The Ticker-Tape Interpretation of Quantum Mechanics

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The Ticker Tape Interpretation of Quantum Mechanics is a Kantian based interpretation, similar to Copenhagen. It examines the everyday properties of measurements and shows that they lead inexorably to Quantum Mechanics as we know it. The classical Quantum Mechanic formalism is derived. Some conjectures are made about the nature of time which allows the above results to be applied generally.

The author has rather playfully borrowed the title of some of Einstein's famous "Principles". Even though the principles in this paper have the same names as those found in Special or General Relativity, *they have nothing to do with any specifics of Special or General Relativity*; the names were chosen however because, at some level, the both versions derive from an even more general principle.

1. Philosophical Underpinnings

This paper adopts a world view that is essentially Kantian. The Kantian world view consists of an external world that is perceived through our sensors; we use the information provided to build mental models of the external world but cannot know its true nature.

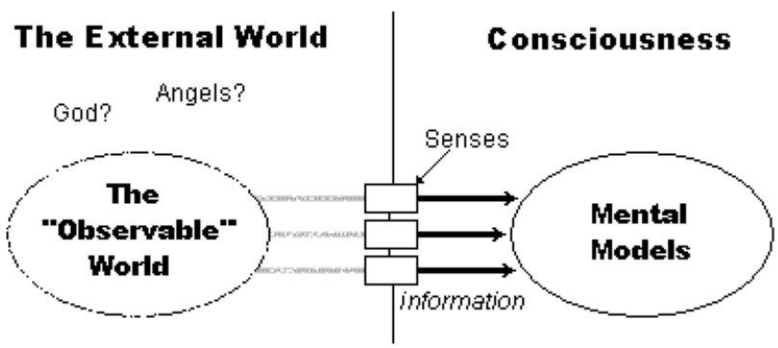

Figure 1.
Kantian World View (circa 1785).

Copenhagen clarifies aspects of the Kantian world view. (It has been argued that the Copenhagen Interpretation is Positivist, however subtle arguments over philosophical classifications are beyond the scope of this paper) Quantum Mechanics replaces the vague idea of sensory input with that of measurement and precise mathematical description. The idea of a conscious agent is replaced by intelligence, which need not be human (although some commentators would dispute this). It could, for example, be a robot.

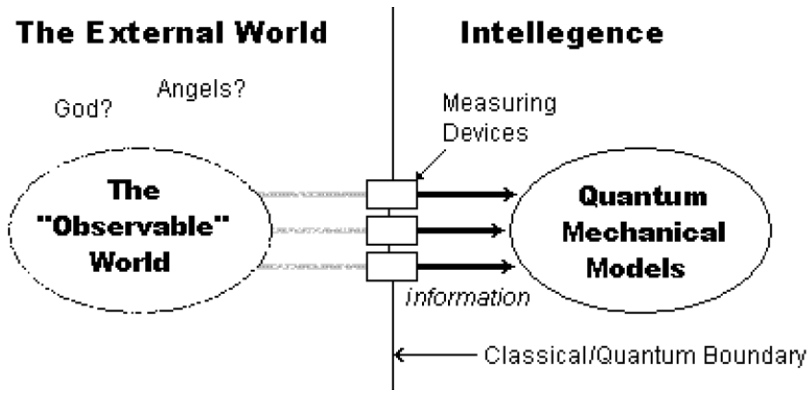

Figure 2.
Copenhagen World View (circa 1928).

The Copenhagen Interpretation acknowledges that our mental models may be incomplete; it may be necessary to apply a wave model in some situations and apply a particle model in another (Complementary Principle). Realism, on the other hand, is rooted in the belief that the world is as it appears, and seeks to preserve macroscopic models, possibly beyond their domain of applicability.

2. Measurement

Measuring devices feed specific observers with measurement streams; measurements are regarded as "elements of reality" by those observers. The observer is logically separate from the measurement itself. A measurement discontinuously changes the probability distribution an observer associates with the next measurement.

Definition: A measuring device that produces a single real number as its output is called a "basic" measuring device, and the measurements it produces are called "basic" measurements. The notation $\mathbf{X} = x$ is used to mean the device **X** has been used to make a measurement and result has been reported as x.

Why a real number? Measurement results can be put "in order" from the smallest to the largest. I.e. If $\mathbf{M}$ = {set of possible measurement values}, then there is a strict total ordering of **M**, which creates an order preserving mapping from **M** to some subset of Real numbers.

