A Proof of Goldbach Conjecture

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(July, 2020)

Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the Goldbach Conjecture.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

We use a modified sieve of Eratosthenes similarly to the method in my *paper*[1].

Let $p_m\sharp = \prod_{i=1...m} p_i$,

From the paper, we have the following, when sieve upto p_m, the total number of the remaining numbers { R_j^m}, inside of (0,$p_m\sharp$) is $\prod_{i=1...m}(p_i - 1)$,

These remaining numbers can be paired up as (x, $p_m\sharp$ -x), here x is a remaining number.

So there are total $\prod_{i=1...m}(p_i - 1)$ /2 pairs of the remaining number pairs.

In general not all the remaining number pairs are primes. We need to sieve more larger primes to get all primes.

Let p_M be the least prime satisfied the $p_m\sharp < p_M^2$, then we sieve upto p_M for the period (0, $p_m\sharp$), then all those still remaining numbers are primes and remaining number pairs are all primes.

From the paper we have the following,

Theorum 1;

For any number d with (d,$p_m\sharp$) =1, no common factor with $p_m\sharp$, when sieve upto p_m, the total number of the remaining numbers inside period (0, $p_m\sharp/d$) is equal approximately to $\prod_{i=1...m}(p_i - 1)$ / d ± 1,

When sieve upto p_M, the total number of the remaining numbers inside period (0, $p_m\sharp$) are those remaining numbers when sieve upto p_{M-1} in the same period (0, $p_m\sharp$) subtract those remaining numbers when sieve upto p_{M-1} in the period (0, $p_m\sharp/p_M$) multiplied by p_M.

We use $\{(a,b)\}^M$ to denote those remaining numbers in period (a, b) when sieve upto p_M. We have,

$$\{(0, p_m\sharp)\}^M = \{(0, p_m\sharp)\}^{M-1} - \{\{(0, p_m\sharp/p_M)\}^{M-1} \times p_M\}, \tag{1}$$

and so on, we have,

$$\{(0, p_m\sharp)\}^{M-1} = \{(0, p_m\sharp)\}^{M-2} - \{\{(0, p_m\sharp/p_{M-1})\}^{M-2} \times p_{M-1}\}, \tag{2}$$

and

$$\{(0, p_m\sharp/p_M)\}^{M-1} = \{(0, p_m\sharp/p_M)\}^{M-2} - \{\{(0, p_m\sharp/p_M p_{M-1})\}^{M-2} \times p_{M-1}\}, \tag{3}$$

and so on and on, we will have,

$$\{(0, p_m\sharp)\}^M = \sum_{d|P} \mu(d)\{\{(0, p_m\sharp/d)\}^m \times d\}, \tag{4}$$

here P $=\prod_{i=m+1...M} p_i$.

There are no remaining number in period (0, $p_m\sharp/d$) when $p_m\sharp/d < 1$, and only one remaining number, 1, when $1 < p_m\sharp/d < p_m$,

We have,

$$|\{(0, p_m\sharp)\}^M| = \sum_{d|P} \mu(d)|\{(0, p_m\sharp)\}^m|/d \pm ER_m \tag{5}$$

we have,

$$|\{(0, p_m\sharp)\}^M| = [\prod_{i=1...m}(p_i - 1)] \times [\prod_{i=m+1,...M}(1 - 1/p_i)] \pm ER_m \tag{6}$$

here, the ER_m is the possible error,

ER_m= $|\{d; d \mid P, p_m < d < p_{m-1}\sharp\}|$,

$$ER_m = |\{(0, p_{m-1}\sharp)\}^m| - |\{(0, p_{m-1}\sharp)\}^M| \tag{7}$$

$$ER_m = \prod_{i=1...m}(p_i-1)/p_m - [\prod_{i=1...m-1}(p_i-1)] \times [\prod_{i=m,...M}(1-1/p_i)] + ER_{m-1} \tag{8}$$

We have,

$$ER_m = \sum_{l=1,m}[\prod_{i=1...l}(p_i - 1)/p_l] \times [1 - \prod_{i=l+1,..M}(1 - 1/p_i)], \tag{9}$$

Then we have,

Theorum 2;

when sieve upto p_M for the (0, $p_m\sharp$), the total number of the remaining primes inside (0, $p_m\sharp$)is equal approximately to $\prod_{i=1...m}(p_i-1)\prod_{j=m+1...M}(1-1/p_j)\ \pm ER_m$,

here ER_m as above.

Similarly process is used for the remaining number pairs we have,

here we modify the Eratosthenes sieve as we sieve all the primes, p, of $\{p_{m+1}$, ... $p_M\}$, for each pair, (x, $p_m\sharp$ -x),we check both x=0, or x= $p_m\sharp$, mod p.

Theorum 3;

when using this modified sieve upto p_M for all the former remaining number pairs in the (0, $p_m\sharp$), the new total number of the remaining number pairs inside (0, $p_m\sharp$)is equal approximately to $\prod_{i=2...m}(p_i-1)/2\prod_{j=m+1...M}(1-2/p_j)\ \pm ER_m$,

and here ER_m is the same as above.

Now we will have at least two remaining number pairs of (x, $p_m\sharp$ -x), and there is at least one prime pair, (p, p'), with p + p' = $p_m\sharp$

In general, a large enough even number, N can be,

N = $\prod_{i=1,..m} p_{l_i}^{j_i}$,

here $j_i \geq 1$, with $l_1 = 1$,

Let set P_1 be $\{p_{l_i}\}$, set P_2 be $\{p; p < p_{l_m}, p \notin P_1\}$,

As before,

Let p_M be the least prime satisfied the $N < p_M^2$, first we use Eratosthenes sieve to sieve all the p in set P_1, for the period (0, N), the total number of the remaining numbers is equal to $\prod_{i=1,..m}(p_{l_i}-1) \times N_0$, here, N_0 is, $\prod_{i=1,..m} p_{l_i}^{(j_i-1)}$

They are in pairs as (x, N-x),

Let set $P_3 = P_2 \cup \{p_{l_m+1}, ..., p_M\}$,

Using modified Eratosthenes sieve as above by checking both x=0, or x=N mod p, for all $p \in P_3$,

sieve all the p of P_3, we will have the total number of the remaining number pairs is equal to,

$\prod_{i=1,..m}(p_{l_i} - 1) \times N_0/2 \times \prod_{p \in P_3}(1 - 2/p) \pm ER$,

Using the same procedure we get the ER as following,

$$\sum_{k=1,m} \sum_{n=1,..j_k} \prod_{i=1,..k} p_{l_i}^{(j_i-1)} \times [\prod_{i=1...k} (p_{l_i} - 1)/p_{l_k}^n] \times [1 - \prod_{i=k+1,..m} (1-1/p_{l_i}) \times \prod_{p \in P_3} (1-1/p)], \quad (10)$$

It is obvious that for a large enough even N, there are at least two remaining prime pairs of which (1, N-1) might be one of them.

So there are at least one prime pair (p, p') as a remaining pair, and p+p' = N.

This proves the Goldbach Conjecture.

Reference:

1. Vixra; Xuan Zhong Ni, "Prime and Twin Prime Theory", 2020