A solution to the black hole information paradox

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This paper shows that the Schwarzschild metric's prediction of black holes violates the equivalence principle. Rindler horizons, which are used to support the idea of black holes, are shown to work differently according to special relativity than is generally accepted today. New equations are derived for escape velocity and gravitational time dilation. A problem is shown in the derivation of the Schwarzschild metric, and solved to derive a new metric for Schwarzschild geometry. The new metric doesn't predict black holes, yet is experimentally confirmed, including by the famous picture that's reportedly of a black hole.

1 The Schwarzschild metric violates the equivalence principle

A formal statement of the Einstein equivalence principle (EP) is

> In any and every locally Lorentz (inertial) frame, the laws of special relativity must hold.

Experiment #1: A free-falling ball is just below the event horizon of a supermassive static black hole, and directly below a rocket that hovers just above the event horizon. The ball is at rest in a local inertial frame (LIF) that contains the rocket.

The EP tells us that only special relativity (SR) is needed to calculate the ball's motion relative to the rocket. See the equations of SR at The Relativistic Rocket, for a rocket having a constant proper acceleration $a > 0$. They work in reverse as well, like for a rocket decelerating to dock at a spaceport. The ball can reach the rocket in principle, in the time given by

$$t = \sqrt{(d/c)^2 + 2d/a} \tag{1}$$

This equation gives the time taken for a decelerating rocket to arrive at a free-falling destination (like a spaceport or the ball) at relative rest. Both t and the initial distance d between them are measured in the destination's LIF that contains the rocket. The speed of light is c. Their initial velocity toward each other in the destination's LIF is found by plugging that value for t into

$$v = \frac{at}{\sqrt{1 + (at/c)^2}} \tag{2}$$

We can let the initial distance d between the ball and the rocket be arbitrarily small. The acceleration a that a rocket needs to hover ever more closely above an event horizon approaches infinity, hence (1) shows the ball can reach the rocket in an arbitrarily short time t in principle. But the Schwarzschild metric predicts that the ball can't pass outward through the event horizon to reach the rocket at all. The metric violates the EP by disagreeing with SR in a LIF.

The laws of physics are the same in all inertial frames, according to SR. Therefore, an argument that the ball can't reach the rocket must find the same in an inertial frame in an idealized, gravity-free universe, while letting the ball approach the rocket at any speed $< c$ initially, or else it violates the EP. But (1) shows that's impossible. The case in which the ball reaches the rocket at relative rest is the reverse of the case (when we "run the film backward") in which the ball had simply dropped from the rocket. The Relativistic Rocket equations apply to both cases.

The rocket's Rindler horizon, a plane from below which nothing can reach the rocket, including light, is at the distance c^2/a below the rocket (see the section "Below the rocket, something strange is happening" at The Relativistic Rocket). As a rocket hovers ever more closely above an event horizon, c^2/a approaches zero. This prediction is used to support the idea of black holes. The thinking is that, in the limit where a rocket hovers at the event horizon, its Rindler horizon would be there too, so that the EP is obeyed in a LIF that straddles an event horizon.

But (1) shows that the ball can reach the rocket in principle, and the EP requires that this is correct. Let's understand this better. See the "general formula" at The Relativistic Rocket:

$$T = \frac{2c}{a} \operatorname{acosh}\left(\frac{ad}{2c^2} + 1\right) \tag{3}$$

This equation gives the aging of a rocket's crew during a trip that covers a distance d as measured in the LIF in which they started, at an acceleration a that's negated at the midpoint so that the rocket brakes to arrive at its destination at relative rest. For example, it predicts that a rocket can launch from Earth and accelerate and decelerate at 1 g to arrive at the Andromeda galaxy, $d = 2$ million light years away from Earth as we measure, while its crew ages just $T \approx 28$ years (use $c = 1$ ly/yr and $a = 1.03$ ly/yr$^2 \approx g$). Notice that Andromeda reaches the rocket that's decelerating toward it, starting from the midpoint that's 1 million light years away as measured in Andromeda's frame, even though the rocket's Rindler horizon is $c^2/a \approx 0.97$ light years below the rocket. The equation from the site for the acceleration or deceleration half of the rocket's trip is

$$T = \frac{c}{a} \operatorname{acosh}(ad/c^2 + 1) \tag{4}$$

Compare (4) to (3). See that (3) just doubles the time returned by (4) for either half of the trip. For the deceleration half of the trip to Andromeda, (4) returns $T \approx 14$ years. Or use (1) for the time t taken in the destination's LIF. Both (1) and (4) show that a free-falling object (like Andromeda or the ball) can in principle reach a rocket having any acceleration a, starting from any distance d below the rocket.

