The Eigen Theory of the Physical World

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Abstract

This paper introduces the Eigen Theory of the physical world in which the fundamental form of a particle of matter is a distribution of points that forms a manifold of its own. Each of these points results from the intersection of a corresponding pair of eigenvelocity vectors. These vectors are functions of a pair of symmetric and antisymmetric tensors characteristic of gravitation and electromagnetism respectively. The theory develops in three stages. In the first stage, an arbitrary infinitesimal perturbation of the point of intersection of a pair of eigenvelocity vectors produces the wave aspect of the particle of matter and the second stage produces the particle itself. The third stage formulates the base manifold, characterised by an eigenfield of a second pair of symmetric and antisymmetric tensors, in which the particle moves as a whole but with only the particle centre remaining in contact with it. The first two stages expose the reality behind matter-antimatter, the quantum mechanical wave-particle duality and the results of the double-slit experiment. The third stage reveals the nature of dark energy and the structure of dark matter. In Eigen Theory Quantum Mechanics becomes restructured; in particular the uncertainty principle ceases to exist. Finally, the Eigen Theory is in agreement with Einstein's physical reality since it establishes that, in contrast to the extrinsic physical reality that exists due to the above-mentioned arbitrary perturbation, an intrinsic physical reality may also exist, which in itself is inaccessible to observation and experimentation.

Key words: Pair of eigenvelocity vectors, Gravitation and electromagnetism, matter-antimatter, Double-slit experiment, Dark matter and Dark energy, Fibre bundle

Introduction

The Eigen Theory (ET) this paper presents establishes that the fundamental form of a particle of matter, or simply the particle, is the key to the complete structure of the physical world. The particle is a distribution of points that forms a manifold of its own. Each of these points results from the intersection of a corresponding pair of eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ that represents the wave aspect of the particle. The vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ are functions of a pseudo-Riemannian symmetric metric tensor $\boldsymbol{g}$ and an antisymmetric tensor $\boldsymbol{h}$. These two tensors are characteristics respectively of:

(***a***) The magnitude of the vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ and their angular separation
(***b***) The concepts of mass and charge
(***c***) Gravitation and electromagnetism

In Minkowski spacetime, $\boldsymbol{v}$ and $\boldsymbol{u}$ turn out to be Lorentzian boosts of each other. Thus, ET subsumes Special Relativity (SR) at the outset and it subsumes General Relativity (GR) in due course. ET has three primary sets of equations $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$. These happen to be the respective ET counterparts of the three laws of Newton (**§4.4**). Therefore, in ET, theoretical physics may have completed a full circle and stands liberated high above the place on which it stood restricted at first.

The above outline of ET suggests that for ET, as it was for SR, no relevant fundamental references are available to cite. ET begins so to speak with a clean slate and finds that pair formation is the primary feature of the physical world and pairs such as $(\boldsymbol{v}, \boldsymbol{u})$, $(\boldsymbol{g}, \boldsymbol{h})$ and more besides are the vocabulary of the physical world.

In this paper, ET develops in 3-stages in **§1, 2** and **3,** as follows.

The first stage in **§1** produces the following.

(*a*) The vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ as functions of $\boldsymbol{g}$ and $\boldsymbol{h}$.

(*b*) The mutual transport equations of $\boldsymbol{v}$ and $\boldsymbol{u}$ resulting from an arbitrary infinitesimal perturbation of the point of intersection of $\boldsymbol{v}$ and $\boldsymbol{u}$ that preserves their angular separation, which is also their identity

(*c*) A set of field equations satisfied by $\boldsymbol{g}$ and $\boldsymbol{h}$, which is a transformation of the above mutual transport equations

(*d*) Wave aspects of both the particle and the pair of fields $\boldsymbol{g}$ and $\boldsymbol{h}$

In the second stage in **§2** a simple transformation of $\boldsymbol{v}$ and $\boldsymbol{u}$ produces the particle. It is made up of the three fundamental pairs of opposites of the physical world, time-space, translation-rotation, and gravitation-electromagnetism. The two components of each of these pairs unify at the particle centre and this triple unification physically manifests as a pair of null photon velocity vectors. Thus, the particle centre is of an extraordinary character and is able to function as a fulcrum on which each of the three pairs of opposites maintain a perpetual state of dynamic balance.

The third stage in **§**3 formulates the base manifold in which the particle moves as a whole. For this motion only the particle centre remains in contact with the base manifold. The characteristic feature of the base manifold is an eigenfield of a second pair of symmetric and antisymmetric tensor fields $\underline{\boldsymbol{g}}$ and $\underline{\boldsymbol{h}}$. In ET this pair of fields is the structure of the conventional dark matter.

The particle consists of two alternative forms that corresponds to the + and – signs of the antisymmetric tensor $\boldsymbol{h}$ (§2.1). However**,** these may co-exist as two particles occupying two separate manifolds one of which is charged negative and the other positive; they are the progenitors of hadron and lepton of particle physics. The fundamental form of this hadron-lepton pair formation is the Hydrogen atom. The internal structure of this atom is virtually electromagnetic and its external structure, virtually gravitational. Hence, these two structures are characterised by a virtual separation of gravitation and electromagnetism. Since gravitational and electromagnetic motions are primarily translational and rotational, these motions also virtually separate. So also do time and space as these are the primary spacetime subdivisions that correspond to translation and rotation, respectively. Thus, these three fold separations are characteristic of the above-mentioned internal and external structures of the Hydrogen atom. These two structures evolve respectively as an atomic region consisting of atoms and a cosmic region consisting of a massive base manifold of 'dark matter' that envelops the particle centres of atoms and an empty spacetime manifold that envelops and separates the atoms.

From the foregoing, it follows that the atomic and cosmic regions, which constitute the physical world, are a pair of opposites. Furthermore, in comparison to the cosmic region the atomic region is tiny. However, the electromagnetic force in this tiny region is mighty and the gravitational force in the massive cosmic region is feeble on average.

Just as the three fundamental pairs of opposites of the particle are perpetually balanced on the fulcrum at the particle centre, the above two fundamental opposite regions may also be perpetually balanced on a fulcrum of 'immense complexity', which is characteristic of the central region of the physical world where life evolves. An analysis of this balancing and the nature of the corresponding fulcrum are outside the scope of this paper as life itself may be the fundamental element of these.

According to the forgoing, a pair of fulcrums may exist, one physical and the other perhaps not.

§5 compute the motion of point-satellites of negligible masses that circle a static spherically symmetric cosmic central body. In §6 an atomic central body system is examined using the Hydrogen atom as a composite structure that represents static spherically symmetric fields.

