

# The spectra of cosmic ray protons and the origin of particles.

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## ABSTRACT

This work quantitatively deals with the fundamental issue of the origin of particles. By focussing initially on the relation between mass and magnetic moments for baryons, we then show that a wealth of experimental data, including the form of the observed flux profiles of cosmic rays protons can be consistently analyzed together if one takes as the starting point of the model that these particles condense from a vacuum, in the form of loops of charge. This brings back the longterm issue of whether particles should strictly be treated theoretically as point-like objects, or otherwise might have an intrinsic motion formally accounted for, which might be related to an effective finite size. Such issue arises in the so-called zitterbewegung solution of the point-like particle's Dirac equation, which introduces the intrinsic motion of a "point" electron in the center of mass frame. The present treatment also applies the Dirac equation, and we obtain regularized loop-like solutions for the fields representing *embryonic* baryons. The fundamental "proton state" is taken as a substrate, as proposed by Barut many years ago. A loop can be regarded as a particle "embryo" and is a sizeable ( wavepacket) rather than a point-like object, tenths of a femtometer big when detached from the vacuum parent state. Such vacuum state is predicted as 2.7 GeV above the proton rest mass, which is consistent with the peak position observed in the energy profile of cosmic rays protons flux. Features of such energy profile can readily be understood from the present analysis.

**Keywords:** Casimir effect, Cosmic Rays.

## INTRODUCTION

The present paper contains the main results of investigations which have directly addressed the longterm problem of describing the genesis of particles. In particular, the issue of the origin of mass is considered[1,2]. Extensive use of ideas previously advanced by Barut, Post, and Jehle [3-5], has been made. For short, the starting point in this treatment, following Evert Post and Herbert Jehle, is that magnetic moments are fundamental properties of leptons and baryons, and that the presence of magnetic moments in particles can be *modelled* by the introduction of an intrinsic closed electrical current loop of finite ( rather than point-like) size. It might be argued that such hypothesis should be incompatible with QED and that electrons behave experimentally as point-like objects. However, as shown later, the present treatment may be regarded as describing the earliest stages of a particle condensation process, in which electromagnetic energy is concentrated in the form of rest mass. The current-loop model refers specifically to this *embryonic* stage, and no internal structure ( probably with a more complex topology) for baryons is explicitly introduced. A multiply connected current path arises, whose possible topological forms were the object of intense discussions by Jehle and Post, but in the absence of more concrete evidence a simple circular loop path is adopted in this treatment. The confinement of magnetic flux within such paths was initially assumed as occurring in numbers of flux quanta determined by the magnetic moments in magneton units, a property easily derived from Barut's semiclassical spinning particle-model, but such assumption is later adjusted to better fit data.

In paper [1] we have shown that it is possible to describe the masses  $m$  of all the baryons of the octet and decuplet in terms of a single formula, involving the magnetic moments  $\mu$  and corresponding numbers of confined flux quanta  $n$ . One might otherwise use this relation to define  $n$  from the experimental masses and moments[2]:

$$n = (2 f c^2 \alpha / e^3 ) \mu m.$$

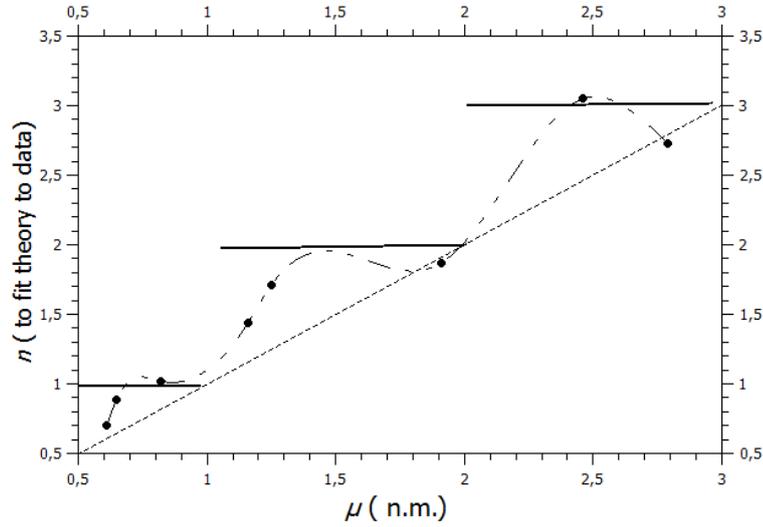


Figure1: Plot of  $n$  against the magnetic moment for the baryons octet ( points) from the definition  $n = (2 c^2 \alpha / e^3 ) \mu m$ . The diagonal line is the classical prediction of one flux quantum per nuclear magneton (n.m.). Nucleons are on the line. The data display undulations, and a tendency to reach for the steps ( traced line as guide)[2].

