

Polynomials Generating Twin Prime Numbers

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Abstract

In the Ulam spiral, there are places where prime numbers appear continuously on line. Integers are arranged in a square spiral in the Ulam spiral. I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315 degrees, etc. Then, prime numbers appeared continuously on line. And usually, integers are arranged, but I wonder what would happen if I arranged odd numbers. I arrange odd numbers in the angles of 45, 90, 135, 180, 225, 270, 315, 360 degrees, etc. Then, twin prime numbers appeared continuously on line etc.. I found many polynomials generating 14 to 4 consecutive twin prime numbers.

Contents

1

1	Introduction	3
2	Polynomials generating prime numbers	3
2.1	The Ulam spiral	3
2.2	Polynomial generating prime numbers 1	3
2.3	Polynomial generating prime numbers 2	4
2.4	Polynomial generating prime numbers 3	4
2.5	Euler's polynomial generating prime numbers	4
2.6	Other Polynomials generating prime numbers	4
3	Polynomials generating twin prime numbers	4
3.1	Polynomial generating twin prime numbers 1	5
3.2	Polynomial generating twin prime numbers 2	6
3.3	Polynomial generating twin prime numbers 3	6
3.4	Polynomial generating twin prime numbers 4	7
3.5	Polynomial generating twin prime numbers 5	8
3.6	Polynomial generating twin prime numbers 6	8
3.7	Polynomial generating twin prime numbers 7	8

3.8	Polynomial generating twin prime numbers 8	8
3.9	Polynomial generating twin prime numbers 9	9
3.10	Polynomial generating twin prime numbers 10	9
3.11	Polynomial generating twin prime numbers 11	9
3.12	Polynomial generating twin prime numbers 12	9
3.13	Other Polynomials generating twin prime numbers	10
4	List of Figures	
4.1	Figure 2.1 The Ulam Spiral	13
4.2	Figure 2.2 180 degrees Arrangement Legendre Polynomial	14
4.3	Figure 2.3 135 degrees Arrangement Brox Polynomial	15
4.4	Figure 2.4 270 degrees Arrangement Frame Polynomial	16
4.5	Figure 2.5 90 degrees Arrangement Euler's Polynomial	17
4.6	Figure 2.6 Hexagonal 90 degrees Arrangement Euler's Polynomial	18
4.7	Figure 3.1 45 degrees Arrangement	19
4.8	Figure 3.2 180 degrees Arrangement	20
4.9	Figure 3.3 270 degrees Arrangement	21
4.10	Figure 3.4 360 degrees Arrangement	22
4.11	Figure 3.5 60 degrees Arrangement	23
4.12	Figure 3.6 180 degrees Arrangement	24
4.13	Figure 3.7 135 degrees Arrangement	25
4.14	Figure 3.8 180 degrees Arrangement	26
4.15	Figure 3.9 160 degrees Arrangement	27
4.16	Figure 3.10 60 degrees Arrangement	28
4.17	Figure 3.11 225 degrees Arrangement	29
4.18	Figure 3.12 360 degrees Arrangement	30
5	Consideration	31
6	Acknowledgment	31
	References	31

1 Introduction

I was interested in prime numbers looking at the Ulam spiral, I analyzed it myself. And I learned that Euler's polynomial generating prime numbers is simple and great. I thought that other polynomials generating prime numbers may be found in other arrangements, I investigate. In addition, I thought that polynomials generating twin prime numbers may be found by arranging odd numbers. I found many polynomials generating 14 to 4 consecutive twin prime numbers, and I collect the results.

These algebraic polynomials have the property that for $n = 0, 1, \dots, m-1$ value of the polynomial, eventually in module, are m primes.

2 Polynomials generating prime numbers

2.1 The Ulam spiral

In the Ulam spiral, there are places where prime numbers appear continuously on line. I noticed that there are places where prime numbers appear continuously in a certain pattern in the Ulam spiral, although they do not appear continuously on line. They are two polynomials, $P(n) = 4n^2 + 2n + 41$ and $P(n) = 4n^2 + 6n + 43$, generates 20 primes, see Figure 2.1. Each value is a value obtained by skipping one of Euler prime numbers. When the values of the two polynomials are inserted alternately, the values are the same as values of Euler prime numbers, see Figure 2.1.

2.2 Polynomial generating prime numbers 1

Integers are arranged in a square spiral in the Ulam spiral, but I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315 degrees, etc., using a computer. Then, prime numbers appeared continuously on line. In 180 degrees arrangement, see Figure 2.2, 29 prime numbers appear continuously. It was prime numbers of Legendre polynomial [1798], $P(n) = 2n^2 + 29$, generates 29 primes.

2.3 Polynomial generating prime numbers 2

In 135 degrees arrangement, see Figure 2.3, 29 prime numbers appear continuously. It was prime numbers of Brox polynomial [2006], $P(n) = 6n^2 - 342n + 4903$ (or $6n^2 + 6n + 31$), generates 29 primes. Also, in Figure 2.3, prime numbers of polynomials, $P(n) = 6n^2 + 6n + p$, p are lucky numbers $p = 5, 7, 11, 17, 31$, are clearly appeared. In addition, prime numbers of polynomials, $P(n) = 6n^2 + 12n + p$, p are lucky numbers $p = 11, 13, 19, 23$, are clearly appeared.

2.4 Polynomial generating prime numbers 3

In 270 degrees arrangement, see Figure 2.4, 22 prime numbers appear continuously. It was prime numbers of Frame polynomial [2018], $P(n) = 3n^2 + 3n + 23$, generates 22 primes.

2.5 Euler's polynomial generating prime numbers

In 90 degrees arrangement, see Figure 2.5 and the hexagonal 90 degrees arrangement, see Figure 2.6 (illustrated as a rectangle for simplification in Figure 2.6), 40 prime numbers appear continuously. It was prime numbers of Euler's polynomial, $P(n) = n^2 + n + 41$, generates 40 primes. Also, prime numbers of polynomial, $P(n) = n^2 + n + p$, p are Euler's lucky numbers $p = 3, 5, 11, 17, 41$, are clearly appeared.

2.6 Other polynomials generating prime numbers

I found polynomials generating prime numbers with small continuous numbers, and I will collect them in the future.

3 Polynomials generating twin prime numbers

Integers are arranged in the Ulam spiral, but I wonder what would happen if I arranged odd numbers. I arrange odd numbers in the angles of 45, 90, 135, 180, 225, 270, 315, 360 degrees, etc., using a computer. I mark the twin prime numbers. (In the figure of 360 degrees, I mark the prime numbers and twin prime numbers.) Then, twin prime numbers appeared continuously on line etc.. I found many polynomials generating twin prime numbers. The generating appearance of prime numbers are diagonal, vertical, and

horizontal lines and evenly spaced, but the generating appearance of twin prime numbers are diagonal, vertical, horizontal, and curved lines and evenly spaced or not evenly spaced.

3.1 Polynomial generating twin prime numbers 1

When odd numbers are arranged in 45 degrees arrangement, see Figure 3.1, continuous twin prime numbers appear.

The produce of polynomial is as follows. (Since the method of obtaining the polynomial in Section 3.1 is difficult to understand, so I recommend to refer to the method of obtaining the polynomial in Section 3.2.) The central values of the twin prime numbers are 12, 42, 102, 192, 312, 462, 642.

12	12	n=0
30		
42	30	42=(12+30)
60		=12+30x1
102	30	102=(12+30)+(30+30)
90		=12+30x2+30x1
192	30	192=(12+30)+(30+30)+(30+30+30)
120		=12+30x3+30x3
312	30	312=(12+30)+(30+30)+(30+30+30)+(30+30+30+30)
150		=12+30x4+30x6
462	30	462=(12+30)+(30+30)+(30+30+30)+(30+30+30+30)
180		+ (30+30+30+30)=12+30x5+30x10
642		642=
		$f(n)=12+30n+30xn(n-1)/2=12+30n+15n^2\cdot 15n=15n^2+15n+12$

This polynomial is twin prime numbers even if n = -1 to -7, so I insert n=n-7,

$$\begin{aligned} f(n) &= 15(n-7)^2 + 15(n-7) + 12 = 15n^2 - 15 \cdot 2 \cdot 7n + 15 \cdot 7 \cdot 7 + 15n - 15 \cdot 7 + 12 = 15n^2 - 210n + 735 + 15n - 105 + 12 \\ &= 15n^2 - 195n + 642 \end{aligned}$$

This is polynomial generating 14 twin prime numbers.

$P(n) = 15n^2 - 195n + 642 \pm 1$, generates 14 twin primes: 641/643, 461/463, 311/313, 191/193, 101/103, 41/43, 11/13, 11/13, 41/43, 101/103, 191/193, 311/313, 461/463, 641/643. But since the same twin prime numbers take twice each, so it is polynomial that 7 succession appear twice.

3.2 Polynomial generating twin prime numbers 2

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.2, continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 60, 150, 270, 420, 600, 810, 1050, 1320, 1620, 1950, 2310.

60 90 150 120 270 150 420 180 600 210 810 240 1050	60 30 150=(60+90) =60+90x1 270=(60+90)+(90+30) =60+90x2+30x1 420=(60+90)+(90+30)+(90+30+30) =60+90x3+30x3 600=(60+90)+(90+30)+(90+30+30)+(90+30+30+30) =60+90x4+30x6 810=(60+90)+(90+30)+(90+30+30)+(90+30+30+30) +(90+30+30+30)=60+90x5+30x10 1050= · · · · ·		n=0 n=1 n=2 n=3 n=4 n=5
	$f(n)=60+90n+30xn(n-1)/2=60+90n+15n^2-15n=15n^2+75n+60$		

This is polynomial generating 11 twin prime numbers.