1 Stage 1 of the Eigen Theory

1.1 The pair of eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$

Appendix A develops the following set of algebraic equations $\mathbf{F_1}$ satisfied by the vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ and their eigenvalue $\boldsymbol{\beta}$**. T**hese vectors are confined to a manifold of their own.

$$\boldsymbol{\beta g_{ij} v^j = (g_{ij} + h_{ij}) u^j} \qquad \textbf{(A14)}$$

$$\boldsymbol{\beta g_{ij} u^j = (g_{ij} - h_{ij}) v^j} \qquad \textbf{(A15)}$$

In $\boldsymbol{n}$-spacetime $\mathbf{F_1}$ produces $\boldsymbol{n}$ pairs of vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ and $\boldsymbol{n}$ corresponding eigenvalues $\boldsymbol{\beta}$ as functions of $\boldsymbol{g}$ and $\boldsymbol{h}$**.** The tensor $\boldsymbol{h}$ **is** the sum of a 2-form and the exterior derivative of a 1-form $\boldsymbol{p}$, which means that $\boldsymbol{h}$ has the same number of elements as the tensor $\boldsymbol{g}$**.** The elements of $\boldsymbol{p}$ are the counterparts of the elements of $\boldsymbol{g}$ that are arbitrary due to the Bianchi identities that $\boldsymbol{g}$ satisfies; hence, the elements of $\boldsymbol{p}$ are likewise primarily arbitrary. The tensors $\boldsymbol{g}$ and $\boldsymbol{h}$**,** being the same for each of the $\boldsymbol{n}$ pairs of eigenvelocity vectors, are the kernel, or the substance, of matter. Therefore, in this introductory paper on ET it suffices to focus on just one pair of vectors, $\boldsymbol{v}$ and $\boldsymbol{u}$. Appendix A also shows that in Minkowski spacetime $\boldsymbol{v}$ and $\boldsymbol{u}$ are Lorentzian boosts of each other.

$\mathbf{F_1}$ produces only squired values of $\boldsymbol{\beta}$**;** hence $\boldsymbol{\beta}$ is positive or negative and these produce the following pairs of vectors.

(*a*) $(\pm\boldsymbol{v}, \pm\boldsymbol{u})$ as a reversal of both $\boldsymbol{v}$ and $\boldsymbol{u}$ also satisfies $\mathbf{F_1}$
(*b*) $(\pm\boldsymbol{v}, \mp\boldsymbol{u})$ as according to $\mathbf{F_1}$ if $\boldsymbol{\beta}$ changes its sign then either $\boldsymbol{v}$ or $\boldsymbol{u}$ reverses

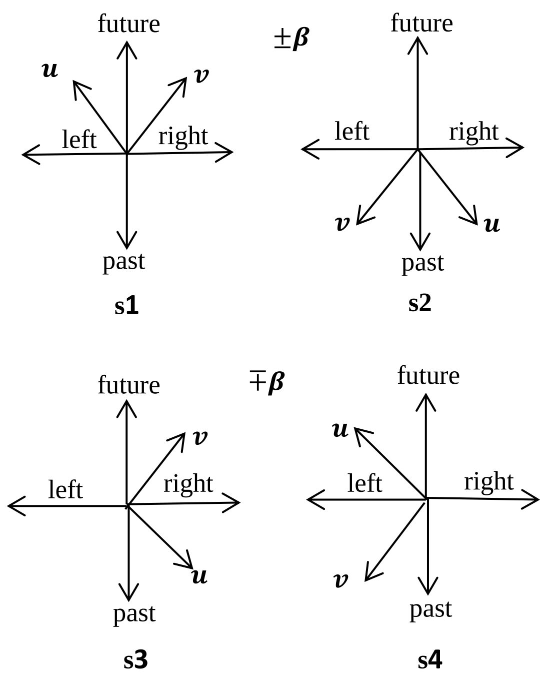

Fig. 1 The two pairs of solutions of $\mathbf{F_1}$

Fig. 1 shows sketches of the pairs of vectors in **(*a*)** as **(s1, s2)** and those in **(*b*)** as (**s3, s4)**. In s1, the coordinate axes at the point of intersection of $\boldsymbol{v}$ and $\boldsymbol{u}$ are so configured that $\boldsymbol{v}$ and $\boldsymbol{u}$ are symmetrically placed with respect to the time-axis and co-planar with it. This coordinate configuration may be considered as a form of MCRF.

According to $\mathbf{F_1}$ if $\boldsymbol{v}$ and $\boldsymbol{u}$ are interchanged then $\boldsymbol{h}$ reverses its sign. In **s1** and **s2** this interchange reverses the spatial direction of each of $\boldsymbol{v}$ and $\boldsymbol{u}$. Accordingly $\mathbf{F_1}$ has CP symmetry since the sign of $\boldsymbol{h}$ also corresponds to the sign of charge. In **s3** and **s4**, if $\boldsymbol{v}$ and $\boldsymbol{u}$ are interchanged then time reverses for each of $\boldsymbol{v}$ and $\boldsymbol{u}$; accordingly, $\mathbf{F_1}$ has CT symmetry. If both $\boldsymbol{v}$ and $\boldsymbol{u}$ are reversed then time reverses for each of these; accordingly $\mathbf{F_1}$ has PT symmetry. Owing to the reversal of time, CT and PT symmetries are 'hidden'.

1.2 The absolute conservation principle

Multiplying **(A14)** by $\boldsymbol{u}$ and **(A15)** by $\boldsymbol{v}$ and combining the results we get

$$\boldsymbol{\beta g_{ij} v^j u^i = g_{ij} v^j v^i = g_{ij} u^j u^i} \qquad \textbf{(1)}$$

These together with $\mathbf{F_1}$ produce the following.

$$(\boldsymbol{g_{ij}} + \boldsymbol{\breve{h}_{ij}})\boldsymbol{u^j v^i} \ = \ \mathbf{0} \qquad \textbf{(2)}$$

$$\boldsymbol{\breve{h} = h/\left(1-\beta^2\right)}, \ \ |\boldsymbol{\beta}| \ \neq \ \mathbf{1} \qquad \textbf{(2}\boldsymbol{a}\textbf{)}$$

The scalars $\boldsymbol{g_{ij}u^j v^i}$ and $\boldsymbol{\breve{h}_{ij}u^j v^i}$ in **(2)** are the ET counterparts of energy and angular momentum that are conserved separately in standard physics. According to **(2)**, in ET the sum of these two counterparts is conserved as zero in perpetuity.