where  $f=1$  for spin  $\frac{1}{2}$  and  $\approx 1/\sqrt{3}$  for spin  $3/2$ , and  $\alpha$  is the fine-structure constant ( one immediately recovers the often-mentioned inverse relation of mass with the constant  $\alpha$ , since  $n$  and  $\mu$  are approximately proportional to each other). The treatment that results into this equation is essentially heuristic, but precise enough for instance to highlight the dependence of mass upon the square-root of the spin angular momentum, as reported in the literature (note that the phenomenological factor  $f$  that corrects for spin is related to kinetic energies rather than to magnetic contributions)( cf. Figure 1 of ref.[1]). This made clear that mass is a property which is essentially determined by magneto-kinetic energies at the early stage of creation of a baryon, not requiring further inputs from other kinds of interactions at shorter range that might dominate at later stages( one may wonder that what we call strong interactions would result from local changes of topology inside the particle, beyond the initial loop stage, which would produce the extreme strength of a proton- and of the electron).

In paper [2], whose main results are reproduced in the following section, we took much further the treatment presented in [1]. A key parameter in this analysis is the number of flux quanta  $n$  arrested inside a current loop. In particular, we obtain in [2] a very revealing result which has previously been reported mainly through Condensed Matter physics investigations, which is that the energy of currents ( here regarded as a particle's rest mass) is a *periodic function* of the confined magnetic flux in multiply connected structures. Consistently with these results from Condensed Matter systems, the periodic behavior of baryons masses ( and confined flux )with the magnetic moments( see Figure 1) can be regarded as a demonstration that the initial hypotheses of the present investigations are sound. That is, indeed mass is a manifestation of magnetodynamic energies ( related to currents) confined in a multiply connected region. Such hypothesis is therefore consistent with experimental data. With this evidence in hand the next step clearly was to advance beyond the initial phenomenological-heuristic argumentation and propose a field-theoretical treatment that would describe the observed mass –energy relations for actual particles.

Such kind of treatment has previously been applied for fermion fields flowing around a closed loop containing magnetic flux( see refs in [2]). Starting from a Lagrangian suitable to these baryon fields( assumed as built upon a proton field “substrate”, following Barut), we then obtain an energy spectrum for the possible current-carrying states around a closed path. To simulate the perturbations coming from the vacuum background which will be added to the proton state, a sum over the states in the energy spectrum of kinetic energies for the loop/fermion is necessary. An Epstein-Riemann zeta-function regularization procedure previously adopted for the Casimir Effect problem is applied to eliminate divergences when the sum upon the energy spectrum states is carried out, and the periodic behavior of the baryons masses with magnetic flux is quantitatively reproduced with no further forms of energies required besides the magnetodynamic terms. A new result of this treatment [2], is the

prediction of a parent state at  $U_0 = 3.7$  GeV, which should be identified with the vacuum, which fluctuation instabilities would give origin to baryons. The present work goes beyond ref. [2] in the search for evidence for the existence of this state. The calculated value of  $U_0$  immediately indicates that protons (of rest mass 0.94 GeV) should become unstable if accelerated to kinetic energies beyond 2.7 GeV if their structure were not strong enough and capable to radiate excess energy. We found out that a very good way to investigate this point is through the analysis of the spectra of protons in cosmic rays, whose energy flux profile peaks at 2.7 GeV kinetic energy( Figure 3 below) for reasons we will discuss.

In the following sections we firstly present the field-theoretic model introduced in [2], alongside the comparison with experimental data for mass and magnetic moments for baryons. In the Analysis section we test the hypothesis of the existence of an energy level for vacuum by examining data collected for protons in cosmic rays. We then present the Conclusions.