$P(n) = 15n^2 + 75n + 60 \pm 1$, generates 11 twin primes: 59/61, 149/151, 269/271, 419/421, 599/601, 809/811, 1049/1051, 1319/1321, 1619/1621, 1949/1951, 2309/2311.

3.3 Polynomial generating twin prime numbers 3

When odd numbers are arranged in 270 degrees arrangement, see Figure 3.3, continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 6, 12, 30, 60, 102.

6 6 12	6 12 12=(6+6)		n=0 n=1
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	18	=6+6x1	
30	12	30=(6+6)+(6+12)	n=2
	30	=6+6x2+12x1	
60	12	60=(6+6)+(6+12)+(6+12+12)	n=3
	42	=6+6x3+12x3	
102		102=(6+6)+(6+12)+(6+12+12)+(6+12+12+12) =6+6x4+12x6	n=4

$$f(n)=6+6n+12xn(n-1)/2=6+6n+6n^2-6n=6n^2+6$$

This polynomial is twin prime numbers even if n = 1 to 4, so I insert n=n·4

$$f(n)=6(n·4)^2+6=6n^2-6x2x4n+6x4x4+6=6n^2·48n+96+6=6n^2·48n+102$$

This is polynomial generating 9 twin prime numbers.

$P(n) = 6n^2 - 48n + 102 \pm 1$, generates 9 twin primes: 101/103, 59/61, 29/31, 11/13, 5/7, 11/13, 29/31, 59/61, 101/103.

But since the same twin prime numbers take twice each, so it is polynomial that 5 succession appear twice.

3.4 Polynomial generating twin prime numbers 4

When odd numbers are arranged in 360 degrees arrangement, see Figure 3.4, continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 18, 12, 150, 432, 858, 1428, 2142, 3000, 4002.

	18	18	n=0
	-6		
12	144	12=(18-6)	n=1
	138	=18-6x1	
150	144	150=(18-6)+(-6+144)	n=2
	282	=18-6x2+144x1	
432	144	432=(18-6)+(-6+144)+(-6+144+144)	n=3
	426	=18-6x3+144x3	
858	144	858=(18-6)+(-6+144)+(-6+144+144)+(-6+144+144+144)	n=4
	570	=18-6x4+144x6	
1428	144	1428=(18-6)+(-6+144)+(-6+144+144)+(-6+144+144+144)	n=5
	714	+(-6+144+144+144+144)= 18-6x5+144x10	

2142

1050=

$$f(n)=18 \cdot 6n + 144n(n-1)/2 = 18 \cdot 6n + 72n^2 - 72n = 72n^2 - 78n + 18$$

This is polynomial generating 9 twin prime numbers.

$P(n) = 72n^2 - 78n + 18 \pm 1$, generates 9 twin primes: 17/19, 11/13, 149/151, 431/433, 857/859, 1427/1429, 2141/2143, 2999/3001, 4001/4003.

3.5 Polynomial generating twin prime numbers 5

When odd numbers are arranged in 60 degrees arrangement, see Figure 3.5, continuous twin prime numbers appear.

Since the method of obtaining the polynomial is the same as the method described above, so it will be omitted below.

This is polynomial generating 7 twin prime numbers.

$P(n) = 75n^2 - 345n + 420 \pm 1$, generates 7 twin primes: 419/421, 149/151, 29/31, 59/61, 239/241, 569/571, 1049/1051.

3.6 Polynomial generating twin prime numbers 6

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.6, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 21n + 18 \pm 1$, generates 6 twin primes: 17/19, 41/43, 71/73, 107/109, 149/151, 197/199.

3.7 Polynomial generating twin prime numbers 7

When odd numbers are arranged in 135 degrees arrangement, see Figure 3.7, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 27n + 72 \pm 1$, generates 6 twin primes: 71/73, 101/103, 137/139, 179/181, 227/229, 281/283.

3.8 Polynomial generating twin prime numbers 8

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.8, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 69n + 198 \pm 1$, generates 6 twin primes: 197/199, 269/271, 347/349, 431/433, 521/523, 617/619.

3.9 Polynomial generating twin prime numbers 9

When odd numbers are arranged in 160 degrees arrangement, see Figure 3.9, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 6n^2 - 30n + 42 \pm 1$, generates 6 twin primes: 41/43, 17/19, 5/7, 5/7, 17/19, 41/43. But since the same twin prime numbers take twice each, so it is polynomial that 3 succession appear twice.

3.10 Polynomial generating twin prime numbers 10

When odd numbers are arranged in 60 degrees arrangement, see Figure 3.10, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 75n^2 - 165n + 102 \pm 1$, generates 6 twin primes: 101/103, 11/13, 71/73, 281/283, 641/643, 1151/1153.

3.11 Polynomial generating twin prime numbers 11

When odd numbers are arranged in 225 degrees arrangement, see Figure 3.11, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 153n^2 - 135n + 180 \pm 1$, generates 6 twin primes: 179/181, 197/199, 521/523, 1151/1153, 2087/2089, 3329/3331.

3.12 Polynomial generating twin prime numbers 12

When odd numbers are arranged in 360 degrees arrangement, see Figure 3.12, continuous twin prime numbers appear.

This is polynomial generating 5 twin prime numbers.

$P(n) = 288n^2 - 180n + 30 \pm 1$, generates 5 twin primes: 29/31, 137/139, 821/823, 2081/2083, 3917/3919.

3.13 Other polynomials generating twin prime numbers

I found many polynomials generating 4 twin prime numbers. The details of diagrams are omitted. The polynomials found in Figure 3.1 to Figure 3.12 are shown in the figures. The figures of polynomials found in the figures other than Figure 3.1 to Figure 3.12 are omitted.

3.13.1 $P(n) = 3n^2 + 69n + 1878 \pm 1$, generates 4 twin primes:

1877/1879, 1949/1951, 2027/2029, 2111/2113.

3.13.2 $P(n) = 3n^2 + 141n + 1788 \pm 1$, generates 4 twin primes:

1787/1789, 1931/1933, 2081/2083, 2237/2239.

3.13.3 $P(n) = 6n^2 + 222n + 2082 \pm 1$, generates 4 twin primes:

2081/2083, 2309/2311, 2549/2551, 2801/2803.

3.13.4 $P(n) = 9n^2 + 3n + 18 \pm 1$, generates 4 twin primes:

17/19, 29/31, 59/61, 107/109.

3.13.5 $P(n) = 12n^2 + 54n + 42 \pm 1$, generates 4 twin primes:

41/43, 107/109, 197/199, 311/313.

3.13.6 $P(n) = 12n^2 + 174n + 1092 \pm 1$, generates 4 twin primes:

1091/1093, 1277/1279, 1487/1489, 1721/1723.

3.13.7 $P(n) = 18n^2 + 240n + 600 \pm 1$, generates 4 twin primes:

599/601, 857/859, 1151/1153, 1481/1483.

3.13.8 $P(n) = 18n^2 + 252n + 1032 \pm 1$, generates 4 twin primes:

1031/1033, 1301/1303, 1607/1609, 1949/1951.

3.13.9 $P(n) = 27n^2 + 453n + 1788 \pm 1$, generates 4 twin primes:

1787/1789, 2267/2269, 2801/2803, 3389/3391.

- 3.13.10 $P(n) = 33n^2 + 519n + 1998 \pm 1$, generates 4 twin primes:
 $1997/1999, 2549/2551, 3167/3169, 3851/3853.$
- 3.13.11 $P(n) = 48n^2 + 150n + 150 \pm 1$, generates 4 twin primes:
 $149/151, 347/349, 641/643, 1031/1033.$
- 3.13.12 $P(n) = 51n^2 + 657n + 570 \pm 1$, generates 4 twin primes:
 $569/571, 1277/1279, 2087/2089, 2999/3001.$
- 3.13.13 $P(n) = 78n^2 + 228n + 42 \pm 1$, generates 4 twin primes:
 $41/43, 347/349, 809/811, 1427/1429.$
- 3.13.14 $P(n) = 90n^2 + 150n + 822 \pm 1$, generates 4 twin primes:
 $821/823, 1061/1063, 1481/1483, 2081/2083.$
- 3.13.15 $P(n) = 99n^2 + 363n + 108 \pm 1$, generates 4 twin primes:
 $107/109, 569/571, 1229/1231, 2087/2089.$
- 3.13.16 $P(n) = 102n^2 + 72n + 18 \pm 1$, generates 4 twin primes:
 $17/19, 191/193, 569/571, 1151/1153.$
- 3.13.17 $P(n) = 150n^2 - 90n + 12 \pm 1$, generates 4 twin primes:
 $11/13, 71/73, 431/433, 1091/1093.$
- 3.13.18 $P(n) = 201n^2 + 57n + 570 \pm 1$, generates 4 twin primes:
 $569/571, 827/829, 1487/1489, 2549/2551.$
- 3.13.19 $P(n) = 255n^2 - 75n + 12 \pm 1$, generates 4 twin primes:
 $11/13, 191/193, 881/883, 2081/2083.$
- 3.13.20 $P(n) = 294n^2 - 462n + 348 \pm 1$, generates 4 twin primes:
 $347/349, 179/181, 599/601, 1607/1609.$
- 3.13.21 $P(n) = 375n^2 - 555n + 420 \pm 1$, generates 4 twin primes:
 $419/421, 239/241, 809/811, 2129/2131.$

3.13.22 $P(n) = 390n^2 + 90n + 138 \pm 1$, generates 4 twin primes:
137/139, 617/619, 1877/1879, 3917/3919.

3.13.23 $P(n) = -12n^2 + 582n + 312 \pm 1$, generates 4 twin primes:
311/313, 881/883, 1427/1429, 1949/1951.

3.13.24 $P(n) = -45n^2 + 555n + 348 \pm 1$, generates 4 twin primes:
347/349, 857/859, 1277/1279, 1607/1609.