It is postulated that meaningful measurements are basic measurements, or logical combinations of such measurements. For example, ($\mathbf{X}=x_1$ or $\mathbf{X}=x_2$) is the possible outcome of a measurement, as is ($\mathbf{X} > x_1$) and ($\mathbf{X} \neq x_2$)

Definition: A history is a sequence of measurements and denoted $\mathcal{H} = (\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n)$ where $\mathbf{m}_i$ are measurements. If all measurements are "basic", the history will look like ($\mathbf{X} = x_0$, $\mathbf{Y}_1 = y_1$, $\mathbf{Z}_2 = z_2$, ..., $\mathbf{W}_n = w_n$).

3. The Equivalence Principle

There is no transition from the Quantum world to the Classical world. The difference between measurement in the Classical world and measurement in the Quantum world is a matter of interpretation.

Quantum Mechanics is a mental model; the waveform and any measurement operator **A** are mental constructions, built on top of the information gathered from "raw" measurements. Classical mechanics is also a mental model (since we know it is not "true", it cannot be otherwise); typically, Classical Mechanics typically deals with quantities that (roughly) correspond with the expected value <**A**> and regards the difference from <**A**> to be “error”.

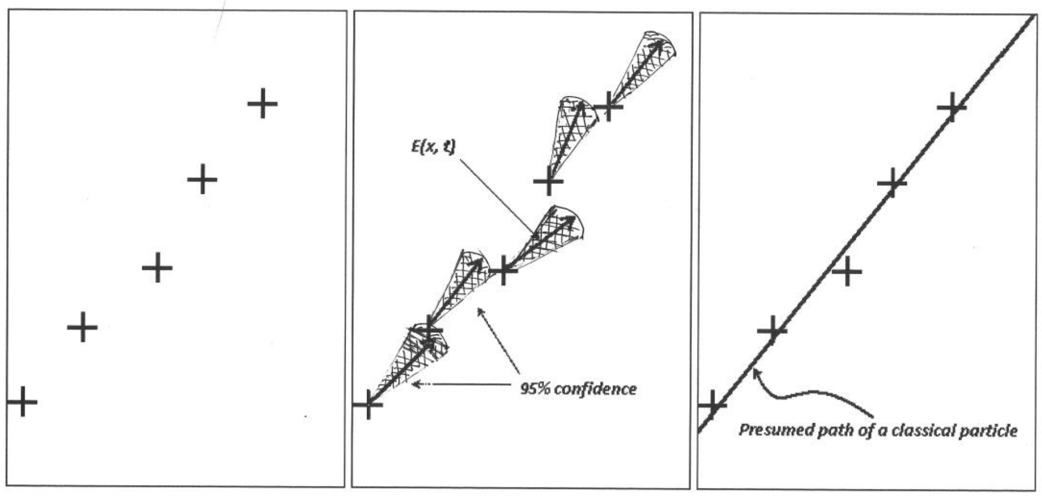

Figure 3
Measurements and Mental Models.

In the diagram above, the left pane shows the “raw” position measurements of a particle moving diagonally from left to right is shown. The central pane *adds* the intellectual machinery of Quantum Mechanics; the *calculated* 95% confidence intervals are shown in gray. The right pane *adds* the intellectual machinery of Classical Mechanics; the presumed classical path is shown in red.

In Quantum Mechanics, the uncertainty in the position of a particle is associated with the Heisenberg Uncertainty Principle and viewed as intrinsic to the system; the measurement themselves are taken at face value (*The Principle of Exact Measurement – If no further information is available, how can a measurement contain error?*).

In Classical Mechanics, it is the measurement that contains “error” (uncertainty) in the position of the particle; the “error” has a multitude of sources external to the system itself, typically related to the construction of the measuring devices and lack of knowledge of initial conditions, but there is always a presumption that if these influences could be eradicated, exact measurements would be possible and that the predictions of Classical mechanics would be confirmed.

4. Mach Devices

Bohr expressed the opinion that measuring devices are “essentially classical” in that they measure classical quantities such as time and space [1]. This implies knowledge of classical mechanics is required before Quantum Mechanical measurements can be understood, yet Quantum Mechanics is presumed to be more fundamental than classical physics. It is also not easy to apply Bohr's vision of essentially classical measuring devices to abstract concepts such as QCD colour.

Rather than follow Bohr, we take a slightly different view of measurement.