Thus an absolute conservation principle of nothingness exists according to which the sum of energy and scalar angular momentum shared by the pair of eigenvelocity vectors remains zero in perpetuity.

By analogy with Euclidean geometry, **(1)** leads to

$$|\boldsymbol{\beta}| = \boldsymbol{sech}\, \boldsymbol{\xi} \qquad \textbf{(3)}$$

where $\boldsymbol{\xi}$ is the hyperbolic angle between $\boldsymbol{v}$ and $\boldsymbol{u}$. Accordingly $|\boldsymbol{\beta}|$ is confined to the closed interval [0, 1]. Then **(1)** indicates that

$$\boldsymbol{g_{ij} v^j u^i = \beta} \qquad \textbf{(4)}$$

Equation **(2)** can now be re-written as

$$\boldsymbol{g_{ij} u^j v^i = -\breve{h}_{ij} u^j v^i \ = \ \beta} \qquad \textbf{(5)}$$

Hence, in ET the fundamental parameters of energy and scalar angular momentum of $\boldsymbol{v}$ and $\boldsymbol{u}$ unify as $\boldsymbol{\beta}$**.** Finally, $\boldsymbol{v}$ and $\boldsymbol{u}$ are:

(***a***) Of equal magnitudes, according to **(1)**
(***b***) Orthogonal with respect to the general tensor $\boldsymbol{g} + \boldsymbol{\breve{h}}$**,** according to **(2)**
(***c***) In a state of nothingness or nullity, according to **(2)**

(*d*) Such that according to $\mathbf{F}_1$ as $|\boldsymbol{\beta}|$ tends to 1, in the limit $\boldsymbol{v}$ and $\boldsymbol{u}$ unify, $\boldsymbol{h}$ reduces to zero and $\mathbf{F}_1$ becomes trivial. Hence, the vector of unification becomes indeterminate and in that sense, it becomes singular.

According to the above features, $\boldsymbol{v}$ and $\boldsymbol{u}$ possess **E**quality, **O**rthogonality, **N**ullity and **S**ingularity, which bear the acronym EONS.

1.3 The CMB radiation

For the condition $\boldsymbol{\beta} = 0$, **(A14)** and **(A15)** produce the following set of equations of the wave aspect of the particle.

$$\mathbf{0} = (\boldsymbol{g}_{ij} + \boldsymbol{h}_{ij})\bar{\bar{\boldsymbol{u}}}^{j} \qquad \textbf{(A19)}$$

$$\mathbf{0} = (\boldsymbol{g}_{ij} - \boldsymbol{h}_{ij})\bar{\bar{\boldsymbol{v}}}^{j} \qquad \textbf{(A20)}$$

$$\det(\boldsymbol{g} \pm \boldsymbol{h}) = \mathbf{0} \qquad \textbf{(A21)}$$

The null velocities $\bar{\bar{\boldsymbol{v}}}$ and $\bar{\bar{\boldsymbol{u}}}$ may represent the CMB radiation. In that case, CMB radiation is integral with the state of relativity between the pair of vectors $\boldsymbol{v}$ and $\boldsymbol{u}$; hence, its presence is not in violation of the principle of relativity, as it appears to be at present, but in conformity with it.

1.4 The transport equations of $\boldsymbol{v}$ and $\boldsymbol{u}$ and the field equation satisfied by $\boldsymbol{g}$ and $\boldsymbol{h}$

In GR, the geodesic, which is a curve that parallel-transports its tangent vector, can be obtained using the criterion that an arbitrary infinitesimal perturbation of the geodesic does not alter its length between two fixed points. In ET, the mutual transport equations of $\boldsymbol{v}$ and $\boldsymbol{u}$ are obtained using a similar procedure, which applies an arbitrary infinitesimal perturbation to the point of intersection of $\boldsymbol{v}$ and $\boldsymbol{u}$. In this case, the perturbation does not alter the angle between $\boldsymbol{v}$ and $\boldsymbol{u}$, which in effect is their relative identity**.** In Appendix B, this procedure applied to **(5)** produces a pair of mutual transport equations satisfied by $\boldsymbol{v}$ and $\boldsymbol{u}$, given below as **(B15)** and **(B16)**. In these $\boldsymbol{v} = \boldsymbol{d}/\boldsymbol{ds}, \boldsymbol{u} = \boldsymbol{d}'/\boldsymbol{ds}$ where $\boldsymbol{s}$ is the path parameter.

$$\boldsymbol{du}^{\boldsymbol{k}}/\boldsymbol{ds} + \{\boldsymbol{ij},\boldsymbol{k}\}\boldsymbol{u}^{i}\boldsymbol{v}^{j} = \mathbf{0} \qquad \textbf{(B15)}$$

$$\boldsymbol{d}'\boldsymbol{v}^{\boldsymbol{k}}/\boldsymbol{ds} + \{\boldsymbol{ij},\boldsymbol{k}\}\boldsymbol{v}^{i}\boldsymbol{u}^{j} = \mathbf{0} \qquad \textbf{(B16)}$$

where

$$\{\boldsymbol{ij},\boldsymbol{k}\} = \{\boldsymbol{ij},\boldsymbol{k}\}^{s} + \{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{a}} \qquad \textbf{(B17)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{s} = ½\boldsymbol{g}^{\boldsymbol{lk}}\left(\boldsymbol{\partial}_{j}\boldsymbol{g}_{il} + \boldsymbol{\partial}_{i}\boldsymbol{g}_{lj} - \boldsymbol{\partial}_{l}\boldsymbol{g}_{ji}\right) \qquad \textbf{(B4)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{a}} = ½\boldsymbol{h}^{\boldsymbol{lk}}\left(\boldsymbol{\partial}_{j}\boldsymbol{h}_{il} + \boldsymbol{\partial}_{i}\boldsymbol{h}_{lj} + \boldsymbol{\partial}_{l}\boldsymbol{h}_{ji}\right) \qquad \textbf{(B9)}$$

Equations **(B15)** and **(B16)** can be re-written as follows.

$$\left[\frac{\boldsymbol{\partial u}^{\boldsymbol{k}}}{\boldsymbol{\partial x}^{j}} + \{\boldsymbol{ij},\boldsymbol{k}\}\boldsymbol{u}^{i}\right]\boldsymbol{v}^{j} = \mathbf{0} \qquad \textbf{(6)}$$

$$\left[\frac{\boldsymbol{\partial v}^{\boldsymbol{k}}}{\boldsymbol{\partial x}^{j}} + \{\boldsymbol{ij},\boldsymbol{k}\}\boldsymbol{v}^{i}\right]\boldsymbol{u}^{j} = \mathbf{0} \qquad \textbf{(7)}$$