3.13.25 $P(n) = 90n + 1608 \pm 1$, generates 4 twin primes:
1607/1609, 1697/1699, 1787/1789, 1877/1879.

Figure 2.1: The Ulam Spiral

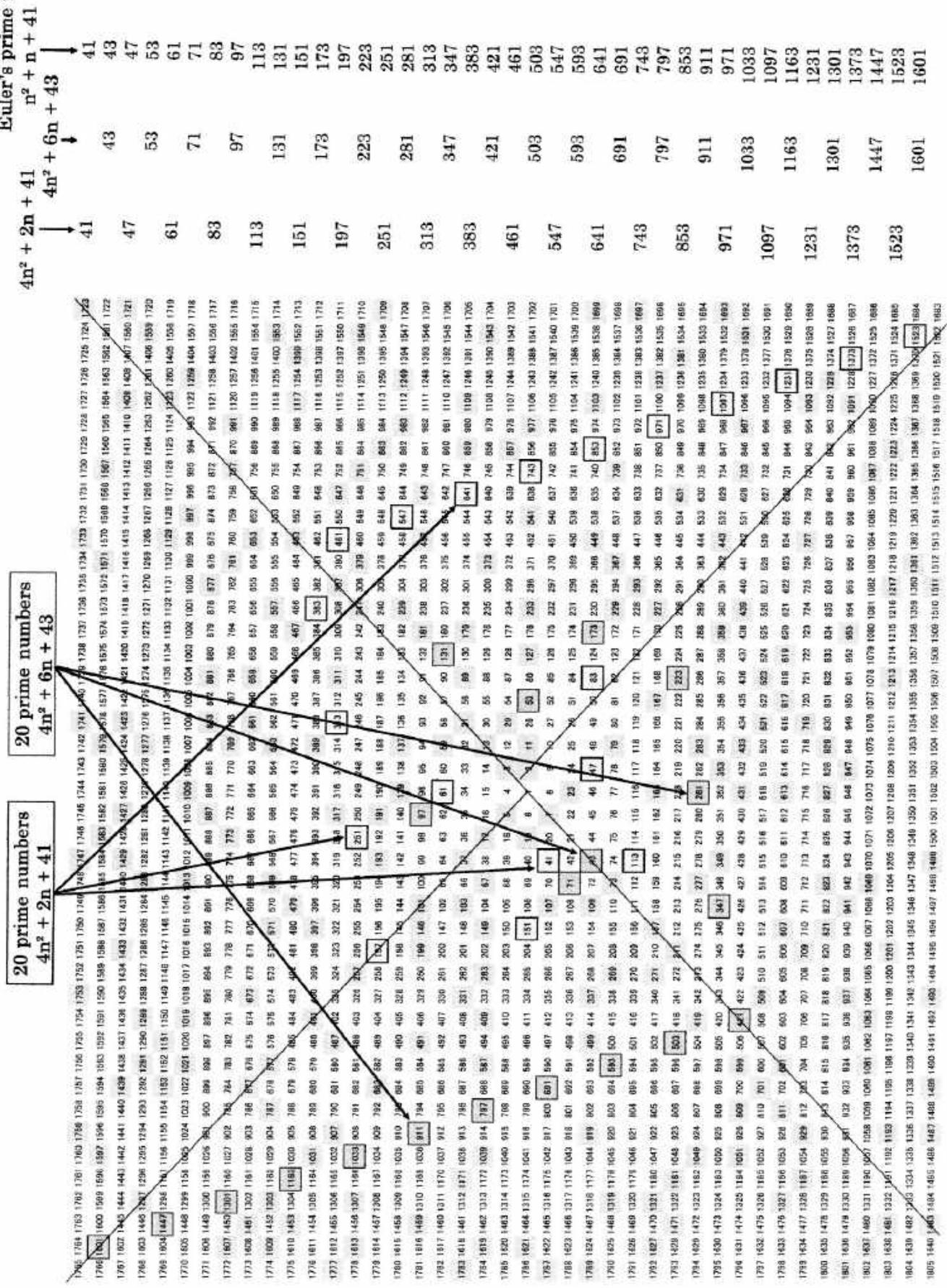
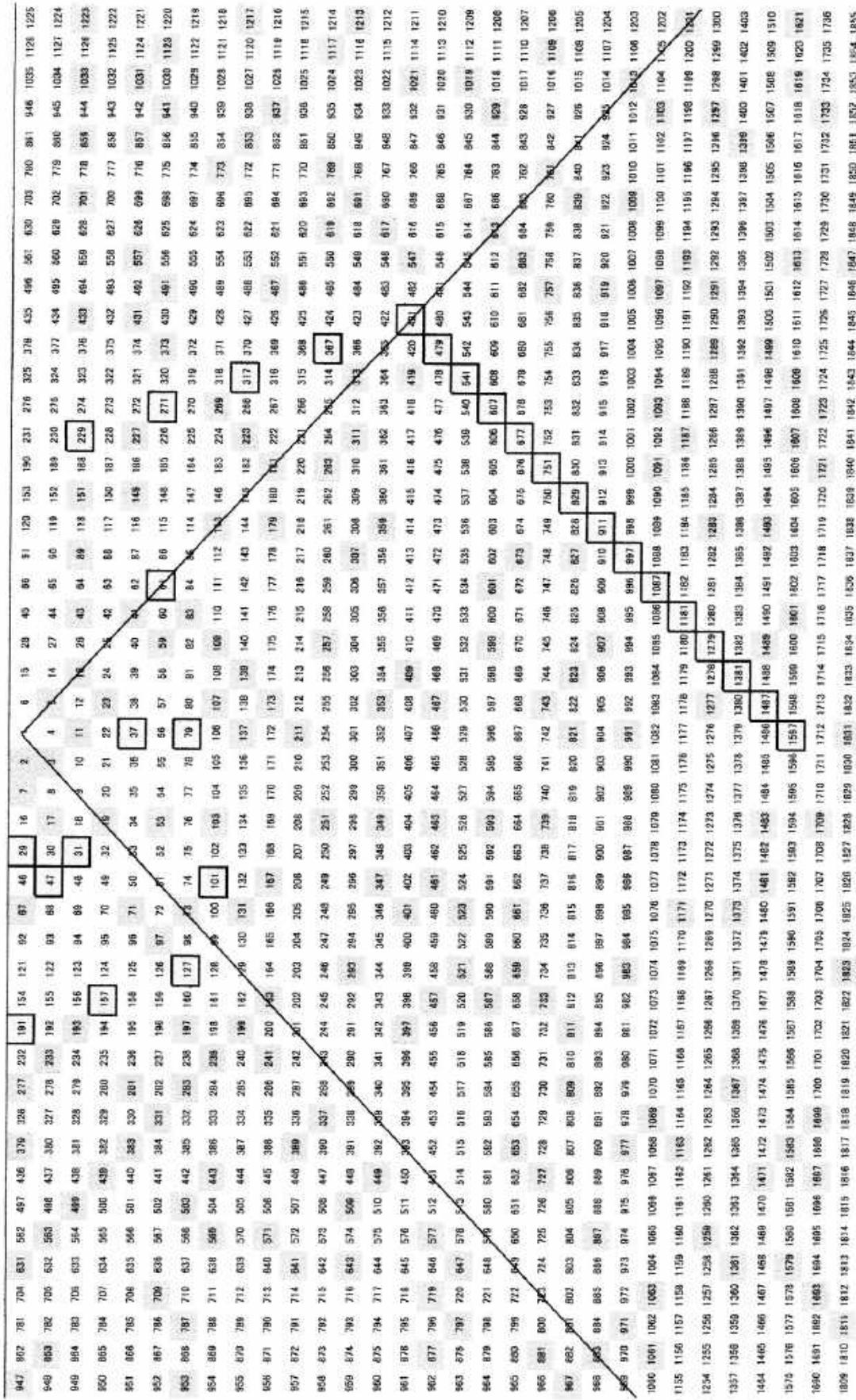