Definition: A measuring device is a Mach device with respect to an observer if the following applies:

(i) The device produces a stream of basic measurements recorded on a tickertape (or equivalent) accessible to the observer. Measurements are recorded in the order they are made.

(ii) The measurements are “repeatable”.

(iii) The device’s internal structure is unknown. It is a “black box”. There is no *a-prior information* about what the numbers it produces mean.

(iv) The observer does not have access to a clock.

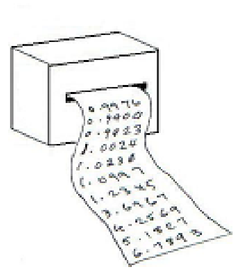

Figure 4.
A Mach device.

A Mach device is the most primitive measurement device possible: it only produces basic measurements.

6.1 When is a measuring device not a measuring device?

Classical "measuring” devices *generally* do not live up to the standard required to be measuring devices. Typical probability distributions associated with an “instantaneous” second measurement of a quantity, $P(\mathbf{X} \mid \mathbf{X}=x_i)$, are shown in figure 5.

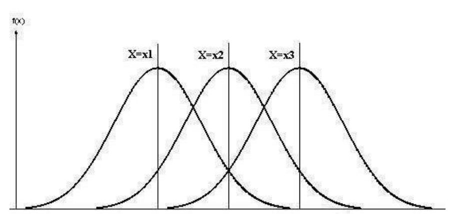

Figure 5.
Probability distributions for a second instantaneous measurement of a quantity

If there is overlap between the probability density functions (as is the case for the probability density functions for $\mathbf{X} = x_1$ and $\mathbf{X} = x_2$ in figure (5)), repeated measurements of the system may result in the different value for $\mathbf{X}$. I.e. if there is any overlap in the probability density functions, the result of any measurement of $\mathbf{X}$ is not repeatable and so the measuring device does not satisfy any reasonable definition of a measuring device.

5. Principle of Relativity (Mach's Principle)

The output from a single Mach device is meaningless.

This is a generalisation of Mach's Principle. Mach’s statement of the Principle of Relativity famously influenced Einstein but the principle itself dates back at least to Galileo. Ernst Mach argued that it would be meaningless to talk about the motion of a single particle in an empty Universe. All motion is relative. In fact, all measurement is relative. If there is no context, a measurement stream becomes a meaningless stream of numbers.

6. Mach Banishes Determinism

Suppose $\boldsymbol{X} = f(t)$ is a classical quantity, and **X** is a measuring device that faithfully returns $\boldsymbol{X}$. If **X** is a Mach device with respect to an observer, it is not possible for that observer to determine what the device measures from the measurement history. Why? There is no way to calibrate the device.

Suppose we construct a second device **Y** whose output is related to the first by

$$\mathbf{Y} = \zeta\,(\mathbf{X}(t))$$

The second device is sealed, mixed up with the first and given to the naive observer so they both become Mach devices. Which measures the "fundamental" quantity? **X** and **Y**? **X**? **Y**? In fact, we can build the device **Y** so it returns any measurement profile we like.

If a measuring device qualifies as a Mach device ***except*** *that it is known that it measures a quantity* **X** *where* **X** *= f(t) for some function f, then the output from that device is meaningless.*

7. State

<u>*Definition:*</u> A system state is *any* representation Ψ such that there is a rule for the calculation of the probability $P(\mathbf{x}\,|\Psi)$ where **x** is a measurement outcome and $P(\mathbf{x}\,|\Psi) = P(\mathbf{x}\,|\,\mathcal{H})$ for all**,** where $\mathcal{H}$ is the known history of the system.

The minimal sub-history H of $\mathcal{H}$ such that $P(\mathbf{x}\,|\,H) = P(\mathbf{x}\,|\,\mathcal{H})$ for all **x** is one possible system state representation**.**

The test of a good theory is whether it can make accurate predictions. In the case of Quantum Mechanics, the obvious question is: How much history is necessary before an observer can make accurate predictions?

The answer cannot be that the observer must know the entire history of a system since the beginning of time since that information will never be available. So, what are the alternatives? One possibility is to only count the last N measurements for some N, perhaps giving the more recent measurements more "weight". But how do we choose N? Why is one choice of N better than another? How do we assign "weights" to newer and older measurements?

Given the problems of choosing any special value for N, it makes sense that the probability of any measurement **X**=x depends at most only on the last value of each measuring device in the system.

