Deeper insight on Existing and New Wave Equations in Quantum Mechanics

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Abstract

In this paper, we discuss in brief the most common wave equations in quantum mechanics and some recent development in wave mechanics. We also present two new quantum wave mechanics equations based on the Compton momentum. We have good reasons to think that the standard momentum is a mathematical derivative of the more fundamental Compton momentum. This will likely simplify interpretations of quantum mechanics significantly. This is a first draft; we will show many more results in a future version.

Key Words: quantum mechanics, de Broglie wavelength, Compton wavelength.

1 Important Fundamentals

1.1 Standard momentum and the de Broglie wave

The relativistic de Broglie wave [1] is given by

$$\lambda_b = \frac{h}{mv\gamma} \tag{1}$$

where h is the Planck constant, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and v is the velocity of the mass. An important note is that the de Broglie wave is not mathematically defined for a rest-mass particle, as setting $v = 0$ means we are dividing by zero. In addition, if we let just v be close to zero, then the de Broglie wave converges towards infinity. Next we solve the de Broglie relation with respect to momentum; this gives

$$p = mv\gamma = \frac{h}{\lambda_b} \tag{2}$$

This means that the momentum is not defined for a rest-mass particle, since λ_b is not defined for a rest-mass particle. This is somewhat new, as modern physics directly and indirectly assumes that the momentum is simply zero when $v = 0$. For any $v > 0$, the formula gives the correct momentum, but again for a rest-mass particle, the standard momentum is not defined.

It seems this has passed the discussion among physicists. We think the likely reason is that the standard momentum was suggested long before the relativistic momentum was conceived. The idea that momentum is mass times velocity was suggested in 1721 by John Jennings [2]. Jennings said that momentum is the quantity of matter multiplied by the velocity, which is the standard momentum: $p \approx mv$, which holds when $v << c$. However, we will claim it is not valid for $v = 0$. The momentum suggested by Jennings came long before the development of relativity theory. So, the relativistic momentum $p = mv\gamma$ was probably derived first by Einstein in 1905.

The de Broglie wave was a hypothesis set out by de Broglie in 1923. As it had been shown that light has a particle-wave duality, de Broglie then speculated that matter had the same characteristics, so he assumed the matter wave was given by $\lambda_b = \frac{h}{mv\gamma}$. That is, the de Broglie wave was derived from the momentum. The fact that something was understood later does not mean that is less fundamental; on the contrary, since we live so far from the quantum world in our everyday lives, physics has mostly developed from the top down. Therefore, we have come up with rules and formulas for macroscopic objects and observations first, then later understood their connection to the particle and quantum world. So, once the quantum world is established, we can just as well derive such things as momentum from them. The point is that the momentum, from a quantum perspective, must be given by the formula 2 and we have shown that this means neither the de Broglie wave nor the standard momentum formula are valid when $v = 0$. As we will see, this is very important for quantum physics, as all the quantum wave equations of modern physics will be impacted by this.

1.2 The Compton wave and the new Compton momentum

Around the same time as de Broglie introduced the hypothesis of his matter wave, Compton [3] calculated and indirectly measured what today is known as the Compton wave. The relativistic Compton wave of an electron is given by[1]

$$\lambda = \frac{h}{mc\gamma} \tag{3}$$

First of all, here we see there are no issues with $v = 0$, as this just means that $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0^2}{c^2}}} = 1$. That is, the Compton wave, unlike the de Broglie wave, is mathematically well defined for any velocity of $v < c$. See also how to derive the Compton wave for any mass without knowledge of h [6].

Next, if we follow a similar approach to the one we used for the de Broglie wave, we get

$$\tilde{p} = mc\gamma = \frac{h}{\lambda} \tag{4}$$

This is what we will call the Compton momentum and it is a new type of momentum recently introduced by Haug [4]. Unlike the standard momentum, this momentum is well defined for $v = 0$ as well, since the Compton wavelength λ is well defined for $v = 0$. Also it dose not have strange properties such as going towards infinity when v is close to zero. The Compton wave is always on the scale of the atomic quantum realm (very very short compared to anything microscopic).

Further, it is important to note that we can always find the de Broglie wave from the Compton wave; we have $\lambda_b = \lambda\frac{c}{v}$. So, if we know the Compton wave, we can calculate the de Broglie wave. The same is true with the standard momentum (the de Broglie momentum); it can always be calculated from the Compton momentum as $p = \tilde{p}\frac{v}{c}$.