The distance c^2/a between a rocket and its Rindler horizon applies only in the rocket's frame. When the ball approaches the rocket fast enough initially, then the initial distance d is length contracted in the rocket's frame to $d/\gamma < c^2/a$ (where γ is the gamma factor), so that the ball is above the rocket's Rindler horizon initially and thus able to reach the rocket. That's also how Andromeda reaches any rocket that decelerates to it. (Calculate d/γ for when the rocket launched from Earth starts braking toward Andromeda, to see that it's slightly $< c^2/a$.) In the current generally accepted view, the distance c^2/a would be applied in the ball's frame in error. Since the ball can reach the rocket in principle, where the escape velocity is $< c$ (and so escape is possible), the escape velocity at the ball's initial location must also be $< c$.

Experiment #2: Modify the barn-pole paradox experiment to replace the near and far doors of the barn with the rockets Achilles and Tortoise, respectively. In the barn's frame, when the switch is thrown, the rockets start their engines and accelerate in the direction that the runner moves. Tortoise has a constant proper acceleration high enough that Achilles starts behind its Rindler horizon and so can't reach it.

The runner passes Achilles before it blasts off, so might be able to reach Tortoise. Relativity of simultaneity helps explain how the runner or a free-falling object (like the ball in experiment #1) can in principle reach a rocket having any acceleration a, starting from any distance d below the rocket.

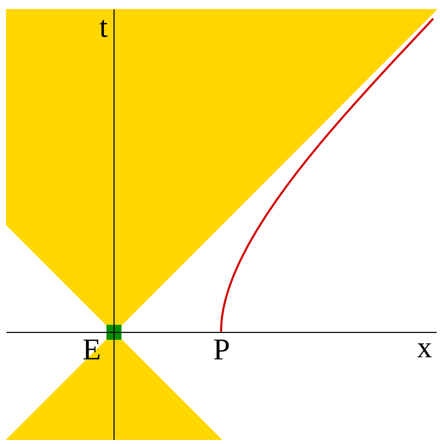

Figure 1: Spacetime diagram showing a uniformly accelerated particle, P, and an event E. The event's future light cone never intersects the particle's world line. By Christopher Thomas / tiZom, CC BY-SA 1.0, via Wikimedia Commons.

Let P in Fig. 1 be the rocket in experiment #1. Then the diagram is for a LIF that momentarily co-moves with the rocket at $t = 0$ (as if the rocket blasts off then). Let the proper distance EP be c^2/a, so that E occurs at the rocket's Rindler horizon. SR lets the ball be between E and P at $t = 0$ in this frame, above the rocket's Rindler horizon, like how the runner in experiment #2 is between Achilles and Tortoise in the barn's frame when the switch is thrown. This is true regardless of the initial distance d between the ball and the rocket, like how the runner could measure any initial distance to Tortoise. The ball can always fit between E and P in principle, due to length contraction, like how the pole fits in the barn in the barn's frame.

The Schwarzschild metric is invalidated by violating the EP, a postulate of general relativity (GR). Moreover, a theory of gravity that obeys the EP can't predict black holes. The EP requires that escape velocity is $< c$ everywhere.

2 A new equation for escape velocity

Equations for a falling body has the velocity of a free-falling object that was dropped in a uniform gravitational field (ignoring air resistance) as

$$v_{\text{old}} = at \tag{5}$$

where a is the acceleration of gravity and t is the elapsed time. We call this v_{old} since it can be $\geq c$.

The EP shows that The Relativistic Rocket's (2) supplants (5):

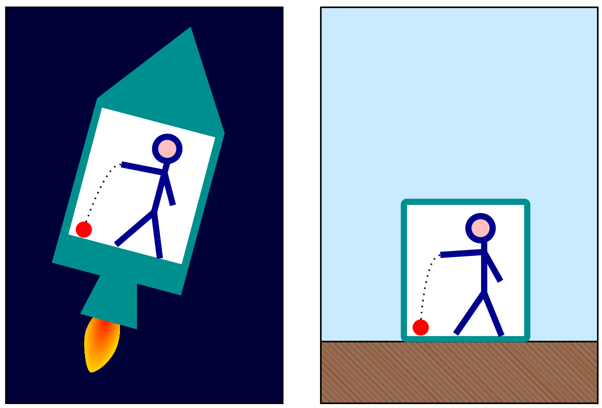

Figure 2: Ball falling to the floor in an accelerating rocket (left) and on Earth (right). By Pbroks13 / Markus Poessel (Mapos), CC BY-SA 3.0, via Wikimedia Commons.

According to the EP, SR's laws hold in both scenarios in Fig. 2. The Relativistic Rocket equations describe the ball's motion relative to the rocket. Then those equations describe the ball's motion relative to the room on Earth as well, where a is the acceleration of gravity. The time t in (2) is measured in the ball's LIF, which momentarily co-moves with the rocket or room when the ball is dropped at $t = 0$ (as if the rocket or room blasts off then).