8. Transition Probabilities and Repeatability

Measurements are expected to be repeatable. I.e. if two measurements are made, one immediately after the other, the results of the two measurements should agree.

Mach devices do not come equipped with a clock. There is no sense of time. It is not possible to know what the interval between any two measurements is. If the results of two consecutive measurements disagree it could be that measuring device is a working Mach device and the interval was so long that the system slowly evolved into another state. It is also possible that the device is "faulty" (not a working Mach device). But it is not possible to tell which is the case.

Since we have no clock, we restrict our attention to systems with stable state transition probabilities. I.e. $P(\Psi|\Psi) = 1$ for any state Ψ. I.e. From the ticker tape, select out measurements of the form ($\mathbf{B} = b$, $\mathbf{A}$ = ?) and calculate the proportion that have $\mathbf{A} = a_i$.

9. How do we know what we are measuring?

If the stream of measurement is deterministic, then there is reason to believe the output from the device is meaningless. It seems that a measurement stream can only be "understood" if it contains a random component.

We postulate the commutator relationships between measurement operators define "what we are measuring".

The proposition has several advantages:

1. The algebraic relationships between operators can be extracted experimentally (with the usual caveats) from a measurement stream, even though this may be very expensive.

2. Commutators do not require Classical Physics to pre-define the concepts required for use by Quantum Mechanics.

3. Commutators naturally define a "scale" and provide a natural mechanism for the introduction of constants. For example, suppose that

$$[\mathbf{X}_i, \mathbf{X}_j] = g_{ij}(\mathbf{X}_1 .. \mathbf{X}_n)$$

   If one of the devices, $\mathbf{X}_i$, is rescaled $\mathbf{X}_i \rightarrow \lambda\mathbf{X}_i$ then, there *may* be a detectable change in the commutator relations

Assuming the measurement space is spanned by $\{\mathbf{X}_i | i=1..n\}$, the simplest restriction would be to limit consideration to

$$[\mathbf{X}_i, \mathbf{X}_j] = a_{ij} + c_{ij}^{\ k}\, \mathbf{X}_k,\ \ i,j,k = 1..n.$$

9.1 Principle of Scale

A set of devices { $\mathbf{X}_i, \ldots \mathbf{X}_n$ } *can be assigned meaning only if any rescaling* (*calibration*) *of the devices is detectable.*

$$[\mathbf{L}_z', \mathbf{L}_x'] = i\,\lambda^2\,\hbar\,\mathbf{L}_y$$

However no further scaling $\mathbf{L}_y \rightarrow \mathbf{L}_y' = \mu\,\mathbf{L}_y$, $\mu \neq 0, \mu \neq 1$ (below) can disguise the original change of scale.

$$[\mathbf{L}_x', \mathbf{L}_y'] = i\,\mu\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_y', \mathbf{L}_z'] = i\,\mu\,\hbar\,\mathbf{L}_z'$$
$$[\mathbf{L}_z', \mathbf{L}_x'] = i\,\lambda^2 \left(\frac{1}{\mu}\right) \hbar\,\mathbf{L}_y'$$

<u>*Conclusion:*</u> A set of Mach devices described by angular momentum operators { $\mathbf{L}_x$, $\mathbf{L}_y$, $\mathbf{L}_z$ } satisfies the requirements of the Principle of Scale and can therefore be regarded as producing meaningful measurements.

Example 2: (Position and Momentum) Two devices are related by commutator relationship $[\mathbf{x}, \mathbf{p}] = i\hbar$.

Rescale $\mathbf{x} \rightarrow \mathbf{x}' = \lambda\mathbf{x}$ (change in the choice of units), then

$$[\mathbf{x}', \mathbf{p}] = [\lambda\mathbf{x}, \mathbf{p}] = i\lambda\hbar.$$

The original scaling however can be hidden by rescaling $\mathbf{p}$. I.e. $\mathbf{p} \rightarrow \left(\frac{1}{\lambda}\right)\mathbf{p}$.

Example 3: (Position and Momentum in 3D + Angular Momentum). Calculations show these nine devices do not satisfy the requirement of the Principle of Scale.

It is not surprising that measurements of position and momentum fail since these quantities are always relative to a second reference "point", absent from the commutator relations, and scaling is relative to the speed of light. It does raise questions about the basis of scaling in the early opaque Universe.

10. The Formalism of Quantum Mechanics

10.1 Feynman's Rules

We follow the argument put forward by Ariel Caticha[2].