Figure 2.2: 180 degrees Arrangement Legendre Polynomial



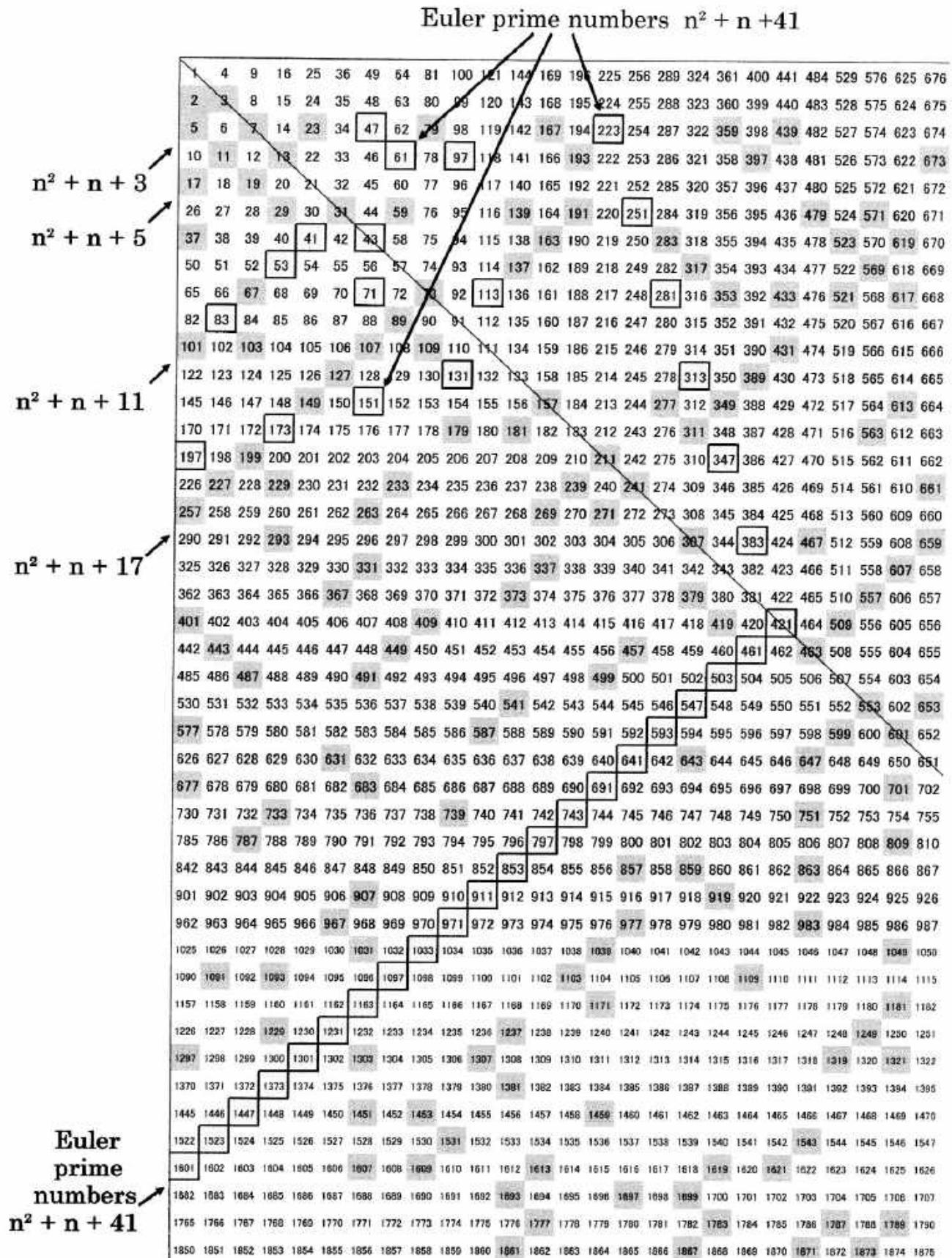
Legendre polynomial $2n^2 + 29$

Figure 2.3: 135 degrees Arrangement Bronx Polynomial

Figure 2.4: 270 degrees Arrangement Frame Polynomial

Frame polynomial $3n^2 + 3n + 23$		Frame Polynomial										Arrangement 270 degrees										
1681	1542	1409	1282	1161	1046	937	834	737	646	561	482	409	342	281	226	177	134	97	66	41	22	9
1682	1543	1410	1283	1162	1047	938	835	738	647	562	483	410	343	282	227	178	135	98	67	42	23	10
1683	1544	1411	1284	1163	1048	939	836	739	648	563	484	411	344	283	228	179	136	99	68	43	24	11
1684	1545	1412	1285	1164	1049	940	837	740	649	564	485	412	345	284	229	180	137	100	69	44	25	26
1685	1546	1413	1286	1165	1050	941	838	741	650	565	486	413	346	285	230	181	138	101	70	45	46	47
1686	1547	1414	1287	1166	1051	942	839	742	651	566	487	414	347	286	231	182	139	102	71	72	73	74
1687	1548	1415	1288	1167	1052	943	840	743	652	567	488	415	348	287	232	183	140	103	104	105	106	107
1688	1549	1416	1289	1168	1053	944	841	744	653	568	489	416	349	288	233	184	141	142	143	144	145	146
1689	1550	1417	1290	1169	1054	945	842	745	654	569	490	417	350	289	234	185	186	187	188	189	190	191
1690	1551	1418	1291	1170	1055	946	843	746	655	570	491	418	351	290	235	236	237	238	239	240	241	242
1691	1552	1419	1292	1171	1056	947	844	747	656	571	492	419	352	291	292	293	294	295	296	297	298	299
1692	1553	1420	1293	1172	1057	948	845	748	657	572	493	420	353	354	355	356	357	358	359	360	361	362
1693	1554	1421	1294	1173	1058	949	846	749	658	573	494	421	422	423	424	425	426	427	428	429	430	431
1694	1555	1422	1295	1174	1059	950	847	750	659	574	495	496	497	498	499	500	501	502	503	504	505	506
1695	1556	1423	1296	1175	1060	951	848	751	660	575	576	577	578	579	580	581	582	583	584	585	586	587

Figure 2.5: 90 degrees Arrangement Euler's Polynomial



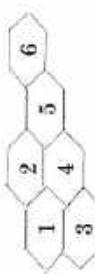


Figure 2.6: Hexagonal 90 degrees Arrangement Euler's Polynomial

Euler prime numbers					
$n^2 + n + 41$			$n^2 + n + 41$		
$n^2 + n + 3$		$n^2 + n + 5$		$n^2 + n + 7$	
2	5	11	29	41	55
1	4	10	18	40	55
3	8	16	27	39	52
7	14	24	36	50	66
13	23	34	46	64	83
21	33	47	62	81	100
31	45	61	79	98	121
43	59	77	98	119	144
57	75	95	118	142	166
73	74	94	116	140	166
73	93	115	139	165	193
91	113	137	164	192	222
111	135	156	182	209	235
133	159	187	216	247	285
157	185	215	247	281	323
183	213	245	279	316	355
211	243	277	313	351	395
241	275	311	349	389	437
273	306	347	387	428	478
307	345	385	427	465	505
344	383	424	466	506	546
343	363	411	452	493	534
362	424	474	514	552	592
381	423	467	513	561	603
422	466	512	560	610	652
421	485	511	565	615	665
464	510	558	606	656	706
483	569	607	646	685	726
508	598	606	632	664	704
537	566	605	647	685	727
553	594	604	656	709	755
601	632	654	696	735	785
651	705	761	816	866	916
704	760	818	878	940	1004
703	789	855	909	945	1009
801	822	854	892	923	959
851	876	904	932	961	987
871	903	937	969	997	1034
913	935	964	993	1024	1054
931	972	997	1034	1063	1093