## FIELD-THEORETICAL MODEL FOR GENERATION OF BARYONS

For the developments that led to this field-theoretic treatment we make reference also to the Annales paper[1]( see also references therein and in ref.[2]). Let's consider a fermion field confined to a circular path of length  $L$ , enclosing an amount of self induced magnetic flux  $\phi$ , in a potential  $A$ . We need to show that such a field corresponds to a state detached from a higher state associated with a sea of excitations in equilibrium, and therefore might be used to represent a "quasiparticle". The relativistic Lagrangian for such a fermion can be modelled through the dressing of a proton of mass  $m_p$  ( as once proposed by A. Barut) in view of the presence of magnetodynamic terms[2]:

$$\mathcal{L} = \bar{\psi} \{i\alpha_\mu (\not{\partial}_\mu - i\frac{e}{c}A_\mu) - \alpha_4 m_p c\} \psi \quad (1)$$

where the  $\alpha_\mu$  are Dirac matrices. This Lagrangian can readily be transformed into a Hamiltonian form. For  $A$  a constant around the ring path, the spectrum of possible energies for a confined fermion are obtained as:

$$\varepsilon_k = c \{(p_k - eA/c)^2 + m_p^2 c^2\}^{1/2} \quad (2)$$

which comes straight from the orthonormalized definition of the Dirac matrices and diagonalization of the Hamiltonian. We now definitely impose a circular closed path. If one takes the Bohr-Sommerfeld quantization conditons, the momentum  $p_k$  ( for integer  $k$ ) is quantized in discrete values  $2\pi\hbar k/L$ . We start from this assumption but the true boundary conditions to close the wave loop might impose corrections to this rule in the form of a phase factor( a phase factor is introduced in the fit to the data below). The potential  $A$  can be replaced by  $\phi/L$ . Such charge motion is affected, for instance, by vacuum polarization and the effects on the kinetic energy are accounted for in a way similar to that used in the analysis of the Casimir Effect, by summing over all possible integer values of  $k$  in eq.(2) [2]. This summation diverges. According to the theory of functions of a complex variable the removal of such divergences requires that the analytic continuation of the terms be taken, which reveals the diverging parts which are thus considered as contributions from the infinite vacuum reservoir. A successful technique for this purpose begins with the rewriting of eq.(2) in terms of Epstein-Riemann Zeta functions[2], including the summation over  $k$  from minus to plus infinity integers, and making a regularization (Reg) transformation. Here  $M(\phi)$  is the flux-dependent dressed mass of a baryon, and  $s \rightarrow -1$ :

$$M c^2 = U_0 + \text{Reg} \sum_k c \{(p_k - e\phi/Lc)^2 + m_p^2 c^2\}^{-s/2} \quad (3)$$

where we have allowed for the existence of a finite energy  $U_0$  to represent an hypothetical state from which the individual baryons would condense, since they would correspond to lower energy states. Such particles should be characterized as states of energy  $Mc^2$  lower than  $U_0$ . It is convenient to define from  $L$  a parameter with units of mass  $m_0 = 2\pi\hbar/cL$ , which will be used to define a scale in the fit to the data. We notice that  $m_0$  is related to the parameter  $L$  in the same way field-theories regard mass as created from broken symmetries of fields, establishing a range for an otherwise boundless field distribution( e.g., as happens with the London penetration depth at the establishment of a superconductor state, that is related to an electromagnetic field "mass" by a similar

expression). For convenience, we define the ratios  $m' = m_p / m_0$  and  $u_0 = U_0 / m_p c^2$ . For comparison with the data analysis in our previous work[1], we must introduce also the number of flux quanta  $n$  ( integer or not) associated to  $\phi$ , such that  $n = \phi / \phi_0$ . In terms of these parameters one may write (3) in the form:

$$M(n)/m_p = u_0 + (1/m') \text{Reg} \sum_k \{(k-n)^2 + m'^2\}^{-s/2} \quad (4)$$

In the analysis of data the experimental values of  $M/m_p$  for baryons will be plotted against  $n$ . The sum in the right side of (4) is a particular case of an Epstein Zeta function  $Z(s)$ , and becomes a Riemann Zeta function, since the summation is over one parameter  $k$  only. The summation diverges but it can be analytically continued over the complex plane, since the Epstein Zeta function displays the so-called reflection property. It has been shown that after the application of reflection the resulting sum is already regularized, with the divergences eliminated. The reflection formula is[2]:

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) Z(s) = \pi^{\frac{s-1}{2}} \Gamma\left(\frac{1-s}{2}\right) Z(1-s) \quad (5)$$

This replaces the diverging  $Z(s)$  straight away by the regularized  $Z(1-s)$ , which converges( since  $\Gamma(-1/2) = -2\sqrt{\pi}$  we see that the regularized sums are negative, like in the Casimir Effect solution).