Definition: If $\mathcal{H}_A = (\mathbf{m}_1,...,\mathbf{m}_k)$ and $\mathcal{H}_B = (\mathbf{m}_{k+1}, ..., \mathbf{m}_{k+m})$ are histories, then

$$\mathcal{H}_A \wedge \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_k, \mathbf{m}_{k+1}, ..., \mathbf{m}_{k+m})$$

The history $\mathcal{H}_A$ must "follow" the history $\mathcal{H}_B$ and not overlap in time. The operator $\wedge$ is read as "and".

Definition: Let $\mathcal{H}_A$ be a history, then $\mathcal{H}_B$ is an alternative history to $\mathcal{H}_A$ if $\mathcal{H}_B$ is the same as $\mathcal{H}_A$ except that some of the measurements, other than the initial and final measurements, have different values.

Definition: If $\mathcal{H}_A = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai}, ..., \mathbf{m}_n)$ and $\mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Bi}, ..., \mathbf{m}_n)$ are alternative histories which differ in the value of the i^{th} measurement, then

$$\mathcal{H}_A \vee \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai} \vee \mathbf{m}_{Bi}, ..., \mathbf{m}_n)$$

The operator $\vee$ is read as "or".

If $\mathbf{m}_{Ai}$ is the measurement $\mathbf{X}= x_i$, and $\mathbf{m}_{Bi}$ is the measurement $\mathbf{X}= x_2$, then

$$\mathcal{H}_A \vee \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{X} \in \{x_1,x_2\}, ..., \mathbf{m}_n)$$

and $\mathbf{X} \in \{x_1,x_2\}$ is regarded as a measurement in its own regard, derived from the basic measurements $\mathbf{X}= x_1$ and $\mathbf{X}= x_2$.

The operators $\wedge$ and $\vee$ obey the following relations:

$$a \vee b = b \vee a$$
$$(a \vee b) \vee c = a \vee (b \vee c)$$
$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

If $\mathcal{H}$ = set of possible histories for a system, the any representation (Ψ, +, x) with Ψ: $\mathcal{H} \rightarrow \Omega$ where Ω is some algebraic system, over a field F, and

$$\Psi(a \vee b) = \Psi(a) + \Psi(b)$$
$$\Psi(a \wedge b) = \Psi(a) \times \Psi(b)$$

would carry these properties across.

Caticha[2] shows that standard addition and multiplication fit the requirements for + and x.

10.2 Derivation of Standard Equations of Quantum Mechanics

The mapping above assumes that each possible outcome is equally likely. We want to describe the case where one state **Ψ'** is more (or less) likely than the other state - it seems sensible to try to represent **Ψ' = (λΨ)**.

1) Feynman's Rules imply that state has an algebraic structure generated by the union and concatenation of histories (and therefore states). In particular,

$$P(\mathbf{\Psi} + \mathbf{\Psi}\,|\mathbf{\Phi}) = P(\mathbf{\Psi} \vee \mathbf{\Psi}\,|\mathbf{\Phi}) = P(\mathbf{\Psi}\,|\mathbf{\Phi})$$

and

$$P(\mathbf{\Phi}|\,\mathbf{\Psi} + \mathbf{\Psi}) = P(\mathbf{\Phi}|\,\mathbf{\Psi} \vee \mathbf{\Psi}) = P(\mathbf{\Phi}|\,\mathbf{\Psi}).$$

This implies

$$P(\boldsymbol{\lambda}\mathbf{\Psi}) = P(\mathbf{\Psi}) \text{ where } \boldsymbol{\lambda}\mathbf{\Psi} = \mathbf{\Psi} + \ldots + \mathbf{\Psi} \ (\boldsymbol{\lambda} \text{ times) for all } \boldsymbol{\lambda} = 1,2,3,4\ldots$$

and

$$P(0.\mathbf{\Psi}) = P(\phi) = 0$$

This result can be extended to

$$P(\boldsymbol{\lambda}\mathbf{\Psi}) = P(\mathbf{\Psi}) \text{ where } \boldsymbol{\lambda} \in F. \qquad (10.2.1)$$

2) Probability is not linear in state. If this was so then suppose $P(\mathbf{\Phi}\,|\,\mathbf{\Psi}) > 0$. It follows

$$P(\mathbf{\Psi} + \mathbf{\Phi}\,|\,\mathbf{\Psi}) = P(\mathbf{\Psi}|\,\mathbf{\Psi}) + P(\mathbf{\Phi}\,|\,\mathbf{\Psi}) = 1 + P(\mathbf{\Phi}\,|\,\mathbf{\Psi}) > 1$$

which cannot be correct.

