li, Q., Xue, C., Liu, J. *et al.* Measurements of the gravitational constant using two independent methods. *Nature* **560,** 582–588 (2018).
[4] Quinn, T. J., Speake, C. C., Richman, S. J., Davis, R. S. & Picard, A. A new determination of G using two methods. Phys. Rev. Lett. 87, 111101 (2001).
[5] Quinn, T. J., Parks, H. V., Speake, C. C. & Davis, R. S. Improved determination of G using two methods. Phys. Rev. Lett. 111, 101102 (2013).
[6] Quinn, T., Speake, C., Parks, H. & Davis, R. The BIPM measurements of the Newtonian constant of gravitation, G. Phil. Trans. R. Soc. A 372, 20140032 (2014).
[7] Parks, H. V. & Faller, J. E. Simple pendulum determination of the gravitational constant. Phys. Rev. Lett. 105, 110801 (2010).
[8] Newman, R., Bantel, M., Berg, E. & Cross, W. A measurement of G with a cryogenic torsion pendulum. Phil. Trans. R. Soc. A 372, 20140025 (2014).
[9] Luo, J. et al. Determination of the Newtonian gravitational constant G with time-of-swing

method. Phys. Rev. Lett. 102, 240801 (2009).
[10] Tu, L. C. et al. New determination of the gravitational constant G with time-of-swing method. Phys. Rev. D 82, 022001 (2010).