For the sake of clarity we describe step by step the regularization of eq.(4) to become eq.(6) ( note that  $s \rightarrow -1$ , and the “reflected” exponent  $-(1-s)/2$  replaces  $-s/2$  of (4) ). In the first passage from the left below, the entire summation argument is replaced by the Mellin integral which results into it. This creates a convenient exponential function to be integrated later. In the second passage the Poisson summation formula is used, in which the summed exponential function is replaced by its Fourier Transform ( note that the same notation  $k$  is used for the index to be summed in the Fourier Transformed quantity). The objective is to replace the  $k^2 t$  in the initial exponential by  $k^2/t$ . In this way, when the integration over  $t$  is carried out a modified Bessel function  $K$  is obtained. In the final line the  $k=0$  term in the sum is separately worked out and appears as the first term between brackets. The remaining summation in  $k$  therefore does not include 0 ( “/0” as shown). The influence of the parameter  $n$  is, as we wanted to prove, to introduce a periodicity depending on the amount of flux confined by the current ring, and the regularized energy is therefore periodic in  $n$ . Therefore,  $Z(1-s)$  is given as:

$$\begin{aligned} & \sum_k \{(k-n)^2 + m'^2\}^{-(1-s)/2} = \\ & = \frac{2}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{\frac{1-s}{2}-1} \left( \sum_k e^{-(k-n)^2 t - m'^2 t} \right) dt = \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{-\frac{s}{2}-1} \left( \sum_k e^{-2\pi i k n} e^{-\frac{\pi^2 k^2}{t} - m'^2 t} \right) dt \\ & = \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \left( \frac{\Gamma\left(-\frac{s}{2}\right)}{m'^{-s/2}} + 2\pi^{-s/2} \sum_{k \neq 0} \left(\frac{k}{m'}\right)^{-\frac{s}{2}} K_{\frac{s}{2}}(2\pi m' k) e^{-2\pi i k n} \right) \end{aligned} \quad (6)$$

for  $s \rightarrow -1$ . From eq. (5), the “Reg” summation in (4) is replaced by  $(\pi^{\frac{2s-1}{2}} / \Gamma\left(\frac{s}{2}\right)) \Gamma\left(\frac{1-s}{2}\right) Z(1-s)$ , and the exponential produces a cosine term.

Since  $\Gamma(-1/2) = -2\sqrt{\pi}$  we see that the regularized sum is negative, corresponding to energies lower than  $U_0$ . In the fitting to the data we will admit that both  $m'$  and  $u_0$  are adjustable parameters.

Figure 2 below shows the data for all baryons in Tables I and II of ref.[2], and the plot of mass in eq.(4) regularized by eq.(6), for  $u_0 = 3.96$  and  $m' = 0.347$  ( corresponding to  $m_0 = 2.88 m_p$  and  $U_0 = 3710$  MeV). The energy 3710 MeV would represent the vacuum state from which the baryons would evolve.

## ANALYSIS

This model produced an entirely new result, which is the proposal of a parent vacuum state at 3.71 GeV. Flux profiles of cosmic rays (CR) protons display important features [6] that seem related to the existence of a threshold energy, corresponding to the difference between 3.71 GeV and the proton's rest energy of 0.94 GeV, that is, an energy difference of about 2.77 GeV, which we will round out to 3 GeV. Let's first of all consider how protons of CR gain such huge amount of energy. At some point in their history protons were subject to immense electromagnetic forces, which their structure resisted without decay. The inner structure of protons (perhaps not in the topological form of the embryonic loops we considered for calculation purposes, as discussed earlier) is most likely formed by the coherent entanglement of at least three major constituents, each of them with 1/3 of the proton rest mass. It is an ensemble of objects strongly connected together and might be characterized by proper elastic constants. For simplicity we shall neglect anything besides bulk isotropic volume distortions in this analysis. Excited by external forces a three-dimensional elastic structure will vibrate at their natural frequencies. The proton might therefore be represented by a three-dimensional quantum harmonic oscillator with three masses. We shall take 3 GeV as the ground state energy of such isolated oscillator, since this number gives according to our theory the maximum kinetic or potential energies. This assumption will produce the natural frequency  $\omega: 3/2 \hbar\omega = 3 \text{ GeV}$  (we neglect corrections in the harmonic oscillator formulas due to ultra-relativistic conditions, as well as any other deformation modes like torsion, e.g.). This produces  $\omega = 3 \times 10^{24} \text{ rad/sec.}$ , an extremely high figure. At this point it is worthwhile to examine some available experimental data, well gathered in ref.[6].