3) The numbers **Ψ** and **Φ** are not members of the field F. If this was not the case, then $\mathbf{\Psi} = \alpha\mathbf{\Phi}$ for some number α, and so

$$P(\mathbf{\Psi}\,|\,\varphi) = P(\alpha\mathbf{\Phi}|\,\varphi) = P(\mathbf{\Phi}|\,\varphi)$$

which cannot be correct.

4) If $\mathbf{\Psi}_i$ are the states associated with distinct outcomes of a measurement, then

$$P(\mathbf{\Psi}_i \,|\, \mathbf{\Psi}_k) = \delta_{ik} \qquad (10.2.2)$$

The above expression looks a lot like an inner product, except that probability is always positive.

Putting it all together, (10.2.1) strongly suggests that the probability is a function of $\frac{\mathbf{\Psi}}{|\mathbf{\Psi}|}$ rather than **Ψ**. I.e.

$$P(\mathbf{\Psi}) = g\left(\frac{\mathbf{\Psi}}{|\mathbf{\Psi}|}\right)$$

(10.2.2) strongly suggests that the presence of an inner product

$$P(\mathbf{\Psi}_i \mid \mathbf{\Psi}_k) = f\left(\left\langle \frac{\mathbf{\Psi}_i}{|\mathbf{\Psi}_i|}, \frac{\mathbf{\Psi}_k}{|\mathbf{\Psi}_k|} \right\rangle\right) \tag{10.2.3}$$

where <.|.> is an inner product and f: $\mathbf{R} \rightarrow [0,1]$ with $f(-1) = 1$, $f(0) = 0$ and $f(1) = 1$.

Choosing $f(x) = |x|^2$, probably the simplest possible choice, and gives the "correct" QM formalization.

The standard representation of measuring devices as operators and measurement values as eigenvalues follows.

10.3 Some Examples - Algebraic Extensions

It is sometimes asked why complex numbers are so prevalent in Quantum Mechanics. They simply occur as the solution to algebraic equations that arise in the calculation of representations

Assume **A** and **B** are conjugate devices that both return two values ↑ and ↓.

Let $\Psi_{\mathbf{A}\uparrow} = \begin{bmatrix}1\\0\end{bmatrix}$, $\Psi_{\mathbf{A}\downarrow} = \begin{bmatrix}0\\1\end{bmatrix}$, then finding a representation for $\Psi_{\mathbf{B}\uparrow}$ and $\Psi_{\mathbf{B}\uparrow}$ is equivalent to solving the equations

$$\Psi_{\mathbf{B}\uparrow} = a\Psi_{\mathbf{A}\uparrow} + b\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } a,b$$
$$\Psi_{\mathbf{B}\downarrow} = c\Psi_{\mathbf{A}\uparrow} + d\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } c,d$$

subject to $P(\Psi_{\mathbf{A}i} \mid \Psi_{\mathbf{B}j}) = 0.5$ for $i\ j$ in {↑,↓}. Solving yields (non-uniquely)

$$\Psi_{\mathbf{A}\uparrow} = \begin{bmatrix}1\\0\end{bmatrix}, \Psi_{\mathbf{A}\downarrow} = \begin{bmatrix}0\\1\end{bmatrix}, \Psi_{\mathbf{B}\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, \Psi_{\mathbf{B}\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

which leads to an obvious geometric interpretation.

If a 3rd 2-valued conjugate device **C** is introduced, then finding a representation is again a case of finding

$$\Psi_{\mathbf{C}\uparrow} = a'\Psi_{\mathbf{A}\uparrow} + b\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } a,b$$
$$\Psi_{\mathbf{C}\downarrow} = c\Psi_{\mathbf{A}\uparrow} + d\Psi_{\mathbf{A}\downarrow} \qquad \text{for some } c,d$$

subject to $P(\Psi_{\mathbf{C}i} \mid \Psi_{\mathbf{A}j}) = 0.5$ and $P(\Psi_{\mathbf{C}i} \mid \Psi_{\mathbf{B}j}) = 0.5$ for $i\ j$ in {↑,↓}.