Since in $\boldsymbol{n}$-spacetime $\boldsymbol{g}$ and $\boldsymbol{h}$ at present consists of $\boldsymbol{n}^2 + \boldsymbol{n}$ free variables, **(6)** can be reduced to

$$\frac{\partial u^k}{\partial x^j} + \{ij,k\}u^i = 0 \tag{8}$$

or **(7)** can be reduced to

$$\frac{\partial v^k}{\partial x^j} + \{ij,k\}v^i = 0 \tag{9}$$

Equations **(8)** and **(9)** effectively represent just one equation as follows.

$$\frac{\partial \overline{w}^k}{\partial x^j} + \{ij,k\}\overline{w}^i = 0 \tag{10}$$

Differentiating **(10)** with respect to x^n we get

$$\frac{\partial^2 \, \overline{w}^k}{\partial x^j \, \partial x^n} + \frac{\partial \{ij,k\}}{\partial x^n} \, \overline{w}^i + \{ij,k\} \frac{\partial \overline{w}^i}{\partial x^n} = 0 \tag{10a}$$

We also have

$$\frac{\partial^2 \, \overline{w}^k}{\partial x^j \, \partial x^n} = \frac{\partial^2 \, \overline{w}^k}{\partial x^n \, \partial x^j} \tag{10b}$$

Equations **(10)**, **(10a)** and **(10b)** lead to the following set of equations $\mathbf{F}_2$.

$$\{R^k_{ijn} + C^k_{ijn}\}\overline{w}^i = 0 \tag{11}$$

where

$$R^k_{ijn} = \{in,k\}^s_{;j} - \{ij,k\}^s_{;n} - \{mn,k\}^s\{ij,m\}^s + \{mj,k\}^s\{in,m\}^s \tag{12}$$

$$C^k_{ijn} = \{in,k\}^a_{;j} - \{ij,k\}^a_{;n} - \{mn,k\}^a\{ij,m\}^a + \{mj,k\}^a\{in,m\}^a \tag{13}$$

Since h is the sum of a 2-form and the exterior derivative of the 1-form p, $\mathbf{F}_2$, which is antisymmetric in the indices j and n, consists of $(n^2 - n)\, n/2$ simultaneous equations in $(n^2 + 2n)$ unknowns; hence n = 4. Therefore, $\mathbf{F}_2$ determines g and h together with a velocity $\overline{w}$ with which the pair of tensor fields g and h moves at a point P in 4-spacetime.

The following equation **(14)**

$$\{R_{ij} + C_{ij}\}\overline{w}^i = 0, \quad \det(R + C) = 0 \tag{14}$$

obtained by contracting $\mathbf{F}_2$ with respect to k and n or k and j determines $\overline{w}$ at P.

According to SR, in flat spacetime symmetric metric tensor consists of 4 unit diagonal elements. The corresponding antisymmetric tensor is just the exterior derivative of the 4-element 1-form of the Maxwellean electromagnetic potentials. For these two forms of g and h, both R and C in $\mathbf{F}_2$ vanish and $\mathbf{F}_2$ is trivially satisfied. Accordingly, $\mathbf{F}_2$ subsumes SR if the 1-form p that h contains is the set of Maxwellean electromagnetic potentials. A simple generalisation of Maxwell's electromagnetic equations in terms of h is given in Appendix C.

1.5 Wave aspects of both the particle and the pair of fields g and h

With the fields g and h that $\mathbf{F}_2$ produces as inputs, $\mathbf{F}_1$ determines an eigenvalues β and the corresponding pair of eigenvelocity vectors $(v,\, u)$ at P, which in ET is primarily the point of intersection of $\boldsymbol{v}$ and $\boldsymbol{u}$. The field of $(\boldsymbol{v}, \boldsymbol{u})$ thus produced generates a dual congruence of world lines, which is the wave aspect of the particle.

Corresponding to the above wave aspect of the particle, the velocity $\overline{\boldsymbol{w}}$ that $\mathbf{F_2}$ produces is the velocity with which the wave aspect of the pair of tensor fields $\boldsymbol{g}$ and $\boldsymbol{h}$ moves at $\boldsymbol{P}$.

2 Stage 2 of the Eigen Theory

2.1 The particle

The pair of vectors $\boldsymbol{v}$ and $\boldsymbol{u}$, which represents the wave aspect of the particle, transforms into a pair of vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ which represents the particle itself, as follows**.**

$$\underline{\boldsymbol{v}} = (\boldsymbol{v} + \boldsymbol{u})/\mathbf{2} \tag{15}$$

$$\underline{\boldsymbol{u}} = \pm(\boldsymbol{v} - \boldsymbol{u})/\mathbf{2} \tag{16}$$

The set of equations $\mathbf{F_1}$ in turn transforms as follows.

$$(1 - \boldsymbol{\beta})\boldsymbol{g}_{ij}\underline{\boldsymbol{v}}^{j} = \pm\boldsymbol{h}_{ij}\underline{\boldsymbol{u}}^{j} \tag{17}$$

$$(1 + \boldsymbol{\beta})\boldsymbol{g}_{ij}\underline{\boldsymbol{u}}^{j} = \pm\boldsymbol{h}_{ij}\underline{\boldsymbol{v}}^{j} \tag{18}$$

According to these

$$\boldsymbol{g}_{ij}\underline{\boldsymbol{v}}^{i}\underline{\boldsymbol{u}}^{j} = \mathbf{0} \tag{19}$$

Combining **(17)**, **(18)** with **(4)** in §1**.2** we get

$$\breve{\boldsymbol{h}}_{ij}\underline{\boldsymbol{v}}^{i}\underline{\boldsymbol{u}}^{j} = \mp\boldsymbol{\beta}/\mathbf{2} \tag{20}$$

Finally **(1)** in §1**.2** becomes

$$\boldsymbol{g}_{ij}\underline{\boldsymbol{v}}^{i}\underline{\boldsymbol{v}}^{j} + \boldsymbol{g}_{ij}\underline{\boldsymbol{u}}^{i}\underline{\boldsymbol{u}}^{j} = \boldsymbol{g}_{ij}\boldsymbol{v}^{j}\boldsymbol{v}^{i} = \boldsymbol{g}_{ij}\boldsymbol{u}^{j}\boldsymbol{u}^{i} \tag{21}$$

In the above system of equations both + $\boldsymbol{\beta}$ and **-**$\boldsymbol{\beta}$ are present on an equal footing.