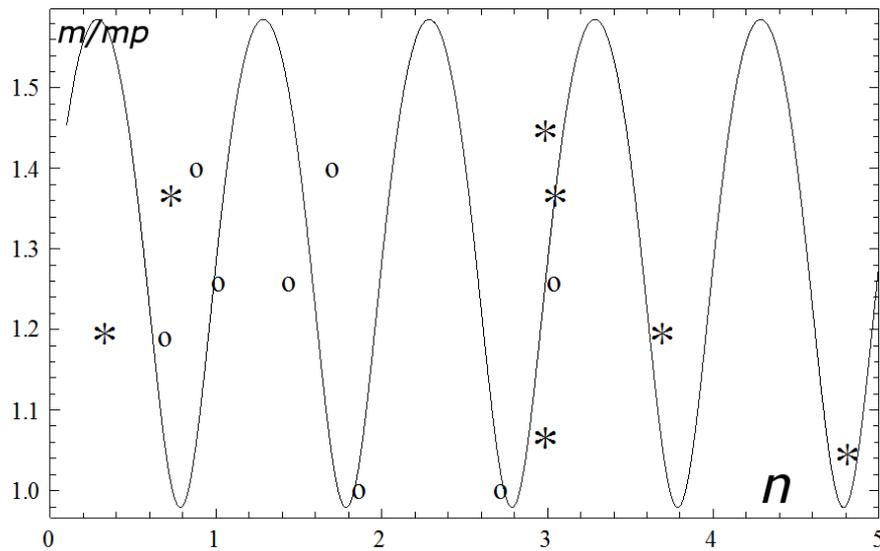


Figure 2: Comparison of baryons masses calculated from eq. (6) (line) as a function of confined flux  $n$ , with data points from Tables 1 and 2 of ref.[2] for octet (open circles) and decuplet particles ( $m_t$  used[2], stars). Nucleons are on the basis of the figure. The points come from the heuristic/phenomenological equation  $n = (2 c^2 \alpha / e^3) \mu m$ . The fit produces  $U_0 = 3710 \text{ MeV}$  as the vacuum parent level.

Figure 3 shows the energy flux profile of protons as detected from interstellar outer space by a space probe[6]. It is clear that a large proportion of protons has kinetic energies below the peak energy. To quantify this point the data are replotted in Figure 4 in a different way, to highlight the actual number density of protons in each energy interval. In Figure 4 the area below the curve is proportional to the number flux of protons detected up to a certain value of logarithmic kinetic energy, in units of number of protons per sq. meter per st. radian and per second. Beyond 90% of detected CR protons are below the 3 GeV kinetic

energy threshold. As shown in the figure the greatest density of protons is observed at 0.6 GeV kinetic energy. This is a very relevant result as discussed now.

Taking ahead our picture of the proton as a harmonic oscillator we should inquire on why statistically the equilibrium collection of oscillators/protons should concentrate at 0.6 GeV kinetic energies, which is substantially below the 3 GeV that would represent a threshold of stability for a single proton. The answer probably is that the proton structure settles at lower energy to reach a relaxed elastic regime of vibrations. Very high energy protons would profusely radiate ( as observed for instance in synchrotron accelerators) until reaching a stable state in the elastic regime. In fact, one may test this explanation by calculating the amplitude of vibrations of the structure and comparing this with the size of a proton. Such amplitude  $\delta$  is obtained by equating a representative elastic energy of a three-mass oscillating system to the measured 0.6 GeV ( from the virial theorem such system-averaged potential energy should match the averaged kinetic energy) :  $(3/2) (m/3) \omega^2 \delta^2 = 0.6 \text{ GeV}$ , with  $\omega$  the natural frequency calculated earlier, and  $m$  the proton mass. One obtains  $\delta = 0.11 \text{ fm}$ , which might be smaller due to relativistic corrections to mass.

