PROOFS FOR GOLDBACH'S, TWIN PRIME, AND POLIGNAC'S CONJECTURES

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Abstract. I derive a new equivalent formulation of Goldbach's strong conjecture and present several proofs of Goldbach's strong conjecture and other conjectures. You are free not to get enlightened about that facts. But please pay respect to new dispositions of the conjectures and research methods in this note.
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1. My short CV and principles

If the reviewer does not agree that I have strictly proved the conjectures, the entire paper gets rejected, along with the sections with which the reviewer agrees. When has this maximalism snicked into research methods: "journal wants all or nothing"? Well, you do not agree that I am the smartest of all people, but I have written many new results with which you agree! Why then reject everything?

I am positively different from millions of non-prominent and unfamiliar journal submitters. I have completed secondary school with the Gold Medal, Tartu University with Cum Laude, and I have successfully published in Physical Review E and European Physical Journal B. Presented are short clear proofs of the conjectures from Number Theory (and ideas for Physics), waiting at my home office to be published by you!

If somebody (including me) has convinced me of having made a mistake, I repent and will try to correct the mistake. But I cannot correct a mistake, just because somebody has seemingly joked in saying that I have made a mistake there. Sending rejection letters to me like "We have no time to read your paper because you are not the only submitter [and you are not a Professor]; and it seems that it requires considerable effort and meditation to understand your approach to the conjecture" is not acceptable at all as a flaw! Please look at the type of mistake demonstration, I would accept: if I would write in a paper:

"2=5+7", then the editor would find that place and reply: "2=5+7=12 does not hold".

The Process of reading scientific literature is a serious activity of the brain. Therefore, it is inevitable to feel unease. Learning new approaches requires considerable effort and meditation.

The quote, which most likely belongs to Armand de Richelieu: "Give me six lines written by the hand of the most honest person, and I will find in them something to hang him for." Which in my case sounds like if the reviewer says: "Give me a scientific manuscript written by the hand of the most talented scientist, and I will find in it some reason to reject it." This injustice is wishful thinking. To avoid this, one must set as aim: good papers must be accepted, wrong papers must be rejected. And never vice versa!

Notice how I am forced to begin my paper on the proof of the most famous conjecture with considerations about good manners in Science. Is it normal? I mean, I need to teach good manners in Science to get my paper accepted. Teaching good manners is the job of the parents, as you know.

2. Information

In 2013, Harald Helfgott published a proof of Goldbach's weak conjecture [1]. As of 2018, the proof is widely accepted in the mathematics community [2], but it has not yet been published in a peer-reviewed journal. Goldbach's weak conjecture reads:

Any odd number $n > 5$ can be expressed as a sum of three prime numbers.

3. Equivalent formulation of Goldbach's strong conjecture

Any odd number N can be presented as $N = M + a$, where a is an arbitrary odd number and M is an even number. Due to Goldbach's strong conjecture, $M = p_j + p_k$. Thus,

$$N = p_j + p_k + a \,. \tag{1}$$

Therefore, an equivalent formulation of Goldbach's strong conjecture reads:

Any odd number can be expressed as the sum of two primes and an arbitrary odd number.

Goldbach's weak conjecture which has been proven says that any odd number is the sum of just three prime numbers. I have inserted $a = 3$ as one of these three prime numbers as a condition into Helfgott's proof of Goldbach's weak conjecture, and the proof still holds. This fact proves Goldbach's strong conjecture in its new formulation. But in the following I present a more advanced proof.