If the set of mutual transport equations **(B15)** and **(B16)** satisfied by $\boldsymbol{v}$ and $\boldsymbol{u}$ is transformed into the set of equations satisfied by $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ then this set does not produce the set of field equations $\mathbf{F_2}$ or an equivalent thereof. Therefore $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$**,** unlike $\boldsymbol{v}$ and $\boldsymbol{u}$, do not transport each other and generate a dual congruence of world lines. The pair of vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ generates only a location in spacetime, which is their point of intersection. Accordingly, the pair of vector fields $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ transforms the dual congruence of line distribution, to a point-distribution, which is the particle. This point-distribution is the microscopic equivalent of the macroscopic point-like atomic-distributions such as solid bodies.

According to **(19),** $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ are orthogonal with respect to the metric tensor. Therefore $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ are of translational and rotational character. In addition, according to **(21)** they appear to have an implicit resultant, which is either $\boldsymbol{v}$ or $\boldsymbol{u}$. Thus, in the presence of $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ or the motion of the particle, $\boldsymbol{v}$ and $\boldsymbol{u}$ or the motion of the wave aspect of the particle appears to reduce to an *implicit state* of $\boldsymbol{v}$ or $\boldsymbol{u}$.

2.2 The particle centre

According to equations **(17)** and **(18)** if $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ unify then $\boldsymbol{\beta}$ **= 0.** This unified velocity, say $\underline{\boldsymbol{w}}$**,** is unique among the pairs of vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$. Hence, its location is special among the distribution of points that makes up the particle. This special point is the particle centre, say $\overline{\boldsymbol{P}}$**.** In terms of $\underline{\boldsymbol{w}}$**,** **(17)** and **(18)** reduce to the following set of equations $\mathbf{F}_3$.

$$\left(\overline{\boldsymbol{g}}_{\boldsymbol{ij}} \pm \overline{\boldsymbol{h}}_{\boldsymbol{ij}}\right)\underline{\boldsymbol{w}}^{\boldsymbol{i}} = \mathbf{0}, \quad \det\left(\overline{\boldsymbol{g}} \pm \overline{\boldsymbol{h}}\right) = \mathbf{0} \tag{22}$$

Tensors $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$ in **(22)** are those present at $\overline{\boldsymbol{P}}$**.** According to **(22),** $\underline{\boldsymbol{w}}$ represents a pair of null photon velocity vectors and in addition they are the velocities with which the pair of tensors $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$ moves at $\overline{\boldsymbol{P}}$**.** Just as $\overline{\boldsymbol{w}}$**,** associated with the wave aspect of the particle, is the velocity of the wave aspect of $\boldsymbol{g}$ and $\boldsymbol{h}$**,** $\underline{\boldsymbol{w}}$**,** associated with the particle, is the velocity of the particle aspect of $\boldsymbol{g}$ and $\boldsymbol{h}$ which behaves as a singular general tensor at the particle centre. Notice that whilst $\underline{\boldsymbol{w}}$ is a distribution spread throughout the wave aspect of the particle, $\underline{\boldsymbol{w}}$ is confined to the particle centre.

Equation **(22)** is likely to be the precursor of the Planck-Einstein relation. A simple insight into this possibility based on a 2-dimensional form of **(22)** is given in Appendix D.

According to the foregoing, the following unifications take place at $\overline{\boldsymbol{P}}$**.**

(a) Time and space, as $\underline{\boldsymbol{w}}$ is null
(b) Translational and rotational components of the particle motion, as $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ unify
(c) Gravitation and electromagnetism, as due to its zero determinant $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$ acts as a singular general tensor

2.3 Integration of the particle centre with the particle manifold

Consider the following set of equations $\mathbf{F}_2$ in **§1.4**

$$\{\boldsymbol{R}^{\boldsymbol{k}}_{\boldsymbol{ijn}} + \boldsymbol{C}^{\boldsymbol{k}}_{\boldsymbol{ijn}}\}\overline{\boldsymbol{w}}^{\boldsymbol{i}} = \mathbf{0} \tag{11}$$

If it is contracted with respect to $\boldsymbol{k}$ and $\boldsymbol{n}$ or $\boldsymbol{k}$ and $\boldsymbol{j}$ we get

$$\{\boldsymbol{R}_{\boldsymbol{ij}} + \boldsymbol{C}_{\boldsymbol{ij}}\}\overline{\boldsymbol{w}}^{\boldsymbol{i}} = \mathbf{0}, \quad \det(\boldsymbol{R} + \boldsymbol{C}) = \mathbf{0} \tag{14}$$

With $\overline{\boldsymbol{R}}$ and $\overline{\boldsymbol{C}}$**,** which correspond to $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$ at the particle centre $\overline{\boldsymbol{P}}$**,** equation **(14)** becomes

$$\{\overline{\boldsymbol{R}}_{\boldsymbol{ij}} + \overline{\boldsymbol{C}}_{\boldsymbol{ij}}\}\overline{\boldsymbol{w}}^{\boldsymbol{i}} = \mathbf{0}, \quad \det\left(\overline{\boldsymbol{R}} + \overline{\boldsymbol{C}}\right) = \mathbf{0} \tag{23}$$

At the particle centre $\overline{\boldsymbol{P}}$ we also have

$$\left(\overline{\boldsymbol{g}}_{\boldsymbol{ij}} \pm \overline{\boldsymbol{h}}_{\boldsymbol{ij}}\right)\underline{\boldsymbol{w}}^{\boldsymbol{i}} = \mathbf{0}, \quad \det\left(\overline{\boldsymbol{g}} \pm \overline{\boldsymbol{h}}\right) = \mathbf{0} \tag{22}$$

Since the particle centre $\overline{\boldsymbol{P}}$ and the null vector $\underline{\boldsymbol{w}}$ present at $\overline{\boldsymbol{P}}$ are unique it follows that

$$\{\overline{\boldsymbol{R}}_{\boldsymbol{ij}} + \overline{\boldsymbol{C}}_{\boldsymbol{ij}}\} = \overline{\boldsymbol{b}}\left\{\overline{\boldsymbol{g}}_{\boldsymbol{ij}} \pm \overline{\boldsymbol{h}}_{\boldsymbol{ij}}\right\} \tag{24}$$

where $\overline{\boldsymbol{b}}$ is a scalar. The particle centre, subject to this set of conditions, behaves as a fulcrum on which the particle, or each of the following three pairs of opposite fields that makes up the particle, balances as a whole.