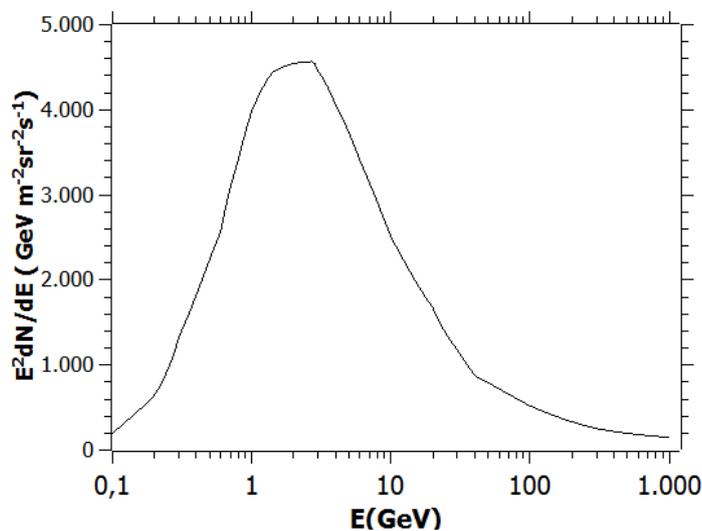


Figure 3: Interstellar energy-flux profile of protons in CR, which peak at 2.7 GeV kinetic energy[6].

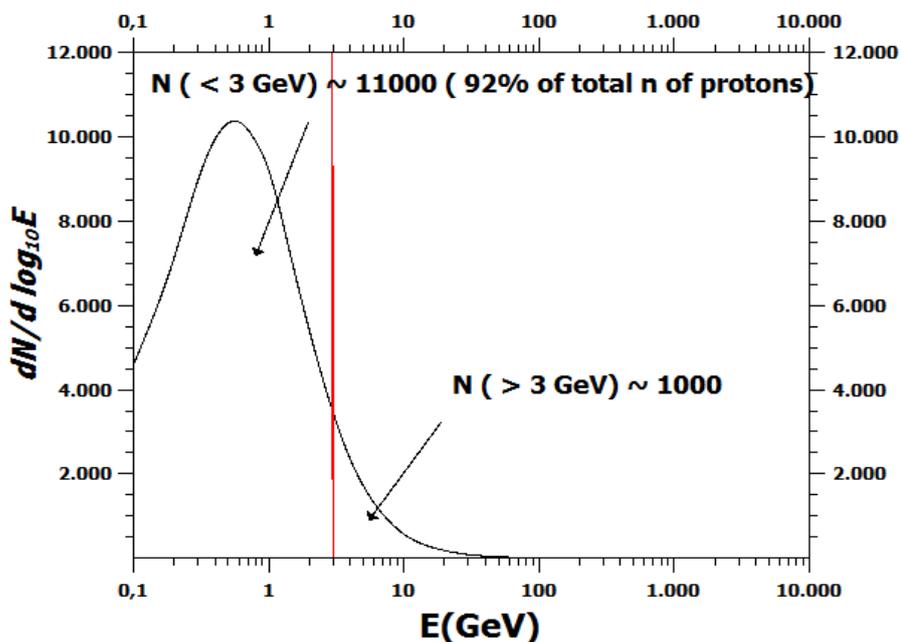


Figure 4: The previous figure is replotted to clearly show the concentration of protons below 3 GeV kinetic energies. The density peaks at 0.6 GeV, with protons reaching a relaxed configuration after radiating their excess energy.

The diameter of a proton determined by scattering experiments is  $\approx 1.8$  fm. This should be taken as the greatest possible spacing between constituents in a “relaxed” proton structure. Therefore, the vibration amplitude  $\delta$  is 6.3 % of such spacing. Criteria have been developed to evaluate whether the deformation of interatomic spacing in a substance might provoke a change of state. The range of deformation between 5 and 10% of the inter-constituent spacing ( in the present case) is usually recognized as within a typical limit for a structure to remain stable. This indicates that the release of excessive energy ( rather than decay or a change of state back into vacuum) leads most of the population of CR protons to an energy range within their elastic regime of vibrations, as compared to the threshold of about 3 GeV which theoretically indicates complete loss of stability. This is consistent with the shape and peak positions of the observed flux profiles. The process of energy release is through EM radiation.

