A solution to the black hole information paradox

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I show that the Schwarzschild metric's prediction of black holes violates the equivalence principle. I show that Rindler horizons, which are used to support the idea of black holes, work differently according to special relativity than is generally accepted today. I derive new equations for escape velocity and gravitational time dilation. I show a problem in the derivation of the Schwarzschild metric, and solve it to derive a new metric for Schwarzschild geometry. The new metric doesn't predict black holes, yet is experimentally confirmed, including by the famous picture that's reportedly of a black hole.

1 The Schwarzschild metric violates the equivalence principle

A formal statement of the Einstein equivalence principle (EP) is

> In any and every locally Lorentz (inertial) frame, the laws of special relativity must hold.

Experiment #1: A free-falling ball is at rest in a local inertial frame (LIF) that straddles the event horizon of supermassive static black hole. The ball is just below the event horizon, directly below a rocket that hovers just above the event horizon.

See the equations of special relativity (SR) at The Relativistic Rocket, for a rocket having a constant proper acceleration $a > 0$. These equations show that the ball can reach the rocket in principle, in the elapsed time given by

$$t = \sqrt{(d/c)^2 + 2d/a} \tag{1}$$

This equation gives the time taken by a decelerating rocket to reach a free-falling destination (like a galaxy or the ball) at relative rest, as measured in the destination's frame (a LIF that contains the rocket), where d is the initial distance between them as measured in the destination's frame. The speed of light is c. Their initial velocity toward each other is given by

$$v = \frac{at}{\sqrt{1 + (at/c)^2}} \tag{2}$$

The initial distance d between the ball and the rocket can be arbitrarily small, hence they can reach each other in an arbitrarily small time t.

The Schwarzschild metric predicts that the ball can't pass outward through the event horizon to reach the rocket. Because the metric's prediction disagrees with SR's in a LIF, the metric violates the EP.

The EP is violated when the ball must fall further below the rocket. All inertial frames are equivalent in SR. In an inertial frame, the velocity v of any object (whether or not it's free-falling) can be any value in the interval $-c < v < c$ in principle. So the rocket and the ball can be moving toward each other initially. An argument that the ball can't reach the rocket must determine the same in an inertial frame in an idealized, gravity-free universe, or else it depends on an EP violation.

The rocket's Rindler horizon, a plane from below which signals can't reach the rocket, is at the distance c^2/a below the rocket. (See The Relativistic Rocket, section "Below the rocket, something strange is happening.") The acceleration a that a rocket needs to hover ever more closely above an event horizon of a black hole approaches infinity, so that c^2/a approaches zero. This prediction is used to support the idea of black holes. The thinking is that, in the hypothetical limit where a rocket hovers at the event horizon, its Rindler horizon would be right at the event horizon too, so that the EP is obeyed in a LIF that straddles an event horizon.

But SR predicts that the ball can reach the rocket despite any Rindler horizon. See the "general formula" at The Relativistic Rocket:

$$T = \frac{2c}{a}\text{acosh}\left(\frac{ad}{2c^2} + 1\right) \tag{3}$$

This equation gives the aging of a rocket's crew during a trip that covers a distance d as measured in the LIF in which they blasted off, at an acceleration a that's reversed at the midpoint so that the rocket brakes to arrive at its destination at relative rest. For example, this equation predicts that a rocket can blast off from Earth, accelerate and decelerate at 1 g to arrive at the Andromeda galaxy, $d = 2$ million light years away from Earth, while its crew ages just $T \approx 28$ years. (Use $c = 1$ ly/yr and $a = 1.03$ ly/yr$^2 \approx g$.) Notice that Andromeda reaches the rocket that's decelerating toward it, starting from the midpoint that's 1 million light years away as measured in Andromeda's frame, even though the rocket's Rindler horizon is $c^2/a =$ 0.97 ly below the rocket. The equation from the site for the acceleration or deceleration half of the rocket's trip is

$$T = \frac{c}{a}\text{acosh}(ad/c^2 + 1) \tag{4}$$

Compare (4) to (3). See that (3) just doubles the time T that's returned by (4) for either half of the trip. So the simpler (4) shows that a free-falling object (like Andromeda or the ball) can reach a rocket having any acceleration a, starting from any distance d below the rocket. For example, (4) returns $T \approx 14$ years for the deceleration half of the trip to Andromeda. Or use (1) for the time t as measured in the free-falling object's frame.

The distance c^2/a between a rocket and its Rindler horizon applies only in the rocket's frame. When the ball in experiment #1 will reach the rocket, then the initial distance d is length contracted in the rocket's frame to $d/\gamma < c^2/a$ (where γ is the gamma factor), so that the ball is initially above the rocket's Rindler horizon and hence able to reach the rocket, the same as how Andromeda reaches any rocket that decelerates to it.

The Einstein field equations (EFE) depend on the EP. The Schwarzschild metric derives from the EFE, which means the EP takes precedence over the metric. When the metric violates the EP, the EP remains presumably valid and the metric is invalidated within the context of the EFE. Therefore, the Schwarzschild metric is invalid, and moreover, a metric for Schwarzschild geometry that obeys the EP can't predict black holes.