(a) Time and space
(b) Translational and rotational components of the particle motion
(c) Gravitation and electromagnetism

In equations **(24)**, which may be referred to as the *particle-centre equation*, the position of $\overline{\boldsymbol{P}}$ and the elements of $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$ at $\overline{\boldsymbol{P}}$ are still unknown as the base manifold in which $\overline{\boldsymbol{P}}$ moves is yet to be formulated. This formulation begins with the following

$$\{\underline{\boldsymbol{R}}_{ij} + \underline{\boldsymbol{C}}_{ij}\} = \underline{\boldsymbol{b}}\{\underline{\boldsymbol{g}}_{ij} \pm \underline{\boldsymbol{h}}_{ij}\},\ \det(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}}) = \boldsymbol{0} \qquad \textbf{(24m)}$$

which is a generalised form of the *particle-centre equation* **(24)** in the sense that at the location of the particle centre $\overline{\boldsymbol{P}}$ it becomes the same as equation **(24).** In other words the values of $\underline{\boldsymbol{R}}$ and $\underline{\boldsymbol{C}}$ and $\underline{\boldsymbol{g}}$ and $\underline{\boldsymbol{h}}$ at $\overline{\boldsymbol{P}}$ are the same as those of $\overline{\boldsymbol{R}}$ and $\overline{\boldsymbol{C}}$ and $\overline{\boldsymbol{g}}$ and $\overline{\boldsymbol{h}}$**.** If the *particle-centre equation* (24) is compared to the fulcrum in a merchant's balance then equation (24m) is comparable to the fulcrum body, which in the case of merchant's balance can be, loosely speaking, as big as the Earth itself.

Now according to equation **(24m)**, $(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}})$ goes through a form of transformation on the left hand side of **(24m)** and maps to itself on the right hand side of **(24m)**. Hence, equation **(24m)** is an eigen-formulation of $(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}})$ with $\underline{\boldsymbol{b}}$ as the corresponding eigenvalue. Just as the eigenvalue $\boldsymbol{\beta}$ is energy, $\underline{\boldsymbol{b}}$ also is likely to be a form of energy a possible candidate for which is dark energy.

In the base manifold, which is developed in terms of equation **(24m)** in §**3**.1 below, the particle moves as a whole but with only the particle centre $\overline{\boldsymbol{P}}$ in coincidence with a point in the base manifold.

3 Stage 3 of the Eigen Theory

3.1 The base manifold

In equation **(24m),** the number of elements of $\underline{\boldsymbol{g}}$ and $\underline{\boldsymbol{h}}$ present exceeds the number of component equations by 4. To rectify this mismatch **(24m)** is first split into its symmetric and antisymmetric components as follows.

$$\underline{\boldsymbol{R}}_{ij} + \underline{\boldsymbol{C}}^{s}_{ij} = \underline{\boldsymbol{b}}\underline{\boldsymbol{g}}_{ij} \qquad \textbf{(25)}$$

$$\underline{\boldsymbol{C}}^{a}_{ij} = \pm\underline{\boldsymbol{b}}\underline{\boldsymbol{h}}_{ij} \qquad \textbf{(26)}$$

$$\det(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}}) = \boldsymbol{0} \qquad \textbf{(24n)}$$

where

$$\underline{\boldsymbol{R}}_{ij} = \{\boldsymbol{ik},\boldsymbol{k}\}^{s}_{,j} - \{\boldsymbol{ij},\boldsymbol{k}\}^{s}_{,k} - \{\boldsymbol{mk},\boldsymbol{k}\}^{s}\{\boldsymbol{ij},\boldsymbol{m}\}^{s} + \{\boldsymbol{mj},\boldsymbol{k}\}^{s}\{\boldsymbol{ik},\boldsymbol{m}\}^{s} \qquad \textbf{(27)}$$

$$\underline{\boldsymbol{C}}^{s}_{ij} = \{\boldsymbol{ik},\boldsymbol{k}\}^{a}_{;j} + \{\boldsymbol{mj},\boldsymbol{k}\}^{a}\{\boldsymbol{ik},\boldsymbol{m}\}^{a} \qquad \textbf{(28)}$$

$$\underline{\boldsymbol{C}}^{a}_{ij} = -\{\boldsymbol{mk},\boldsymbol{k}\}^{a}\{\boldsymbol{ij},\boldsymbol{m}\}^{a} - \{\boldsymbol{ij},\boldsymbol{k}\}^{a}_{,k} \qquad \textbf{(29)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{s} = ½\underline{\boldsymbol{g}}^{lk}(\boldsymbol{\partial}_j\underline{\boldsymbol{g}}_{il} + \boldsymbol{\partial}_i\underline{\boldsymbol{g}}_{lj} - \boldsymbol{\partial}_l\underline{\boldsymbol{g}}_{ji}) \qquad \textbf{(30)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{a} = ½\underline{\boldsymbol{h}}^{lk}(\boldsymbol{\partial}_j\underline{\boldsymbol{h}}_{il} + \boldsymbol{\partial}_i\underline{\boldsymbol{h}}_{lj} + \boldsymbol{\partial}_l\underline{\boldsymbol{h}}_{ji}) \qquad \textbf{(31)}$$

The 4 excess elements that equations **(25)** and **(26)** contain separate from the rest if the two components of $\underline{\boldsymbol{h}}$**,** the 2-form and the exterior derivative of the 1-form $\boldsymbol{p}$, separate. With this separation **(25)** and **(26)** split into two sets of equations that represent two separate manifolds, as follows.

Set 1: $\underline{\boldsymbol{h}}$ consists only of the exterior derivative of the 1-form $\boldsymbol{p}$**.** Consequently $\underline{\boldsymbol{C}}^{\boldsymbol{s}}$ and $\underline{\boldsymbol{C}}^{\boldsymbol{a}}$ vanish and **(25)** reduces to the empty spacetime field equation in GR and **(26)** reduces to $\underline{\boldsymbol{bh}}_{\boldsymbol{ij}} = \mathbf{0}$**.** Since, as outlined at the end of **§1.4,** the 1-form $\boldsymbol{p}$ is an integral feature of empty spacetime, $\underline{\boldsymbol{b}}$ in **(26)** becomes zero. Then **(25)** and **(26)** reduce to just the following with respect to which the1-form $\boldsymbol{p}$ remains arbitrary.

$$\overline{\overline{\boldsymbol{g}}}_{\boldsymbol{ij}} = \mathbf{0} \tag{32}$$

As in GR, this equation produces the tensor field $\overline{\overline{\boldsymbol{g}}}$ characteristic of the gravitational field that the particle produces in its surrounding empty spacetime.