Why should there be two wavelengths linked to matter? And why should we have two types of momentum? We will suggest that the standard momentum and the de Broglie wave only are derivatives of the true matter wave and the true and deeper physical momentum, namely what we call Compton momentum. If we should connect the standard energy definition to the Compton momentum, we simply get

$$E = \tilde{p}c \tag{5}$$

That is, the total energy is equal to the Compton momentum multiplied by the speed of light; this is then a new (additional) relativistic energy momentum relation. This means we can also derive a new quantum mechanical wave equation from this new relation; this will be the relation between energy and the Compton momentum.

It is worth mentioning that the standard momentum is never observed directly – it is a mathematical construct. First, assume we have a brass ball; we can measure its relative weight relative to one kg and then find its mass relative to one kg. Second, we can put this brass ball in motion. We can then measure its velocity, but we cannot directly observe mv. What we can observe is the impact from its kinetic energy. We can drop a brass ball, for example, and measure its velocity just before it hit a brick of "soft" clay. Most of the kinetic energy will then be used to make a indent in the clay. Gravesande [7] did this and confirmed that experimentally the kinetic energy was a function of v^2 and not just of v. At that time, the question of whether the kinetic energy was a function of v or v^2 had been a debate among leading physicists for many years. And at least when $v << c$, the kinetic energy is a function of v^2. So, indeed we can measure the kinetic energy of a moving body, and the mass of a body easily when it is at rest, and we can easily measure the velocity of a body, but we cannot measure mv; this is a mathematical entity, that is, however, linked to real observable entities, so it can be very useful. What about our new Compton momentum? Can it be observed more directly? It should, because as we have explained here, we think the standard momentum is a derivative of the true (more real) momentum.

First, looking at our new Compton momentum. when $v = 0$ we get

$$\tilde{p} = mc\gamma = mc \tag{6}$$

[1] Actually this is a relativistic extension of Comptons work, see [4, 5].

In other words, we get a rest-mass momentum that is mc. This is not easy to observe, as it is an embedded momentum, a rest mass momentum. This may sound strange, as we are not used to thinking of rest-mass momentum, and some may even say that this is impossible, as momentum is related to something that is moving. However, that is the standard momentum that indeed only is defined for something that is moving. This is nothing more strange than rest-mass energy. If the rest-mass momentum is $\tilde{p}_r = mc$, then we must also have what we can call a kinetic momentum, and this must be

$$\tilde{p}_k = \tilde{p} - \tilde{p}_r = mc\gamma - mc \tag{7}$$

This formula holds for any $v < c$. It would require advanced laboratory equipment to test this when v is significantly close to the speed of light c. However, when $v << c$ what do we expect to observe? When $v << c$, we can approximate the formula above with the first term of a Taylor expansion, and we then get

$$\tilde{p}_k \approx \frac{1}{2} m \frac{v^2}{c} \tag{8}$$

That is, our Compton momentum is a function of v^2 and not of v, so it is kinetic energy divided by c. Our Compton momentum is exactly the same function of v as kinetic energy, but it is simply our standard energy defenition divided by c, a significant finding (that we will come back to in a new update of this paper). Our momentum is observable through measurements of impacts, while standard momentum is not as it is a function of v and not v^2. This supports our view that our newly defined Compton momentum is the real momentum and that the standard momentum is a derivative of this momentum. While the reader may not wish to take this for granted, it will be helpful to be open to the thought that there can be a momentum (or another term could be chosen and defined) that is linked to the Compton wavelength, and next we will look at the Relativistic Energy Momentum Relation in more detail.

2 Relativistic Energy Momentum Relation

The standard relativistic energy momentum of Einstein [8] is given by

$$E^2 = p^2c^2 + m^2c^4 \tag{9}$$

the standard momentum and this relation have played a central role in developing the well known quantum mechanical wave equations. If our analysis is correct and the standard momentum (de Broglie momentum) not is valid for rest-mass particles and further, if it is a mathematical derivative of the Compton momentum, then any quantum mechanical wave equation derived from it will be a wave equation linked to a derivative and all such quantum mechanical wave equations will probably also not be valid for rest-mass particles.

Our new relativistic energy momentum relation is

$$E = \tilde{p}c \tag{10}$$

When deriving a quantum mechanical wave equation consistent with this, the equation should also be valid for rest-mass particles, and should also be more directly linked to the depth of the reality, and therefore likely easier to interpret.

Next, we will shortly discuss a series of well-known quantum mechanical wave equations and also show a few new quantum mechanical wave equations.

