Could a new equivalent formulation of Goldbach's conjecture lead us to the final proof?

Dmitri Martila

Tartu University, 4 Tähe Street, 51010 Tartu, Estonia[*]

(Dated: July 15, 2020)

Abstract

This paper is an important step towards the final proof, namely my attempt to demonstrate the validity of the Goldbach's strong conjecture. Using the proof of Goldbach's weak conjecture, on a single page I derive the new simple equivalent formulation of Goldbach's strong conjecture. In addition, I show that the proof of Goldbach's weak conjecture satisfies the equivalent formulation.

Keywords: Goldbach's conjecture; Goldbach's weak conjecture; number theory; primes.

[*]Electronic address: `eestidima@gmail.com`

I. AND BECAUSE MISTAKES AND FAKES SHALL ABOUND, THE WAY OF TRUTH WILL BE EVIL SPOKEN OF

This section can be removed from the paper on request of the referee. It is not meant as a proposal to modify the peer-review process, but as an argument for the referee to use goodwill.

The goal "to find mistakes" could be a bad attitude. The final goal should be to enjoy reading the publication. If flaws are seen, they must be reported. However, this report should be given without any laughs and sadistic enjoyment. Instead, the flaws should be reported with some sadness.

The psychologists have conducted a social experiment: they told the probants that the man on the photo is a serial killer. The probants testified that he is looking like one. The next day they told another group of probants that the man on the same photo is an American national hero; these probants have confirmed his heroic look.

In conclusion, having the "mistakes desire" as your default position while reading the manuscript of an unknown author increases the chances for the paper to be unjustly rejected. The scientific skepticism should be the readiness to deal with mistakes, but not the expectation – by desire – to find them.

Why do I ask as an author for detailed reports from the referee system? The referee must convince me that I have done mistakes. Otherwise, I would not accept them. Yes, it seems like living in an "utopian" perfect world. But I cannot repent a hypothetical mistake. I can only repent if the mistake is demonstrated to me and I am convinced that it is not the usual fake-news, trolling or bullying. This research principle is my personal "guiding star" during my quest for the objective truth. As an example, the absolute majority of scientists have accepted the proof for Goldbach's weak conjecture, but not all of the scientists have accepted it yet, mainly because it is not published in a journal. [3] Therefore, one needs to have personal convictions and opinions to move forward. [4]

To navigate in Science, you need to have a personal point of view and convictions you should not rush to abandon. Otherwise, you will soon be disoriented. Only then you will realize the objective truth. That is the subjective search for the objective truth because you are choosing what is right and what is not.

II. INTRODUCTION

Top journals usually receive a large number of submissions concerning famous open problems like the Riemann hypothesis, the Goldbach conjecture, Fermat's last theorem, or Collatz' problem or the twin prime conjecture. Extraordinary claims require extraordinary evidence, especially in view of the very many failed attempts to prove these types of problems (or disprove these theorems). Therefore, please consider this paper as an important step towards the final proof, namely my attempt to demonstrate the validity of the strong Goldbach's conjecture.

III. EQUIVALENT FORMULATION OF GOLDBACH'S STRONG CONJECTURE

In 2013, Harald Helfgott published a proof of Goldbach's weak conjecture [1]. As of 2018, the proof is widely accepted in the mathematics community [2], but it has not yet been published in a peer-reviewed journal. Goldbach's weak conjecture reads:

Any odd number $n > 5$ can be expressed as a sum of three prime numbers.

Let us take an arbitrary even number $N > 2$ and add to it an arbitrary prime number $p > 2$. Then $M = N + p$ is an odd number. Therefore, due to Goldbach's weak conjecture, one has

$$M = p_i + p_j + p_k \,. \tag{1}$$

Now, for

$$p = p_i \,, \tag{2}$$

one has $N = p_j + p_k$ which proves Goldbach's strong conjecture. Therefore, one can reformulate Goldbach's strong conjecture:

Any odd number can be expressed as a sum of three prime numbers,
one of which is odd and arbitrary.

In particular, Goldbach's strong conjecture is true if an arbitrary odd number n can be expressed as a sum of three prime numbers, one of which is three (in this case one has $p_i = 3$).

Goldbach's strong conjecture is false if one cannot express this n as a sum of two prime numbers and the number three. Indeed, because n is odd, $n - 3$ is even. Therefore, if Goldbach's strong conjecture is true, n is expressible as the sum of three prime numbers.

IV. THE SHORT PROOF

Dr. Helfgott's result was published without any conditions; namely, the result reads simply: "any odd number can be expressed as a sum of three prime numbers". The result is not, e.g. "any odd number can be expressed as a sum of three prime numbers, but not if one of these numbers is three". Therefore, in 2013 my new formulation was proven.

V. THE LONG PROOF

Consider the following **Lemma**: any odd number n_1 is expressible as

$$n_1 = p_v + p_u + p_t - a + b\,, \tag{3}$$

where a, b are arbitrary prime numbers, and p_v, p_u and p_t are prime numbers.

Proof: it holds that

$$n_2 = n_3 - a\,, \tag{4}$$

where n_2 is an arbitrary even number, a is an arbitrary odd prime number, and n_3 is the resulting odd number (this because $n_2 + a$ must be odd).

And it holds that

$$n_4 = n_5 + b\,, \tag{5}$$

where n_4 is an arbitrary odd number, b is an arbitrary odd prime number, and n_5 is the resulting even number (because $n_4 - b$ is always even). The Lemma is proven because n_3 is expressible as sum of three prime numbers (p_v+p_u+p_t), due to Helfgott's proof of Goldbach's weak conjecture.

However, as a is arbitrary, it could be chosen to be $a = p_t$. In this case, an arbitrary odd number is expressible as a sum of three prime numbers, one of which is the arbitrary number b.

Therefore, any number can become a function (i.e. a plain sum) of the position in the multi-dimensional space, where coordinates are prime numbers. Please note that due to my equivalent formulation, the dimension of space for even numbers and the dimension of space for odd numbers coincide and is the minimum possible, namely two. That makes my theory esthetical.

Let us suppose now for a moment that Goldbach's strong conjecture is false. Because we can suppress the dimension to minimally two [5] by the free choice of a

$$min := \min_a |p_t - a| \,, \tag{6}$$

there are only two independent coordinates, p_v and p_u, pointing to two-dimensionality and, thus, to the validity of Goldbach's strong conjecture. Therefore, any number (n_2 if even, and n_4 if odd; see above) can be represented as a function of just two coordinates

$$n_2 = p_v + p_u \,, \quad n_4 = p_v + p_u + 3 \,. \tag{7}$$

[1] Harald A. Helfgott, "The ternary Goldbach conjecture is true", arXiv:1312.7748 [math.NT].

[2] "Alexander von Humboldt-Professur – Harald Andrés Helfgott", www.humboldt-professur.de.

[3] Harald A. Helfgott, "The ternary Goldbach conjecture is true", arXiv:1312.7748 [math.NT].

[4] Massimiliano Proietti *et. al.*, Experimental test of local observer-independence, Science Advances 5(9), eaaw9832 (2019), arXiv:1902.05080 [quant-ph]; Ian T. Durham, Observer-independence in the presence of a horizon, arXiv:1902.09028 [quant-ph]

[5] The role of p_t in $n_1 = p_v + p_u \pm min + b$ with the smallest min is suppressed.