Figure 3.1: 45 degrees Arrangement

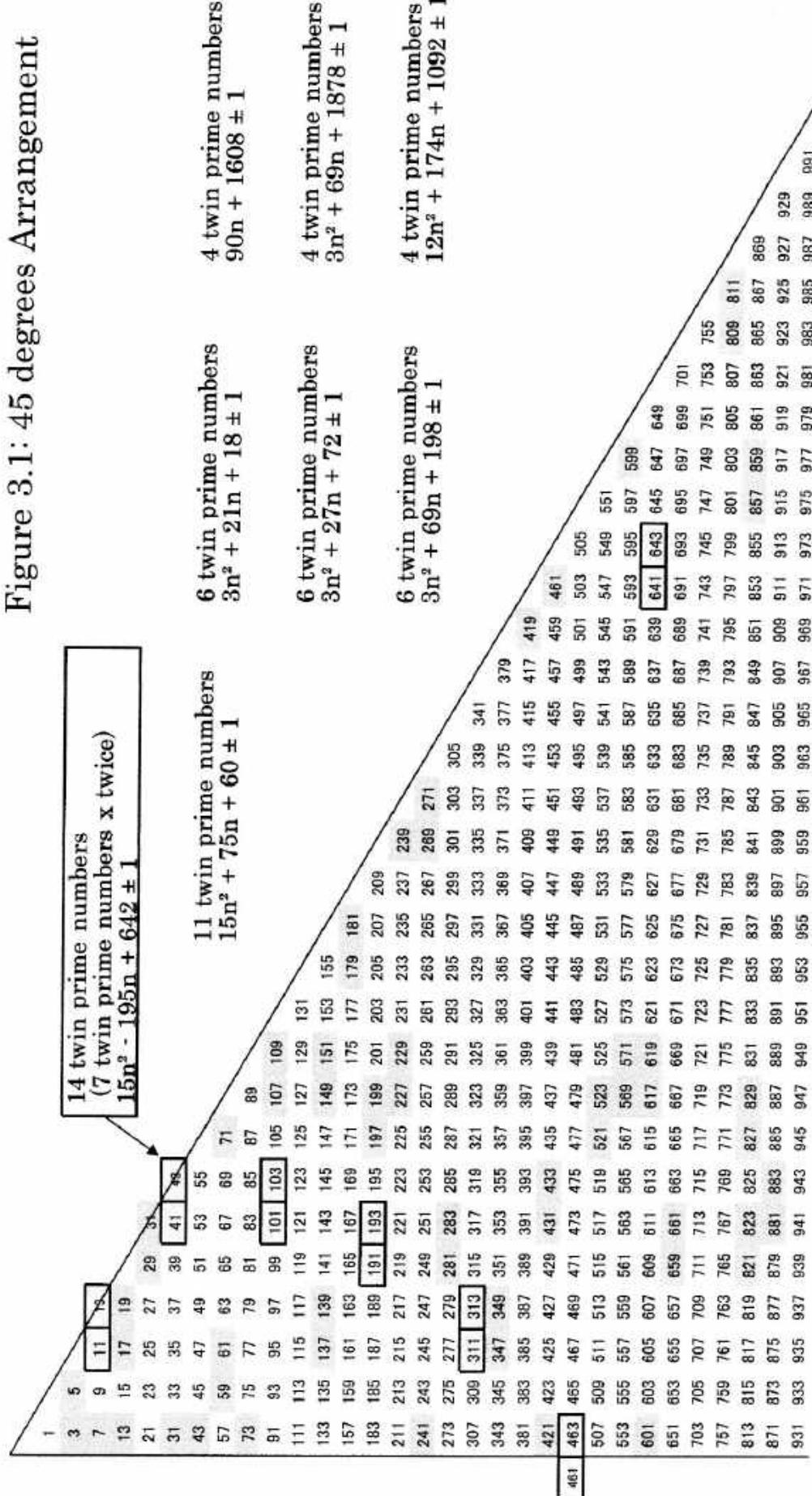


Figure 3.2: 180 degrees Arrangement

11 twin prime numbers $15n^2 + 75n + 60 \pm 1$										
2451	2257	2071	1893	1723	1561	1407	1261	1123	993	871
2453	2259	2073	1895	1725	1563	1409	1263	1125	995	873
2455	2261	2075	1897	1727	1565	1411	1265	1127	997	875
2457	2263	2077	1899	1729	1567	1413	1267	1129	999	877
2459	2265	2079	1901	1731	1569	1415	1269	1131	1001	879
2461	2267	2081	1903	1733	1571	1417	1271	1133	1003	881
2463	2269	2083	1905	1735	1573	1419	1273	1135	1005	883
2465	2271	2085	1907	1737	1575	1421	1275	1137	1007	885
2467	2273	2087	1909	1739	1577	1423	1277	1139	1009	887
2469	2275	2089	1911	1741	1579	1425	1279	1141	1011	889
2471	2277	2091	1913	1743	1581	1427	1281	1143	1013	891
2473	2279	2093	1915	1745	1583	1429	1283	1145	1015	893
2475	2281	2095	1917	1747	1585	1431	1285	1147	1017	895
2477	2283	2097	1919	1749	1587	1433	1287	1149	1019	897
2479	2285	2099	1921	1751	1589	1435	1289	1151	1021	899
2481	2287	2101	1923	1753	1591	1437	1291	1153	1023	901
2483	2289	2103	1925	1755	1593	1439	1293	1155	1025	1027
2485	2291	2105	1927	1757	1595	1441	1295	1159	1029	1031
2487	2293	2107	1929	1759	1597	1443	1297	1299	1030	1305
2489	2295	2109	1931	1761	1599	1445	1447	1449	1451	1453
2491	2297	2111	1933	1763	1601	1603	1605	1607	1609	1611
2493	2299	2113	1935	1765	1607	1605	1607	1609	1611	1613
2495	2301	2115	1937	1767	1609	1607	1609	1611	1613	1615
2497	2303	2117	1939	1769	1611	1609	1611	1613	1615	1617
2499	2305	2307	1941	1771	1613	1609	1613	1615	1617	1619
2501	2503	2505	2507	1773	1615	1609	1615	1617	1621	1623

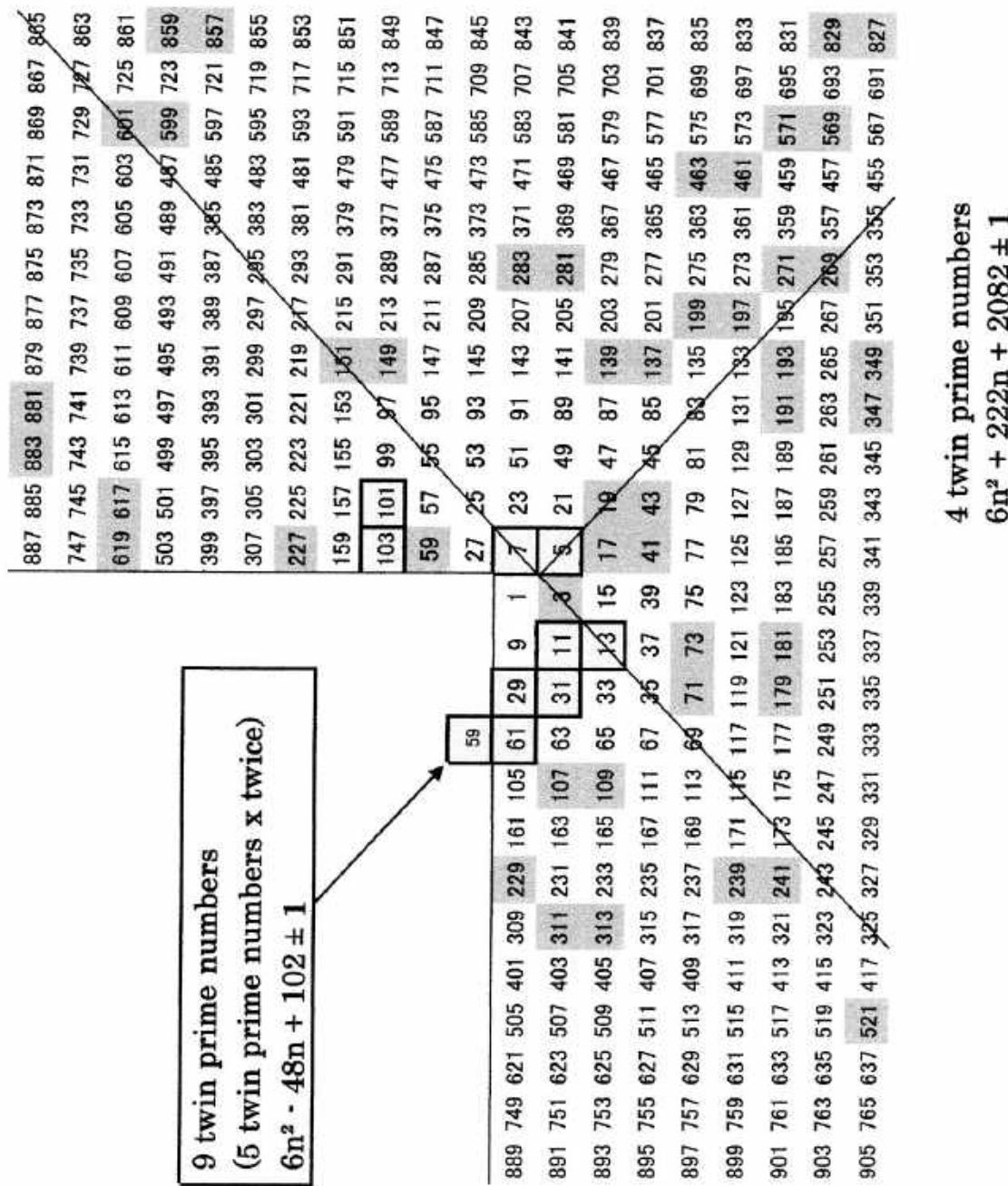
6 twin prime numbers
 $3n^2 + 21n + 18 \pm 1$