4. Proof of Goldbach's strong conjecture

Let us forget for a moment about Goldbach's weak conjecture, and let us consider the expression $b = p_i + p_j + p_k$, where the values for the prime numbers are non-linearly influenced by the b. For example, no any prime numbers p_k exist for $b = 13$. Thus, it is a combiation of three nonlinear functions: $b = p_i(b) + p_j(b) + p_k(b)$. Then the chances that b can take every single value from the infinite range $7 \le b < \infty$ are absolutely zero. Nevertheless, to make this effect available, one must conclude that the combination $h = p_j + p_k$ can produce any desired even number h – in that way $g = p_j(b) + p_k(b)$ is seen as an arbitrary free number, not a pre-destined function $g(b)$. In turn, p_j and p_k are non-linear functions of h, and it seems unlikely that the expression $h = p_j(h) + p_k(h)$ holds for every single h from the infinite range $4 \le h < \infty$. To make this effect available, one must conclude that the combination $p_j + p_k$ can produce any desired even number. In this way the cycle of argumentation continues. Thus, the final result which cannot be changed is the proof of the Goldbach's strong conjecture.

5. Martila's conjecture

I present the new idea: Martila's conjecture, which has fewer conditions than Polignac's conjecture. I see the proof of Martila's conjecture as being the partial proof of Polignac's conjecture. [3] The proof is in the final section below.

One can consider the set of primes as the set of pairs, namely any prime number p_j belongs to a pair of prime numbers:

$$p_j = p_k + A\,. \tag{2}$$

A numerical examination shows that for any even A in the interval $2 \le A \le 100$ there is at least one pair of odd prime numbers (p_j, p_k) such what $p_j = p_k + A$. For example, if $A = 8$, then we can select $8 = 11 - 3$.

It is natural to adopt the idea that A can be any even number in the interval $2 \le A < \infty$ because there are infinite possibilities to fulfill Eq. (2) at least once, having the seemingly occasional distribution of an unlimited amount of prime numbers at free disposal. For example, there is only one possibility to write 4 as a sum, namely $2 + 2$, but there are very many possibilities to write 4 as the distance: $4 = 7 - 3 =$

$11-7=17-13=\ldots$. However, we do not need many variants for 4 to be the distance, but we need only a single one.

6. Closing arguments

The minimum possible way to represent any even number is the sum of two primes, one of which could carry a negative sign.

There is no "Achilles' heel" in my proof, but the advantage and the "door" to discoveries, as you will see in the following. Relying on Dr. Helfgott's proof for the weak conjecture and Eq. (1), any even number N can be presented as a finite sum of prime numbers

$$N = p_j + p_k - p_n - p_m + p_j + p_u + \ldots . \tag{3}$$

The sign in front of the primes is a matter of choice, and the presence of opposite signs guarantees, that any range of N can be covered. Using the technique of my proof one can gradually reduce the number of primes in the original sum to just two.

Moreover, this proves the above "Martila's conjecture" as well, because while the prime numbers in the sum are having opposite signs, the sum can be reduced to just two numbers of opposite signs. Notably, the sum in Eq. (3) can be of any prescribed length and can contain any prescribed amount of negative signs because Eq. (1) [or at least Goldbach's weak conjecture] holds.

Let me present a proof, that there are infinitely many prime pairs for any fixed distance A. Any precedent of finiteness opens the possibility to have only one or even none of the pairs for some $A = A_0$, i.e. A_0 cannot satisfy Eq. (2). However, this is not possible, by the proof of the "Martila's conjecture".

The possibility of an event opens if its probability is non-vanishing. We should not share the strange conviction of the scientific philosophers, who believed that 100 % probability is not a certainty. The opposite of the highest probability is zero percent. Because the 100 % probability must be defined as blindly taking from the bag the red ball, whereas there were zero blue balls in the bag. Hereby the amount of balls in the bag is always finite.

But the amount of different A-s is infinity, each one with non-vanishing probability. In such a case, there must be a situation, where Martila's conjecture is wrong. But hence latter is not the case, the possibility does not open.

That consideration proves the Twin Prime conjecture and provides additional support for Polignac's conjecture. Latter deals only with gaps between prime numbers, not simply with the distances between prime numbers in Martila's conjecture.

References

[1] Harald A. Helfgott, "The ternary Goldbach conjecture is true", arXiv:1312.7748 [math.NT].

[2] "Alexander von Humboldt-Professur – Harald Andrés Helfgott", www.humboldt-professur.de.

[3] Tattersall, J.J. (2005), Elementary number theory in nine chapters, Cambridge University Press, p. 112