3 Two New Relativistic Wave Equations

We have that

$$E = \tilde{p}c \tag{11}$$

This can be rewritten as

$$E = \tilde{p}_k c + mc^2 \tag{12}$$

where $\tilde{p}_k = mc\gamma - mc$, in other words the kinetic Compton momentum. From this we get the following quantum wave equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(i\hbar c\nabla + mc^2\right)\Psi \tag{13}$$

This we can rewrite as

$$\begin{aligned} i\frac{\partial\Psi}{\partial t} &= \left(ic\nabla + \frac{mc^2}{\hbar}\right)\Psi \\ i\frac{\partial\Psi}{\partial t} &= \left(ic\nabla + \frac{c}{\bar{\lambda}}\right)\Psi \end{aligned} \tag{14}$$

This is relativistic quantum wave equation consisting of a first order PDE where ∇ is the operator linked to the kinetic Compton momentum.

We also get a quantum wave equation linked to the total momentum instead of the kinetic momentum, this is because

$$E = \tilde{p}_k c + mc^2 = \tilde{p}c \tag{15}$$

This gives

$$i\hbar\frac{\partial\Psi}{\partial t} - i\hbar c\nabla\Psi = 0 \tag{16}$$

which can be simplified to (see also [4])

$$\frac{\partial\Psi}{\partial t} - c\nabla\Psi = 0 \tag{17}$$

That is, we have two new quantum mechanical wave equations

4 The Klein-Gordon Equation

Another well-known relativistic quantum equation is the Klein-Gordon equation, which is given by

$$E^2 = p^2c^2 + m^2c^4 \tag{18}$$

where p is the relativistic (de Broglie) momentum and $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. When replacing E and p with their energy, $i\hbar\frac{\partial}{\partial t}$, and momentum operator, $i\hbar\nabla$, we get

$$\begin{aligned} i^2\hbar^2\frac{\partial^2\Psi}{\partial t^2} - i^2\hbar^2\nabla^2\Psi - m^2c^4\Psi &= 0 \\ -\hbar^2\frac{\partial^2\Psi}{\partial t^2} + \hbar^2\nabla^2\Psi - m^2c^4\Psi &= 0 \\ \frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi + \frac{m^2c^4}{\hbar^2}\Psi &= 0 \\ \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi &= 0 \end{aligned} \tag{19}$$

The last line is how the Klein-Gordon equation is often presented, but the lines are all the same. Since the reduced Compton wave is given by $\bar{\lambda} = \frac{\hbar}{mc}$, we can replace $\frac{m^2c^2}{\hbar^2}$ in the equation above with $\frac{1}{\bar{\lambda}^2}$ and we get

$$\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi + \frac{1}{\bar{\lambda}^2}\Psi = 0 \tag{20}$$

The Klein-Gordon equation is indirectly liked to the de-Broglie momentum (standard momentum), $p = mv\gamma$, which is a derivative of the real momentum. The Klein-Gordon equation is therefore unnecessarily complex. Yet, it cannot be simplified further if we want a relativistic wave equation from the de-Broglie momentum. It is also linked to a unnecessarily complex definition of energy. The formula is likely not valid for a rest-mass particle, since it is derived from the de Broglie momentum.

The Klein-Gordon equation is often written as

$$(\Box + \mu^2)\Psi = 0 \tag{21}$$

where $\Box$ is the d'Alembert operator: $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$, and $\mu = \frac{mc}{\hbar} = \frac{1}{\bar{\lambda}^2}$. Do not let unfamiliar notation stop you from exploring the mysteries of quantum mechanics.

5 The Schrödinger Equation

We have

$$\begin{aligned} E &\approx \frac{1}{2}mv^2 + mc^2 \\ E &\approx \frac{p^2}{2m} + mc^2 \end{aligned} \tag{22}$$

Based on this, we get the following wave equation, namely the Schödinger [9] equation

$$i\hbar\frac{\partial\Psi}{\partial t} \approx \left(\frac{i^2\hbar^2}{2m}\nabla^2 + mc^2\right)\Psi \tag{23}$$

This we can rewrite further

$$\begin{aligned} i\hbar\frac{\partial\Psi}{\partial t} &\approx \left(\frac{-\hbar^2}{2m}\nabla^2 + mc^2\right)\Psi \\ i\frac{\partial\Psi}{\partial t} &\approx \left(\frac{-\hbar}{2m}\nabla^2 + \frac{mc^2}{\hbar}\right)\Psi \\ i\frac{\partial\Psi}{\partial t} &\approx \left(\frac{-\bar{\lambda}c}{2}\nabla^2 + \frac{c}{\bar{\bar{\lambda}}}\right)\Psi \end{aligned} \tag{24}$$

Note that when rewritten this way, there is no Planck constant in the Schrödinger equation. Also note that the imaginary number not goes away.