Substituting the two terms at in (2) with the v_{old} from (5) gives the conversion formula

$$v = \frac{v_{\text{old}}}{\sqrt{1 + (v_{\text{old}}/c)^2}} \tag{6}$$

This is the same as dividing v_{old} by its gamma factor γ, as shown by

$$\gamma = \sqrt{1 + (at/c)^2} \tag{7}$$

from The Relativistic Rocket. That is, a Newtonian speed v_{old} converts to a relativistic speed v by dividing it by its gamma factor, using (7), which accepts a Newtonian speed.

The equation for escape velocity in both Newtonian mechanics and GR is

$$v_{\mathrm{e}} = \sqrt{\frac{2GM}{r}} \tag{8}$$

The new equation for escape velocity, derived by converting (8) using (6), is

$$v_{\mathrm{e}} = c\sqrt{\frac{2GM}{c^2 r + 2GM}} \tag{9}$$

This equation returns a value $< c$ for an r-coordinate (radial coordinate or reduced circumference) $r > 0$. So it doesn't predict black holes, or give reason to think a star or other massive body can collapse to a singularity. GR's escape velocity (8) better approximates the new escape velocity (9) as gravity weakens.

3 A new equation for gravitational time dilation

Imagine nested spherical shells concentric to a massive body. An observer drops from an infinite distance, falling freely toward the ground while measuring, as a fraction x of their own rate of time, the rate of clocks at each shell as they pass right by. Escape speed is also the speed of an object that was dropped from an infinite distance, so each shell passes at the escape velocity v_{e} there. SR tells us that inputting that velocity into the reciprocal of the gamma factor equation gets the value x for that shell. The ground accelerates like a rocket toward the observer, so the observer's rate of time remains the rate of time at an infinite distance, as if they're the stationary twin in a twin paradox experiment.* Then the gravitational time dilation factor, the time t_0 between two events as measured at an r-coordinate r, as a fraction of the time t_{f} between the events as measured at an infinite distance (i.e. by a faraway observer), is given by

$$\frac{t_0}{t_{\mathrm{f}}} = (\text{gamma factor for } v_{\mathrm{e}})^{-1} = \left(\frac{1}{\sqrt{1 - v_{\mathrm{e}}^2/c^2}}\right)^{-1} = \sqrt{1 - v_{\mathrm{e}}^2/c^2} \tag{10}$$

We verify this equation by deriving GR's equation for gravitational time dilation from it, using GR's escape velocity (8):

$$\frac{t_0}{t_{\mathrm{f}}} = \sqrt{1 - \left(\sqrt{\frac{2GM}{r}}\right)^2 / c^2} = \sqrt{1 - \frac{2GM}{c^2 r}} \tag{11}$$

The new equation for gravitational time dilation, derived using (10) and the new escape velocity (9), is

$$\frac{t_0}{t_{\mathrm{f}}} = c\sqrt{\frac{r}{c^2 r + 2GM}} \tag{12}$$

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* See Fig. 2. The rocket/room accelerates to reach the ball when it hits the floor. The ball is a stationary twin; the time t in The Relativistic Rocket equations is measured in the ball's LIF. The person is a traveling twin; their aging is given by T in those equations.

4 A new metric for Schwarzschild geometry

There's a problem in the derivation of the Schwarzschild metric, at the step

$$g_{44} = K\left(1 + \frac{1}{Sr}\right) \approx -c^2 + \frac{2GM}{r} = -c^2\left(1 - \frac{2GM}{c^2 r}\right) \tag{13}$$

in the section "Using the weak-field approximation to find K and S". GR's equation for gravitational time dilation (11) is in (13) as

$$\left(\frac{t_0}{t_\mathrm{f}}\right)^2 = 1 - \frac{2GM}{c^2 r} = 1 + \left(-\frac{GM}{r^2}\right)\frac{2r}{c^2} \tag{14}$$

This equation is invalid. Newton's term $-GM/r^2$ there and in the diagram in the derivation contains elapsed time values that are measured at that r-coordinate r, for which gravitational time dilation isn't accounted. For example, suppose $-GM/r^2 = -9.8\ \mathrm{m}/\mathbf{s^2}$. Nothing in the equation accounts for the fact that observers situated at varying r-coordinates measure different elapsed times between two ticks of a clock at r, making the seconds in bold ambiguous. Multiplying the term by $2r/c^2$ doesn't resolve the ambiguity. The gravitational time dilation factor that does that must be dimensionless and include the mass M; the M within the term can't be reused. Newton's concept of absolute time defaults for the term, so that the equation approximates the valid equation only in weak enough gravity, and (13) reads like

$$g_{44} \approx -c^2\,(\text{invalid gravitational time dilation factor})^2 \tag{15}$$