Set 2: Equations **(25)** and **(26)** are unchanged except that $\underline{\boldsymbol{h}}$ consists only of the 2-form. Let these equations be numbered differently to avoid future confusion as follows:

$$\underline{\boldsymbol{R}}_{\boldsymbol{ij}} + \underline{\boldsymbol{C}}^{\boldsymbol{s}}_{\boldsymbol{ij}} = \underline{\boldsymbol{b}}\underline{\boldsymbol{g}}_{\boldsymbol{ij}} \qquad \textbf{(25m)}$$

$$\underline{\boldsymbol{C}}^{\boldsymbol{a}}_{\boldsymbol{ij}} = \pm\underline{\boldsymbol{bh}}_{\boldsymbol{ij}} \qquad \textbf{(26m)}$$

$$\det\left(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}}\right) = \mathbf{0} \qquad \textbf{(24nm)}$$

The manifold that corresponds to this set is the base manifold of the particle. Due to the absence of the 1-form $\boldsymbol{p}$ of the Maxwellean electromagnetic potentials, base manifold is dark and its content, which is a pure field, is the ET equivalent of dark matter.

While the set 1 is the structure of the empty spacetime that envelops the particle as a whole externally, the set 2 is that of the very opposite as the base manifold envelops only the particle centre $\overline{\boldsymbol{P}}$. Therefore, the sets 1 and 2 are the outermost and the innermost structures of the particle, respectively which exists in addition to its own structure. Henceforth the empty spacetime manifold and the base manifold of the particle will be referred to as the outermost and the innermost manifolds of the particle manifold, respectively.

3.2 The complete particle structure

The three sets of equations $\mathbf{F}_2$, {**(25m)**, **(26m), (24nm)}** and **(32)** are the *field equations* of the particle manifold and its innermost and outermost manifolds, respectively. Now recall that in **§2.2** the characteristic feature of the particle centre $\overline{\boldsymbol{P}}$ is the unification thereat of each of the following three pairs of opposites.

. **(*a*)** Time and space
(b) Translational and rotational components of the particle motion
(c) Gravitation and electromagnetism

Because of these unifications, the *particle-centre equation* (24) is unique. The simultaneous solution of the three sets of equations **(24)**, $\mathbf{F}_2$ and {**(25m)**, **(26m), (24nm)}** is the complete particle structure which includes the particle-centre position in the base manifold and the corresponding 1–form thereat. In this case (32) is redundant. While the particle centre position is characteristic of the base manifold, the 1-form thereat is characteristic of the particle manifold. Therefore the *particle-centre equation* (24) integrates 'field' and 'particle'. These position and 1-form are the natural counterparts of those of the point-particle in standard physics, which are arbitrary in comparison.

Notice that the *particle-centre equation* (24) has 8 component equations in excess of the 8 unknowns, which are the four elements of position and the four elements of the 1-form. These additional component equations correspond to the arbitrary nature of four of the elements of the base manifold metric tensor, and the arbitrary nature of the 1-form of the particle manifold, mentioned in §1.1. The presence of excess equations may cause quantum jumping of the particle position in the base manifold and quantisation of the 1-form in the particle manifold.

3.3 The spherically symmetric cosmic central body system

If the particle is spherically symmetric then the particle centre $\overline{\boldsymbol{P}}$ is the same as the origin of the spherical polar coordinate system ($\mathbf{t}, \boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi}$). If this spherically symmetric particle is the central body of a cosmic central body system, then a satellite would be a point particle of negligible mass in comparison to the mass of the central body.

The point-satellite moves in the innermost and the outermost manifolds of the central body, outlined in §3.1. In accord with the spherical symmetry of the central body, if the satellite moves in a circle, then simply both the innermost and the outermost manifolds contribute to it (§5.3). In this context, notice that virtually all satellites in practice move in approximate circular paths around the central body. However, in general, the motion of the satellite would not be strictly circular and as a result, in the case of the simplest form of the central body system mentioned above, the satellite motion would be constrained as follows.

(a) When the satellite is in close proximity to the central body, it effectively moves only in the outermost manifold. An example is the motion of a satellite in the solar system (§5.3)
(b) When the satellite is sufficiently far away from the central body, it effectively moves only in the innermost manifold. An example is a satellite in a galactic system, which is sufficiently far away from the galactic centre (§5.3).

The occurrence of these constraints is due to the reduction of the satellite to a point. Otherwise the combined solution of the six sets of equations, consisting of the pair of *particle-centre equations* (24), pair of particle manifold field equations $\mathbf{F_2}$, and the pair of field equations of the innermost and the outermost manifolds that the two particles share determines the motions of both particles, the central body and the satellite, if indeed each of these two bodies is approximately represented in terms of the particle in ET.

4 Simple applications of ET

4.1 The principle of uncertainty

In **§3.2** it became clear that the particle and its base manifold are in a state of integration such that position of the particle centre in the base manifold and the corresponding 1-form thereat are completely determined. Accordingly, the quantum mechanical uncertainty of position and momentum of a particle is primarily due to the unnatural representation of a particle of matter as a point, which by its nature has no corresponding base manifold.

4.2 Matter-antimatter

Whilst the pair of equations **(A14)** and **(A15)** represents the wave aspect of the particle, the following pair of equations represents the particle.

$$(1-\boldsymbol{\beta})\boldsymbol{g}_{ij}\underline{\boldsymbol{v}}^{j} = \pm\boldsymbol{h}_{ij}\underline{\boldsymbol{u}}^{j} \quad \textbf{(17)}$$

$$(1+\boldsymbol{\beta})\boldsymbol{g}_{ij}\underline{\boldsymbol{u}}^{j} = \pm\boldsymbol{h}_{ij}\underline{\boldsymbol{v}}^{j} \quad \textbf{(18)}$$

According to equations **(17)** and **(18)**, corresponding to the + or – signs of $\boldsymbol{h}$, the particle consists of two alternative components rotating in opposite directions and possessing the same mass but opposite charges. These are the properties of a pair of conventional matter and antimatter particles. Therefore, in ET, a pair of matter and antimatter particles has its origin as two alternatives that are as the two sides of the proverbial coin.

These alternatives can co-exist in two separate spacetime manifolds of their own, producing a pair of matter and antimatter particles.

4.3 The double-slit experiment

4.3.1 Particles of matter

According to §3.2 the particle state is one of singular wholeness of the gravitational and electromagnetic fields due to the unification of these fields at the particle centre. This state becomes disturbed by the external electromagnetic fields with which the particle is fired at the two slits. The resulting gravitational and electromagnetic fields are able to represent only the wave aspect of the particle, which according to §1.1 may result from any given pair of gravitational and electromagnetic tensor fields. This wave aspect would go through both slits.