4 twin prime numbers
 $18n^2 + 25n + 1032 \pm 1$

4 twin prime numbers
 $90n^2 + 150n + 822 \pm 1$

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

Figure 3.3: 270 degrees Arrangement



4 twin prime numbers
 $33n^2 + 1998 \pm 1$
 $288n^2 - 180n + 30 \pm 1$

Figure 3.4: 360 degrees Arrangement

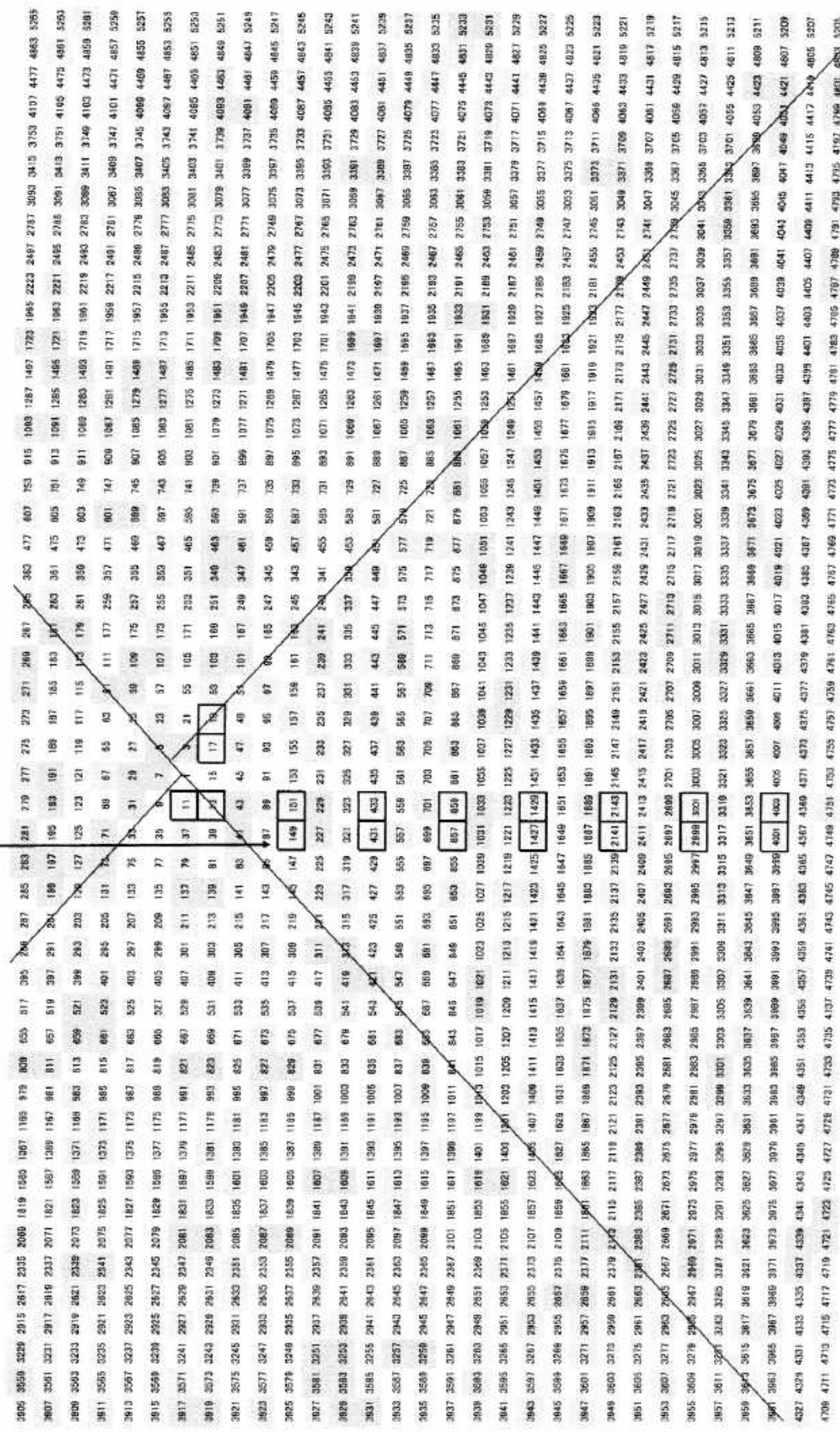


Figure 3.5: 60 degrees Arrangement

6 twin prime numbers $3n^2 + 27n + 72 \pm 1$		7 twin prime numbers $75n^2 - 345n + 420 \pm 1$		4 twin prime numbers $3n^2 + 141n + 1788 \pm 1$	
1					
3	5	7			
9	11	13	15	17	19
21	23	25	27	29	31
33	35	37			
61	63	65	67	69	71
73	75	77	79	81	83
85	87				
91					
129	131	133	135	137	139
141	143	145	147	149	151
153	155	157	159	161	163
165	167	169	171	173	175
177	179	181	183	185	187
197	199	201	203	205	207
209	211	213	215	217	
261	263	265	267	269	271
297	301	303	305	307	309
311	313	315	317	319	321
327	329	331			
375	377	379	381	383	385
387	389	391	393	395	397
423	425	427	429	431	433
435	437	439	441	443	445
447	449	451	453	455	457
459	461	463	465	467	
509	511	513	515	517	519
521	523	525	527	529	531
535	537	539	541	543	545
547					
563	565	567	569	571	573
575	577	579	581	583	585
587	589	591	593	595	597
599	601	603	605	607	609
611	613	615	617	619	621
623	625	627	629	631	633
635	637	639	641	643	645
647	649	651	653	655	657
659	661	663	665	667	669
671	673	675	677	679	681
683	685	687	689	691	693
695	697	699	701	703	705
709					
751	753	755	757	759	761
763	765	767	769	771	773
775	777	779	781	783	785
787	789	791	793	795	797
801	803	805	807	809	811
813	815	817	819	821	823
825	827	829	831	833	835
837	839	841	843	845	847
849	851	853	855	857	859
861	863	865	867	869	871
873	875	877	879	881	883
885	887				
889	891	893	895	897	899
897					
901	903	905	907	909	911
913	915	917	919	921	923
927	929	931	933	935	937
939	941	943	945	947	949
951	953	955	957	959	961
963	965	967	969	971	973
975	977	979	981	983	985
987	989	991	993	995	997
1053	1055	1057	1059	1061	1063
1065	1067	1069	1071	1073	1075
1077	1079	1081	1083	1085	1087
1089	1091	1093	1095	1097	1099
1101	1103	1105			
1143	1147	1149	1151	1153	1155
1157	1159	1161	1163	1165	1167
1169	1171	1173	1175	1177	1179
1181	1183	1185	1187	1189	1191
1193	1195	1197	1199	1201	1203
1205	1207	1209	1211	1213	1215
1217	1219				
1263	1267	1269	1271	1273	1275
1277	1279	1281	1283	1285	1287
1289	1291	1293	1295	1297	1299
1301	1303	1305	1307	1309	1311
1313	1315	1317	1319	1321	1323
1325	1327	1329	1331	1333	1335
1337	1339				
1441	1443	1445	1447	1449	1451
1453	1455	1457	1459	1461	1463
1465					
1521	1523	1525	1527	1529	1531
1533	1535	1537	1539	1541	1543
1545	1547	1549	1551	1553	1555
1557	1559	1561	1563	1565	1567
1569	1571	1573	1575	1577	1579
1581	1583	1585	1587	1589	1591
1593	1595	1597			
1659	1661	1663	1665	1667	1669
1669	1671	1673	1675	1677	1679
1681	1683	1685	1687	1689	1691
1693	1695	1697	1699	1701	1703
1705	1707	1709	1711	1713	1715
1717	1719	1721	1723	1725	1727
1729	1731	1733	1735		
1803	1805	1807	1809	1811	1813
1815	1817	1819	1821	1823	1825
1827	1829	1831	1833	1835	1837
1839	1841	1843	1845	1847	1849
1851	1853	1855	1857	1859	1861
1863	1865	1867	1869	1871	1873
1875	1877	1879	1881	1883	1885
1887	1889	1891	1893	1895	1897
1899	1901	1903	1905	1907	1909
1911	1913	1915	1917	1919	1921
1923	1925	1927	1929	1931	1933
1935	1937	1939	1941	1943	1945
1947	1949	1951	1953	1955	1957
1959	1961	1963	1965	1967	1969
1971	1973	1975	1977	1979	1981
1983	1985	1987	1989	1991	1993
1995	1997	1999	2001	2003	2005
2007	2009	2011	2013	2015	2017
2019	2021	2023	2025	2027	2029
2109	2111	2113	2115	2117	2119
2121	2123	2125	2127	2129	2131
2133	2135	2137	2139	2141	2143
2145	2147	2149	2151	2153	2155
2157	2159	2161	2163	2165	2167
2169	2171	2173	2175	2177	2179
2181	2183	2185	2187	2189	2191
2193	2195	2197	2199	2201	2203
2205	2207	2209	2211	2213	2215
2217	2219	2221	2223	2225	2227
2229	2231	2233	2235	2237	2239
2241	2243	2245	2247	2249	2251
2253	2255	2257	2259	2261	2263
2265	2267	2269	2271	2273	2275
2277	2279	2281	2283	2285	2287
2289	2291	2293	2295	2297	2299
2301	2303	2305	2307	2309	2311
2313	2315	2317	2319	2321	2323
2325	2327	2329	2331	2333	2335
2337	2339	2341	2343	2345	2347