The relevant equations admit no real-valued solutions. (Non-unique) solutions do exist if the "co-ordinate" domain is expanded to included complex numbers (a standard practice in mathematics over the centuries). The standard solution looks like:

$$\Psi_{\mathbf{C}\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}, \Psi_{\mathbf{C}\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

The geometric justification for this is that it is not possible to have 3 sets of right-angles in 2-dimensional Real space which all are at 45 degrees to each other; moving to complex 2-dimensional space provides the additional degree of freedom.

The corresponding standard measurement operators for Angular Momentum are

$$\mathbf{L}_z = 1/2.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{L}_x = 1/2.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{L}_y = 1/2.\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

If we add a 4th 2-valued conjugate quantity, then a solution requires quaternions. Re-solving to choose symmetric solutions, yields

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm i \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm j \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \pm k \end{bmatrix}$$

where ***i, j, k*** are quaternions. The corresponding operators are

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -k \\ k & 0 \end{bmatrix}$$

11. Time

Naive observers, equipped only with Mach devices, do not have access to a clock. How would such an observer measure time?

11.1 Pauli's Theorem

Pauli's Theorem strongly suggests that there is no such thing as a time operator in Quantum Mechanics. No-go theorems always need to be treated with caution. The best attempts at building a quantum clock so far are statistical in nature. We go with the assumption that that's as good as it gets.

11.2 Ensemble Clocks (Quantum Egg-Timers)

An ensemble quantum clock is a device consisting of many quantum subsystems ("particles") that are initially prepared so that they are in identical states, denoted ↑. The "particles" spontaneously change state to a second state, denoted ↓. The transitions are not under the control of an observer, except perhaps that the observer may be able to switch the whole state transition process on or off. Each transition is believed, from the analysis of transition probabilities, to be statistically independent from any other.

For a practical clock, it should also be possible to reset the clock so that the all the particles move back to the original ↑ state, and the whole process repeated (Flipping the Egg-Timer).

A statistical estimate of the time past since the assembly of particles was prepared ($t = 0$) can be made by counting the number of particles that have changed state ($t > 0$). A clock can be designed with arbitrarily high confidence by increasing the number of particles in the ensemble.

Typically if $N(t)$ = expected proportion of "particles" in the ↑ state at time t, then the "measured" time t

$$t \approx N^{-1}(n)$$

where n is the measured proportion of particles in the ↑ state, and t is suitable restricted.

If there are N particles in the ensemble and the particles are ordered by the order in which they make their transition, the expected position in the ordering for any particle should be N/2. Repeatedly reset and run the clock. A sieve can be constructed to exclude any particles that do not appear to be independent of each other to within a specific confidence.

Stepping back into a world equipped with measurable time..

Once we have a clock, we can relax any requirement that transition probabilities are stable, and that always $P(\mathbf{\Psi}(t+\delta t) \mid \mathbf{\Psi}(t)) = 1$. The Hamiltonian can be introduced by "differentiating" the state vector $\mathbf{\Psi}(t)$ with respect to time.

12. The Ticker-Tape Conjecture

Time is a statistical concept.

The implication is that in extreme conditions, time may cease to be measurable and therefore the equations of Quantum Mechanics break down, in much the same way that continuous fluid flow equations break down approaching the atomic level.

13. Summary

The Ticker-Tape Interpretation is based on Copenhagen (I would hope that Bohr, Heisenberg and Born would recognise it and approve) but rejects the view of Bohr that classical notions of space-time and Physics are a necessary pre-requisite for the formulation of Quantum Physics.

It demonstrates that every-day properties of measurement lead to Quantum Mechanics as we know it. It is difficult to imagine how the Universe could be any other way – the mathematics of Quantum Mechanics are the outcome of analysing measurement streams along with taking practical steps to limit the scope of the analysis to that part of the Universe that can be simply understood. It also explains why entanglement is more fundamental than notions of space-time.

14. Request for Assistance

This paper was part of a website that ranked #2 in Google and Bing searches for "Quantum Interpretations" in early 2000s. Any assistance leasing to the publication of this paper in a recognised journal would be appreciated.

15. References

[1] Copenhagen Interpretation of Quantum Mechanics, Stanford Encyclopaedia of Philosophy, Section 2 - Classical Physics, Jan 24 2004, http://plato.stanford.edu/entries/qm-copenhagen.

[2] Ariel Caticha, "Consistency and Linearity in Quantum Theory", arXiv:quant_ph/9803086 v1 31 Mar 1998.