6 Summary

Table 1 shows a summary of three well-known wave equations in quantum mechanics, as well as two new ones. The three older equations are all rooted in standard momentum and therefore the de Broglie wavelength. The standard momentum is not defined for $v = 0$, that is to say, for rest-mass particles, so we suggest that these three traditional wave equations likely not are valid for rest-mass particles. In addition, these three wave equations are, to a large degree, modeling mathematical derivatives of reality rather than the deeper reality because the de Broglie momentum is a derivative of the more fundamental Compton momentum. The two new wave equations are linked directly to the Compton momentum; therefore, they are simpler and also hold for $v = 0$.

- There is no Planck constant in the any of the wave equations except for the Dirac equation that we will soon comment on separately. The apparent Planck constant in the Schrödinger and Klein-Gordon all cancel out against a Planck constant that is hidden in the mass. What we obtain is the Compton-wavelength in the Schrödinger and Klein-Gordon equations, or more precisely, the Compton frequency is also embedded in these equations. One might think that such a line of thought is wrong, if one believed that the Planck constant is needed to find the Compton wavelength. However, this is not the case, even from the Compton 1923 paper it is clear that one can find the Compton wavelength without any knowledge of the Planck constant. Actually, one can find the Compton wave for any mass without any knowledge of any fundamental constant, see [4]. In the Dirac equation, the Planck constant will indirectly cancel out as well, as the momentum operator on the wave function returns the momentum, and the momentum embedded contains the Planck constant in the mass will then cancel out against the Planck constant we see here.

	Normal form:	**Comments :**	**Comments**
Schrödinger	$i\frac{\partial\Psi}{\partial t} \approx \left(\frac{-\hbar}{2m}\nabla^2 + \frac{mc^2}{\hbar}\right)\Psi$	de Broglie momentum	Non-relativistic
deeper level	$i\frac{\partial\Psi}{\partial t} \approx \left(\frac{-c\bar{\lambda}}{2}\nabla^2 + \frac{c}{\bar{\lambda}}\right)\Psi$		
Klein-Gordon (Spin 0)	$\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi = 0$	de Broglie momentum	Relativistic
deeper level	$\frac{\partial^2\Psi}{\partial t^2} - c^2\nabla^2\Psi + \frac{c^2}{\bar{\lambda}^2}\Psi = 0$		Relativistic
Dirac (Spin 1/2)	$i\hbar\frac{\partial\Psi}{\partial t} - \left(c\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta mc^2\right)\Psi = 0$	de Broglie momentum	Relativistic
deeper level	$i\frac{\partial\Psi}{\partial t} - \left(\frac{c}{\hbar}\sum_{i=n}^{3}\alpha_n\mathbf{p}_n - \beta\frac{c}{\bar{\lambda}}\right)\Psi = 0$		
Haug-2: (Spin ?)	$i\frac{\partial\Psi}{\partial t} = \left(ic\nabla + \frac{mc^2}{\hbar}\right)\Psi$	Kinetic Compton momentum	Relativistic
deeper level	$i\frac{\partial\Psi}{\partial t}\Psi = \left(ic\nabla + \frac{c}{\bar{\lambda}}\right)\Psi$		
Haug-1: (Spin ?)	$\frac{1}{c}\frac{\partial\Psi}{\partial t} - \nabla\Psi = 0$	Compton total momentum	Relativistic

Table 1: The table shows a summary of three well-known quantum mechanical wave equations derived from standard momentum (de Broglie) momentum, and two new quantum mechanical wave equations derived from the Compton momentum.

- The Schrödinger, Klein-Gordon, and Dirac equations all use a momentum operator on the standard momentum. The standard momentum is, at a quantum level, actually directly linked to the de Broglie wave. The de Broglie wave is not mathematically defined for a rest-mass particle. Second, the de Broglie wave and also the standard momentum are just mathematical derivatives of the more fundamental Compton wave and what we call Compton momentum. In other words, in the traditional equations, we are taking partial derivatives of mathematical functions of reality, not of the deepest entities. This makes the Schrödinger, Klein-Gordon, and Dirac equations all very hard to interpret at a deeper level. Of course, they are "easy" to interpret at the surface of the exotic zoo of terminology that has evolved in physics and quantum physics, as long as the analysis of modern physics does not go too deep.

- We have strong reasons to believe that our new quantum mechanical wave equations are better suited to understanding certain aspects of the depth of reality. They are mathematically correct for $v = 0$, and they are more directly linked to the depth of reality, as we are modeling from the Compton momentum directly instead of the derivative of it, which is the de Broglie momentum and its corresponding de Broglie wave.

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