This dependence on Newton's absolute time is the root cause of the EP violation shown in section 1. To fix it, first we change (14) to

$$\left(\frac{t_0}{t_\mathrm{f}}\right)^2 = x = 1 - x\frac{2GM}{c^2 r} \tag{16}$$

and solve for x, the square of the gravitational time dilation factor. This standardizes the term $-GM/r^2$ to the faraway observer's measurement for it, by effectively dividing each time value in it by the gravitational time dilation factor $\sqrt{x}$. Alternatively, each instance of GR's escape velocity (8) that's embedded in (14) is converted to the new escape velocity (9), by effectively dividing it by its gamma factor; see (10). Now the ground's speed v relative to objects falling freely toward it approaches an asymptote of c, in the same way a rocket's does. This obeys the EP as depicted by Fig. 2. The square of (9) is used to build x. Solving for x gives

$$x = \frac{c^2 r}{c^2 r + 2GM} \tag{17}$$

See that the new equation for gravitational time dilation that derives from (17):

$$\frac{t_0}{t_\mathrm{f}} = \sqrt{x} = \sqrt{\frac{c^2 r}{c^2 r + 2GM}} = c\sqrt{\frac{r}{c^2 r + 2GM}} \tag{18}$$

matches (12), as expected. The new escape velocity (9) then derives by solving (10) for v_e.

Next we incorporate x into (13):

$$g_{44} = K\left(1 + \frac{1}{Sr}\right) \approx -c^2\left(1 - x\frac{2GM}{c^2 r}\right) \tag{19}$$

This effectively changes the term $-GM/r^2$ to $-xGM/r^2$. (Ultimately that's the only change made herein.)

Completing the derivation, it's found that

$$K = -c^2 \tag{20}$$

and

$$\frac{1}{S} = -x\frac{2GM}{c^2} \tag{21}$$

Hence

$$A(r) = \left(1 - x\frac{2GM}{c^2 r}\right)^{-1} = x^{-1} = \frac{c^2 r + 2GM}{c^2 r} \tag{22}$$

and

$$B(r) = -c^2\left(1 - x\frac{2GM}{c^2 r}\right) = -c^2 x = -c^2\frac{c^2 r}{c^2 r + 2GM} = -\frac{c^4 r}{c^2 r + 2GM} \tag{23}$$

So, the new metric for Schwarzschild geometry is

$$ds^2 = \frac{c^2 r + 2GM}{c^2 r}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - \frac{c^4 r}{c^2 r + 2GM}dt^2 \tag{24}$$

or, in geometric units:

$$ds^2 = \frac{r + 2M}{r}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - \frac{r}{r + 2M}dt^2 \tag{25}$$

5 Experimental confirmation of the new metric

The only change made to the Schwarzschild metric was to the term for escape velocity that's built into it. The old escape velocity (8) better approximates the new escape velocity (9) as gravity weakens. So the Schwarzschild metric better approximates the new metric (24) as gravity weakens.

For the Schwarzschild precession of Mercury, both metrics predict 42.9799″ per Julian century, in agreement with observations.

For the Schwarzschild precession in the orbit of the star S2 around Sgr A*, both metrics predict 12.1′ per orbit, in agreement with observations. If the Schwarzschild metric predicted 12.100′ per orbit for S2's Schwarzschild precession, then the new metric would predict 12.095′ per orbit.

More confirmation is in Why the JWST won't see the first galaxies. The numerical integration program therein shows that the Relativistic Rocket equations approximate the new metric locally, as required for the metric to obey the EP. This is indirect experimental confirmation since SR is well tested. Conversely, the Schwarzschild metric fails that test (see the section "More on the Schwarzschild metric's violation of the equivalence principle" therein).

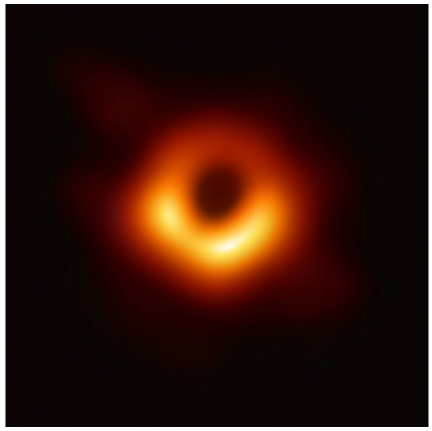

Figure 3: Direct image of a supermassive black hole(?) at the core of Messier 87. By the European Southern Observatory (ESO), CC BY 4.0, via Wikimedia Commons.

The new metric lets an object of any mass M have any radius (reduced circumference) $r > 0$. The new gravitational time dilation factor (12) goes to zero as r goes to zero. Gravitational redshift indicates gravitational time dilation, so a star can look black when viewed from afar. This explains the object in Fig. 3, without invoking a black hole. That is, the image is experimental confirmation of the new metric.