Figure 3.6: 180 degrees Arrangement

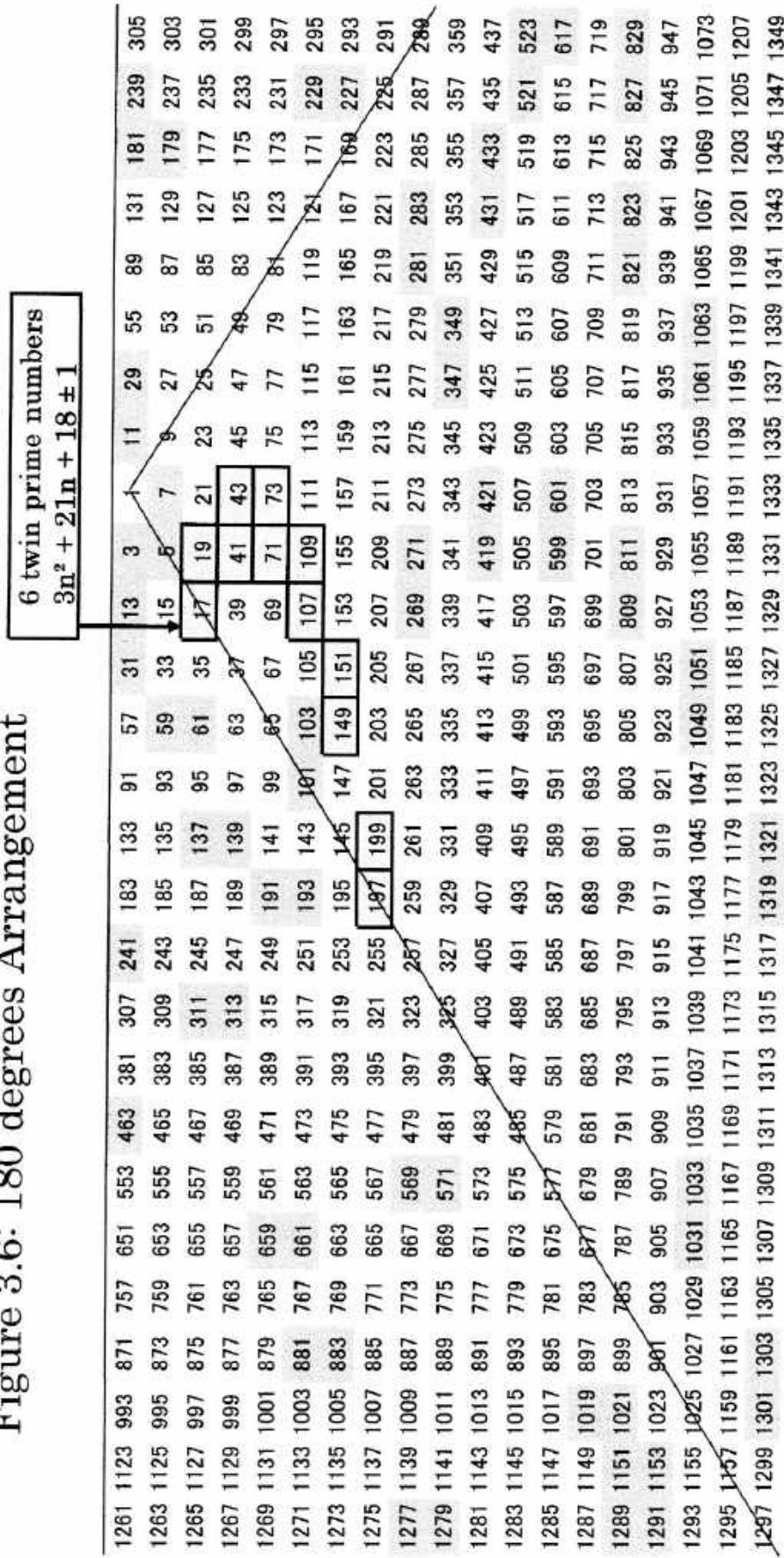


Figure 3.7: 135 degrees Arrangement

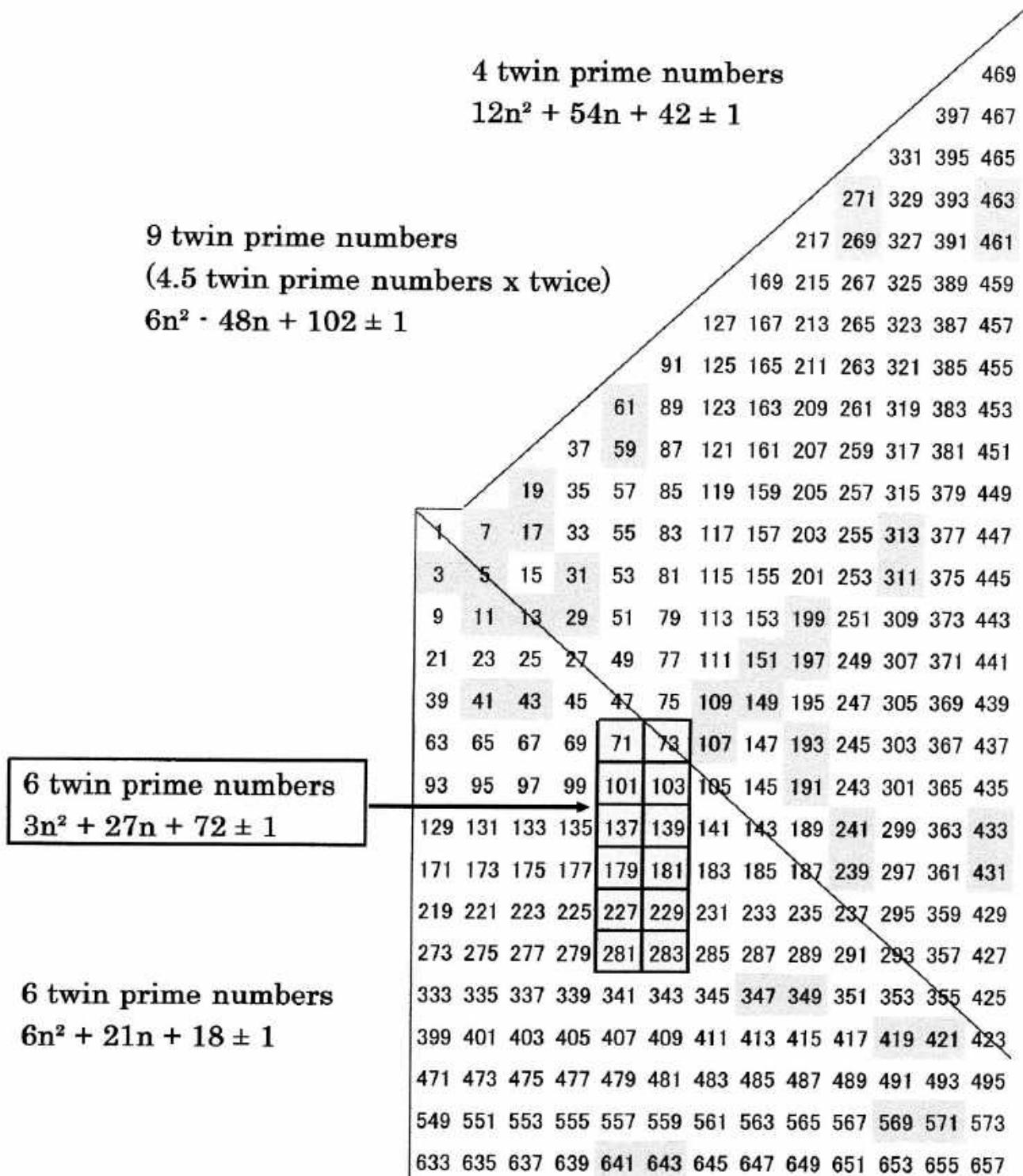


Figure 3.8: 180 degrees Arrangement

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

757	651	553	463	381	307	241	183	133	91	57	31	13	3	1	11	29	55	89	131	181	239	305	379	461	551	649
759	653	555	465	383	309	243	185	135	93	59	33	15	5	7	9	27	53	87	129	179	237	303	377	459	549	647
761	655	557	467	385	311	245	187	137	95	61	35	17	19	21	23	25	51	85	127	177	235	301	375	457	547	645
763	657	559	469	387	313	247	189	139	97	63	37	19	41	43	45	47	49	83	125	175	233	299	373	455	545	643
765	659	561	471	389	315	249	191	141	99	65	67	69	71	73	75	77	79	81	123	173	231	297	371	453	543	641
767	661	563	473	391	317	251	193	143	101	103	105	107	109	111	113	115	117	119	121	171	229	295	369	451	541	639
769	663	565	475	393	319	253	195	145	147	149	151	153	155	157	159	161	163	165	167	169	227	293	367	449	539	637
771	665	567	477	395	321	255	197	199	201	203	205	207	209	211	213	215	217	219	221	223	225	291	365	447	537	635
773	667	569	479	397	323	257	259	261	263	265	267	269	271	273	275	277	279	281	283	285	287	289	363	445	535	633
775	669	571	481	399	325	327	329	331	333	335	337	339	341	343	345	347	349	351	353	355	357	359	361	443	533	631
777	671	573	483	401	403	405	407	409	411	413	415	417	419	421	423	425	427	429	431	433	435	437	439	441	531	629
779	673	575	485	487	489	491	493	495	497	499	501	503	505	507	509	511	513	515	517	519	521	523	525	527	529	627
781	675	577	579	581	583	585	587	589	591	593	595	597	599	601	603	605	607	609	611	613	615	617	619	621	623	625
783	677	679	681	683	685	687	689	691	693	695	697	699	701	703	705	707	709	711	713	715	717	719	721	723	725	727
785	787	789	791	793	795	797	799	801	803	805	807	809	811	813	815	817	819	821	823	825	827	829	831	833	835	837
903	905	907	909	911	913	915	917	919	921	923	925	927	929	931	933	935	937	939	941	943	945	947	949	951	953	955
1029	1031	1033	1035	1037	1039	1041	1043	1045	1047	1049	1051	1053	1055	1057	1059	1061	1063	1065	1067	1069	1071	1073	1075	1077	1079	1081
1163	1165	1167	1169	1171	1173	1175	1177	1179	1181	1183	1185	1187	1189	1191	1193	1195	1197	1199	1201	1203	1205	1207	1209	1211	1213	1215
1305	1307	1309	1311	1313	1315	1317	1319	1321	1323	1325	1327	1329	1331	1333	1335	1337	1339	1341	1343	1345	1347	1349	1351	1353	1355	1357

4 twin prime numbers
 $18n^2 + 252n + 1032 \pm 1$
 $90n^2 + 150n + 822 \pm 1$

Figure 3.9: 160 degrees Arrangement

4 twin prime numbers
 $45n^2 + 555n + 348 \pm 1$

4 twin prime numbers
 $375n^2 - 555n + 420 \pm 1$

6 twin prime numbers
 $(3 \text{ twin prime numbers} \times \text{twice})$
 $6n^2 - 30n + 42 \pm 1$

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

413	503	603														
411	501	601														
329	409	499	599													
327	407	497	597													
325	405	495	595													
253	323	403	493	593												
251	321	401	491	591												
249	319	399	489	589												
187	247	317	397	487	587											
185	245	315	395	485	585											
183	243	313	393	483	583											
131	181	241	311	391	481	581										
129	179	239	309	389	479	579										
127	177	237	307	387	477	577										
85	125	175	235	305	385	475	575									
83	123	173	233	303	383	473	573									
81	121	171	231	301	381	471	571									
49	79	119	169	229	299	379	469	569								
47	77	117	167	227	297	377	467	567								
45	75	115	165	225	295	375	465	565								
23	43	73	113	163	223	293	373	463	563							
21	41	71	111	161	221	291	371	461	561							
19	39	69	109	159	219	289	369	459	559							
1	7	17	37	67	107	157	217	287	367	457	557					
3	5	15	35	65	105	155	215	285	365	455	555					
9	11	13	33	63	103	153	213	283	363	453	553					
25	27	29	31	61	101	151	211	281	361	451	551					
51	53	55	57	59	99	149	209	279	359	449	549					
87	89	91	93	95	97	147	207	277	357	447	547					
133	135	137	139	141	143	145	205	275	355	445	545					
189	191	193	195	197	199	201	263	273	353	443	543					
255	257	259	261	263	265	267	269	271	351	441	541					
331	333	335	337	339	341	343	345	347	349	439	539					
417	419	421	423	425	427	429	431	433	435	437	537					