2 A new equation for escape velocity

Equations for a falling body gives the velocity of a free-falling object that was dropped in a uniform gravitational field (ignoring air resistance) as

$$v = at \tag{5}$$

where a is the acceleration of gravity and t is the elapsed time.

The EP shows that The Relativistic Rocket's (2) supplants (5):

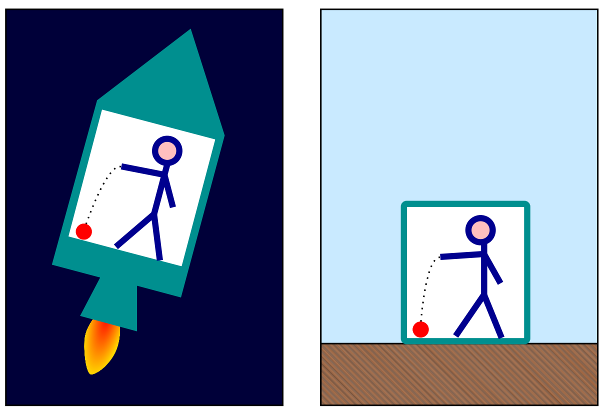

Figure 1: Ball falling to the floor in an accelerating rocket (left) and on Earth (right). By Pbroks13 / Markus Poessel (Mapos), CC BY-SA 3.0, via Wikimedia Commons.

According to the EP, SR's laws hold in both scenarios in Fig. 1. The Relativistic Rocket equations describe the ball's motion relative to the rocket. Then those equations describe the ball's motion relative to the room on Earth as well, where a is the acceleration of gravity. The time t in (2) is measured in the ball's LIF, in which the rocket (or room) blasted off when the ball was dropped.

The equation for escape velocity in general relativity (GR) is

$$v_\mathrm{e} = \sqrt{\frac{2GM}{r}} \tag{6}$$

Escape velocity is also the velocity of an object that was dropped from an infinite distance. For the first small segment of the object's fall, in which the acceleration of gravity a is constant, (5) approximates GR's escape velocity (6), and The Relativistic Rocket's (2) approximates the new escape velocity. So, substituting the two terms at in (2) with GR's escape velocity gives the new escape velocity. That is,

$$v_{\text{new}} = \frac{v_{\text{old}}}{\sqrt{1 + (v_{\text{old}}/c)^2}} \tag{7}$$

where v_{new} is the new escape velocity, and v_{old} is GR's escape velocity (6). This is the same as dividing v_{old} by the gamma factor, since

$$\gamma = \sqrt{1 + (at/c)^2} \tag{8}$$

from The Relativistic Rocket. The new equation for escape velocity, derived from (6) and (7), is

$$v_{\text{e}} = \sqrt{\frac{2GMc^2}{c^2 r + 2GM}} \tag{9}$$

This equation returns a value $< c$ for a radial coordinate $r > 0$. So black holes aren't predicted. GR's escape velocity (6) better approximates the new escape velocity (9) as gravity weakens.

3 A new gravitational time dilation factor

Imagine nested spherical shells concentric to a massive body. An observer drops from an infinite distance, falling freely toward the massive body while measuring, as a fraction x of the observer's own rate of time, the rate of clocks at each shell as they pass right by. Each shell passes at the escape velocity v_{e} there. Inputting that velocity into the reciprocal of the gamma factor gets the value x for that shell. The escape velocity at an infinite distance is zero, so $x = 1$ there. The observer remains stationary relative to the falling space, so the observer's own rate of time remains the rate of time at an infinite distance. Then the gravitational time dilation factor, the rate of time at a radial coordinate r, as a fraction of the rate of time at an infinite distance, is given by the pseudo equation

$$\text{gravitational time dilation factor} = 1/\left(\text{gamma factor for } v_{\text{e}}\right) \tag{10}$$

This equation is verified by deriving GR's gravitational time dilation factor from it, using GR's escape velocity (6):

$$\frac{t_0}{t_{\text{f}}} = 1\Big/\frac{1}{\sqrt{1 - v_{\text{e}}^2/c^2}} = \sqrt{1 - v_{\text{e}}^2/c^2} = \sqrt{1 - \left(\sqrt{\frac{2GM}{r}}\right)^2 /c^2} = \sqrt{1 - \frac{2GM}{c^2 r}} \tag{11}$$