## CONCLUSIONS

This work is a development upon our previous publications [1,2]. In these previous papers the concept that the mass of baryons comes from the relation between magnetic moment and confined flux in a multiconnected current distribution was described, initially in a heuristic/phenomenological way based upon experimental data for baryons. This produced a very useful relation in which the confined number of flux quanta, mass, and moment were gattered in a single expression. In ref [2] a solution for the Dirac equations was developed in the form of a loop of fields, associated with the confinement of flux. The purpose was to compare the masses obtained from the two methods as a function of confined flux. Based upon previous experience with Condensed Matter closed current systems, one expected a periodic behavior of mass as a function of flux, which was actually obtained. The field-theoretical treatment produced the additional information that for the whole treatment to be internally consistent a “parent” state for baryons should exist at 3710 MeV , which we associated with the EM vacuum. What we have done in the present paper has been to analyze data for protons in Cosmic Rays in search for evidence for such latter prediction of the model. Even without detailed analysis, energy- and number profiles of protons in Cosmic Rays are characterized by a steep drop beyond approximately 3 to 4 GeV kinetic energies. We considered some of the possible consequences of a parent state existing at 3.7 Gev. One of them would be the prediction of some sort of break-up or decay taking place at energies in the range 2- 4 GeV. In fact all baryons with exception of the proton ( and the neutron also) have mean lives in the range  $10^{-12}$  to  $10^{-24}$  sec. Statistically speaking, as the rest masses of baryons and mesons increase towards 3 GeV, the shorter the mean lives are. That is, for these hadrons their internal structure is not as resilient as the proton’s and they decay as the “parent” state energy is approached. In view of the fact that protons do not decay, these stable particles must display another mechanism to release energy. This is actually the principle of operation of proton-synchrotron accelerators. Protons are actually accelerated to the range between 3 to 6 GeV and the resulting radiation is used in several applications. Coming back to the analysis of Cosmic Rays, we considered in detail a plot of the energy flux profile of interstellar protons available in ref. [6]( Figure 3). By dividing the flux data by the corresponding values of kinetic energy one obtains a revealing plot of the distribution in energy of the number density of protons, which peaks at 0.6 GeV( Figure 4). Beyond 90% of the protons release energy to settle below 3 GeV. In view of the composite structure of the proton, a range of elastic behavior is expected. The accumulation of particles below 3 GeV and peaking at 0.6 Gev indicates that this would represent the achievement of some stable elastic state. We finished the paper by showing that at 0.6 GeV the range of internal vibrations inside the proton structure should just return to the elastic regime.

In the Introduction we tried to make a distinction between the kind of particle which this model might be applicable to, and the actual protons. Jehle[5] associated each kind of quark to a topologically different “loopform” as he called it. We here considered only circular loops of current. One might wonder that such loops would evolve into more complex entangled structures, which would provide the observed strength of

protons. Another point that deserves consideration is how the present treatment might be related to more sophisticated field theories proposed in the 1980s and 90s(for instance, [7-8]). At that time there were consistent attempts to unify the treatment of quarks and leptons. A quark mass formula related to the one proposed by Barut[3] for leptons was suggested[7]. Both these formulas contain a quantum number  $n$ , and in the case of Barut's, equivalent to the  $n$  in this present treatment. It must be pointed out that in [7]  $n$  is associated with the number of self-interacting bubble diagrams in a perturbation expansion. Of course as  $n$  drops towards 1 one gets into the static regime, with no corrections due to dynamic correlation effects. This indicates the possible reason why a treatment as simple as the present one has been able to describe mass with a considerable precision. Mass appears as an essentially "effective" or "mean" property in structural terms, or in other words, a property that depends very little upon detailed consideration of correlation effects between individual constituents( and for that matter, the regularization process utilized in this treatment in fact leaves aside any details of interactions, concentrating essentially on the elimination of the diverging parts of an expansion).

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