Figure 3.10: 60 degrees Arrangement

6 twin prime numbers $3n^2 + 27n + 72 \pm 1$							7 twin prime numbers $75n^2 + 345n + 420 \pm 1$							4 twin prime numbers $3n^2 + 141n + 1788 \pm 1$							
1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
63	65	67	69	71	73	75	77	79	81	93	95	97	99	101	103	105	107	109	111	113	115
129	131	133	135	137	139	141	143	145	147	149	151	153	155	157	159	161	163	165	167	169	171
171	173	175	177	179	181	183	185	187	189	191	193	195	197	199	201	203	205	207	209	211	213
219	221	223	225	227	229	231	233	235	237	239	241	243	245	247	249	251	253	255	257	259	261
273	275	277	279	281	283	285	287	289	291	293	295	297	299	301	303	305	307	309	311	313	315
333	335	337	339	341	343	345	347	349	351	353	355	357	359	361	363	365	367	369	371	373	375
399	401	403	405	407	409	411	413	415	417	419	421	423	425	427	429	431	433	435	437	439	441
471	473	475	477	479	481	483	485	487	489	491	493	495	497	499	501	503	505	507	509	511	513
549	551	553	555	557	559	561	563	565	567	569	571	573	575	577	579	581	583	585	587	589	591
633	635	637	639	641	643	645	647	649	651	653	655	657	659	661	663	665	667	669	671	673	675
723	725	727	729	731	733	735	737	739	741	743	745	747	749	751	753	755	757	759	761	763	765
819	821	823	825	827	829	831	833	835	837	839	841	843	845	847	849	851	853	855	857	859	861
921	923	925	927	929	931	933	935	937	939	941	943	945	947	949	951	953	955	957	959	961	963
1029	1031	1033	1035	1037	1039	1041	1043	1045	1047	1049	1051	1053	1055	1057	1059	1061	1063	1065	1067	1069	1071
1143	1145	1147	1149	1151	1153	1155	1157	1159	1161	1163	1165	1167	1169	1171	1173	1175	1177	1179	1181	1183	1185
1263	1265	1267	1269	1271	1273	1275	1277	1279	1281	1283	1285	1287	1289	1291	1293	1295	1297	1299	1301	1303	1305
1389	1391	1393	1395	1397	1399	1401	1403	1405	1407	1409	1411	1413	1415	1417	1419	1421	1423	1425	1427	1429	1431
1521	1523	1525	1527	1529	1531	1533	1535	1537	1539	1541	1543	1545	1547	1549	1551	1553	1555	1557	1559	1561	1563
1659	1661	1663	1665	1667	1669	1671	1673	1675	1677	1679	1681	1683	1685	1687	1689	1691	1693	1695	1697	1699	1701

Figure 3.11: 225 degrees Arrangement

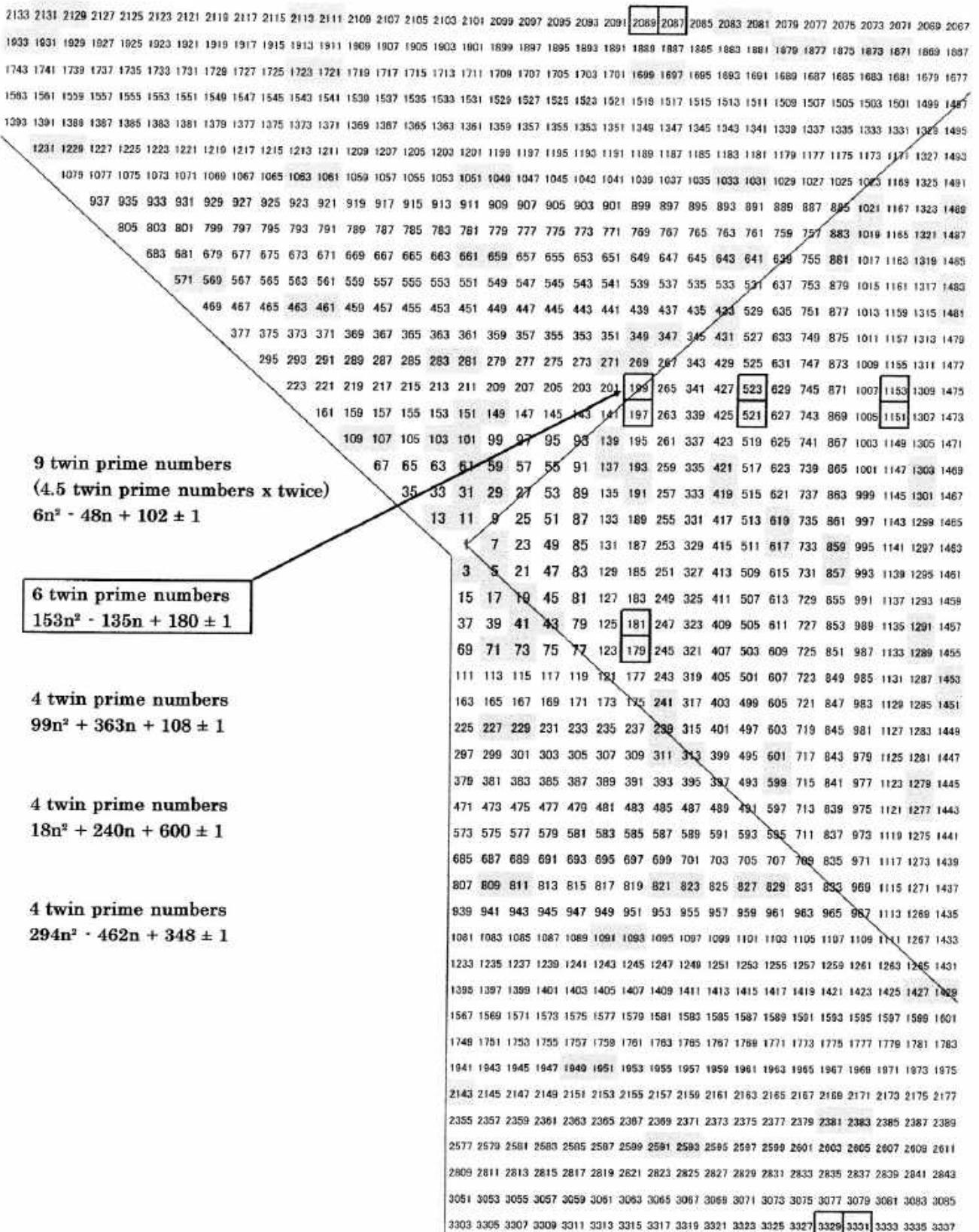


Figure 3.12: 360 degrees Arrangement

4 twin prime numbers
 $33n^2 + 519n + 1998 \pm 1$

5 twin prime numbers
 $288n^2 - 180n + 30 \pm 1$

5859 5443 5033 4639 4261 3899 3553 3223 2909 2611 2329 2063 1813 1579 1361 1159 973 803 649 647 645 643 641 639 637 635 633 631 629 627 625 623 621
5871 5445 5035 4641 4263 3901 3555 3225 2911 2613 2331 2065 1815 1581 1363 1161 975 805 651 513 511 509 507 505 503 501 499 497 495 493 491 489 487
5873 5447 5037 4643 4265 3903 3557 3227 2913 2615 2333 2067 1817 1583 1365 1163 977 807 653 515 393 391 389 387 385 383 381 379 377 375 373 371 369
5875 5449 5039 4645 4267 3905 3559 3229 2915 2617 2335 2069 1819 1585 1367 1165 979 809 655 517 395 289 287 285 283 281 279 277 275 273 271 269 267
5877 5451 5041 4647 4269 3907 3561 3231 2917 2619 2337 2071 1821 1587 1369 1167 981 811 657 519 397 291 204 199 197 195 193 191 189 187 185 183 181
5879 5453 5043 4649 4271 3909 3563 3233 2919 2621 2339 2073 1823 1589 1371 1169 983 813 659 521 399 293 203 129 127 125 123 121 119 117 115 113 111
5881 5455 5045 4651 4273 3911 3565 3235 2921 2623 2341 2075 1825 1591 1373 1171 985 815 661 523 401 295 205 131 73 71 69 67 65 63 61 61 111 117
5883 5457 5047 4653 4275 3913 3567 3237 2923 2625 2343 2077 1827 1593 1375 1173 987 817 663 525 403 297 207 133 75 33 31 29 27 25 23 21 21 55 105 175
5885 5459 5049 4655 4277 3915 3569 3239 2925 2627 2345 2079 1829 1595 1377 1175 989 819 665 527 405 299 209 135 77 35 9 7 6 23 57 107 173
5887 5461 5051 4657 4279 3917 3571 3241 2927 2629 2347 2081 1831 1597 1379 1177 991 821 667 529 407 301 211 137 79 37 11 1 1 3 21 55 105 171
5889 5463 5053 4659 4281 3919 3573 3243 2929 2631 2349 2083 1833 1599 1381 1179 993 823 669 531 409 303 213 139 81 39 13 15 17 19 53 103 169
5891 5465 5055 4661 4283 3921 3575 3245 2931 2633 2351 2085 1835 1601 1383 1181 995 825 671 533 411 305 215 141 83 41 43 45 47 49 51 101 167
5893 5467 5057 4663 4285 3923 3577 3247 2933 2635 2353 2087 1837 1603 1385 1183 997 827 673 535 413 307 217 143 86 87 89 91 93 95 97 99 166
5895 5469 5059 4665 4287 3925 3579 3249 2935 2637 2355 2089 1839 1605 1387 1185 999 829 675 537 415 309 219 145 147 149 151 153 155 157 159 161 163
5897 5471 5061 4667 4289 3927 3581 3251 2937 2639 2357 2091 1841 1607 1389 1187 1001 831 677 539 417 311 221 223 225 227 229 231 233 235 237 239 241
5899 5473 5063 4669 4291 3929 3583 3253 2939 2641 2359 2093 1843 1609 1391 1189 1003 833 679 541 419 313 315 317 319 321 323 325 327 329 331 333 335
5901 5475 5065 4671 4293 3931 3585 3255 2941 2643 2361 2095 1845 1611 1393 1191 1005 835 681 543 421 423 425 427 429 431 433 435 437 439 441 443 445
5903 5477 5067 4673 4295 3933 3587 3257 2943 2645 2363 2097 1847 1613 1395 1193 1007 837 683 545 547 549 551 553 555 557 559 561 563 565 567 569 571

5 Consideration

It is expected that polynomials generating twin prime numbers may be found by devising other arrangements or by arranging up to large odd numbers by the method described in this research work.

6. Acknowledgment

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