The new gravitational time dilation factor, derived using the pseudo equation (10) and the new escape velocity (9), is

$$\frac{t_0}{t_{\text{f}}} = \sqrt{\frac{c^2 r}{c^2 r + 2GM}} \tag{12}$$

4 A new metric for Schwarzschild geometry

There's a problem in the derivation of the Schwarzschild metric, at the step

$$g_{44} = K\left(1 + \frac{1}{Sr}\right) \approx -c^2 + \frac{2GM}{r} = -c^2\left(1 - \frac{2GM}{c^2 r}\right) \tag{13}$$

in the section "Using the weak-field approximation to find K and S". GR's gravitational time dilation factor (11) is in (13) as

$$\left(\frac{t_0}{t_\mathrm{f}}\right)^2 = 1 - \frac{2GM}{c^2 r} \tag{14}$$

But (14) is invalid. Eqs. (10) and (11) show that

$$\frac{t_0}{t_\mathrm{f}} = 1/\left(\text{gamma factor for } v_\mathrm{e}\right) = \sqrt{1 - v_\mathrm{e}^2/c^2} \tag{15}$$

where v_e is GR's escape velocity (6). This velocity isn't relativistic because it can be >= c. The gamma factor equation requires a relativistic velocity, which is always $< c$. So the gravitational time dilation factor that derives from (14) only approximates the valid factor. The approximation worsens as gravity strengthens and the value for v_e strays further from the valid value, until finally the EP is unmistakably violated at $v_\mathrm{e} = c$. Thus (13) reads like

$$g_{44} \approx -c^2\left(\text{invalid gravitational time dilation factor}\right)^2 \tag{16}$$

We can solve this problem without changing the EFE. The right side of (13) can be changed such that its gravitational time dilation factor can be valid. To do this, we first change (14) to

$$\left(\frac{t_0}{t_\mathrm{f}}\right)^2 = w = 1 - w\frac{2GM}{c^2 r} \tag{17}$$

and solve for w, the square of the gravitational time dilation factor. This converts GR's escape velocity (6), that's embedded in (14), to the new escape velocity (9). This is done by effectively dividing GR's escape velocity by the gamma factor; see the comment for (8). (Alternatively, this is done by changing Newton's $-GM/r^2$, that's in the diagram in the derivation, to the faraway observer's measurement for it.) The new escape velocity (9) is used to build the gravitational time dilation factor. Solving for w gives

$$w = \frac{c^2 r}{c^2 r + 2GM} \tag{18}$$

See that the new gravitational time dilation factor that derives from (18):

$$\frac{t_0}{t_\mathrm{f}} = \sqrt{w} = \sqrt{\frac{c^2 r}{c^2 r + 2GM}} \tag{19}$$

matches (12), as expected. The new escape velocity (9) derives by solving (15) for v_e.

Next we incorporate w into (13), while changing the $\approx$ symbol to $=$ since its gravitational time dilation factor can now be valid.

$$g_{44} = K\left(1 + \frac{1}{Sr}\right) = -c^2\left(1 - w\frac{2GM}{c^2 r}\right) = -c^2 w \tag{20}$$

Note that now

$$g_{44} = -c^2\,(\text{gravitational time dilation factor})^2 \tag{21}$$

and compare this to (16) that we had before.

Completing the derivation:

$$K = -c^2 \tag{22}$$

and

$$1 + \frac{1}{Sr} = 1 - w\frac{2GM}{c^2 r} = w \tag{23}$$

Hence

$$A(r) = w^{-1} \tag{24}$$

and

$$B(r) = -c^2 w \tag{25}$$

So, the new metric for Schwarzschild geometry is

$$ds^2 = \left(\frac{c^2 r}{c^2 r + 2GM}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) - c^2\left(\frac{c^2 r}{c^2 r + 2GM}\right) dt^2 \tag{26}$$

5 Experimental confirmation of the new metric

The only change made to the Schwarzschild metric was to the term for escape velocity that's built into it. The old escape velocity (6) better approximates the new escape velocity (9) as gravity weakens. So, the Schwarzschild metric better approximates the new metric (26) as gravity weakens.

For the Schwarzschild precession in the orbit of the star S2 around Sgr A*, both metrics predict 12.1′ per orbit, in agreement with observations. When the Schwarzschild metric predicts 12.100′ per orbit for S2's Schwarzschild precession, the new metric predicts 12.095′ per orbit.

For the Schwarzschild precession of Mercury, both metrics predict 42.9799″ per Julian century, in agreement with observations.

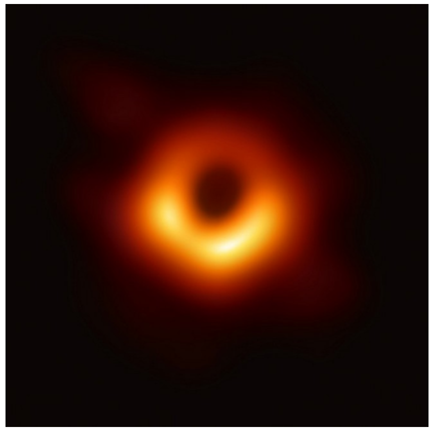

Figure 2: Direct image of a supermassive black hole(?) at the core of Messier 87. Via Wikimedia Commons.

The new metric (26) allows an object of any mass M to have a surface at any radial coordinate $r > 0$. The new gravitational time dilation factor (12) goes to zero as r goes to zero. Gravitational redshift indicates gravitational time dilation, so a star can look black when viewed from afar. This explains the object in Fig. 2, without invoking a black hole.