However, unlike the particle, the wave aspect of the particle is inherently unstable, as it has no fulcrum on which it can balance. Hence, if observed this wave aspect invariably becomes the particle as observation entails photons that only the particle can emit. Since wave- and particle aspects are mutually exclusive, the interference pattern of the wave aspect disappears.

4.3.2 Photons

Because of the integration of the wave and particle aspects of the pair of fields $\boldsymbol{g}$ and $\boldsymbol{h}$ in §2.3, the particle motion (or the photon motion) of this field at the particle centre changes over to a wave-motion elsewhere, which includes both the particle manifold and the outermost manifold which is empty spacetime. This behaviour accords with Planck's view on radiation-behaviour.

Since for the photon velocity vector each of the three fundamental pairs of opposites are in a state of unification (**§2.2),** bizarre double-slit experimental may result under unusual experimental setups.

4.4 ET and Newtonian Mechanics

ET has three fundamental sets of equations $\mathbf{F_1}$**,** $\mathbf{F_2}$, and $\mathbf{F_3}$. A one-to-one correspondence exists between these and the 3 laws of motion of Newton, as follows.

F_1 determines the primary motion as a pair of translational velocities	First law states that the primary motion is a single uniform translational velocity
F_2 determines the force field that determines the primary motion.	Second law states the force that alters the primary motion
F_3 determines a pair of photons oppositely directed in space.	Third law states that forces occur in pairs that are equal and opposite.

The above comparison between ET and Newton's physics indicates that theoretical physics may now have journeyed a full circle and stands liberated high above the place on which she stood restricted at first.

5 Application of ET to a cosmic central body system

5.1 Planetary and galactic systems

The simplest cosmic central body system is one in which the central body and its innermost and outermost manifolds are static and spherically symmetric and the satellites are point-particles of masses negligible in comparison to the mass of the central body. In this case, the central body and its innermost manifold, share a common centre, which is at spatial rest.

Now because of the spherical symmetry the Newtonian forces of gravity cancel at the common centre. Therefore, at this centre spacetime is well behaved from the Newtonian viewpoint, which however is contrary to what the Schwarzschild metric of GR predicts. Nonetheless, fundamentally this metric is valid only in the outermost manifold which surrounds a gravitating body and extending its validity to the interior of the gravitating body is no more than an act of pure assumption. In §5.2 below spacetime is indeed found to be well behaved at the above-mentioned common centre. In addition, the metrics of the central body and its innermost manifold may have the same form under static spherically symmetric conditions and this form is the exact opposite of the Schwarzschild metric characteristic of the outermost manifold of the central body.

5.2 The field solutions of the three manifolds

5.2.1 Field solution of the outermost manifold of the central body

The solution of **(32)** in terms of spherical polar coordinates $(\boldsymbol{t},\ \boldsymbol{r},\ \boldsymbol{\theta},\ \boldsymbol{\phi})$ is the following Schwarzschild metric. The parameter $\boldsymbol{m}$ is the length equivalent of the central body mass.

$$\bar{\bar{g}}_{ij} = diag\{\left(1-\frac{2m}{r}\right), -1/\left(1-\frac{2m}{r}\right), -r,^2$$

$$-r^2 sin^2\theta\} \tag{33}$$

5.2.2 Field solution of the innermost manifold of the central body

Appendix E produces the following solution of **(25m), (26m)** and **(24nm)** in terms of spherical polar coordinates $(\boldsymbol{t}, \boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$. The eigenvalue $\underline{\boldsymbol{b}}$ is zero.

$$\underline{g}_{ij} = diag\{-\left(1-\tfrac{r}{2M}\right), 1/\left(1-\tfrac{r}{2M}\right), \tag{E20}$$

$$-r^2, -r^2 sin^2\theta\}$$

$$\underline{h}_{ij} = \{\ \underline{h}_{01} = -\underline{h}_{10} = -1, \underline{h}_{23} = -\underline{h}_{32} = r^2 sin\theta\} \tag{E21}$$

The parameter $\boldsymbol{M}$ is the length equivalent of the 'dark matter' mass. Only the non-zero elements of $\underline{\boldsymbol{h}}$ are shown in **(E21)**. At the manifold centre $\underline{\boldsymbol{g}}$ is **(-1,1, 0,0)** and in $\underline{\boldsymbol{h}}$ only the two elements $\underline{h}_{01} = -\underline{h}_{10} = -1$ are non-zero.

5.2.3 Field solution of the central body manifold

This field, in general, is given by the following set of equations $\mathbf{F_2}$.

$$\{R^k_{ijn} + C^k_{ijn}\}\overline{w}^i = 0 \tag{11}$$

However, solution of this equation is unnecessary at this stage as the fields $\boldsymbol{g}$ and $\boldsymbol{h}$ play no part in the present case of 'massless' satellites as these satellites only move in the outermost and innermost manifolds. Nonetheless, as a matter of interest these fields $\boldsymbol{g}$ and $\boldsymbol{h}$ appear to be the same as **(E20)** and **(E21)** with $\boldsymbol{M}$ replaced by $\boldsymbol{m}$, for the following reasons.

According to **(E20),** the innermost manifold has an event horizon of radius double the 'dark matter' mass. According to **(33)** the outermost manifold that envelops the central body also has an event horizon of radius double the central body mass. However, **(33)** can also represent the outermost manifold that envelops the innermost manifold in which case $\boldsymbol{m}$ becomes replaced by $\boldsymbol{M}$**.** Therefore, each of these two event horizons, in effect, refers to the interface between two manifolds one of which is empty and surrounds the other, which is a dark matter manifold.

From the foregoing it may be inferred that the central body manifold for the condition of static spherical symmetry is the same as the dark matter manifold with $\boldsymbol{M}$ replaced by $\boldsymbol{m}$. Accordingly, spacetime is well-behaved at the centre of spherical symmetry of the central body

The general solution of (11) may actually involve a barrier interface that corresponds to the singularity of the wave aspect of matter, mentioned in §1.2 which corresponds to the condition of $|\boldsymbol{\beta}|$ tending to 1.

Finally, according to **(E20)** the spacetime curvature, or the gravity force, increases with increasing radial distance, which is also the way the strong force acts. Therefore, it is reasonable to surmise that strong force is of gravitational origin, and that it is the fundamental force that operates at the heart of matter.

5.3 Circular motion of a satellite of mass, negligible in comparison to the central body mass

Let $\boldsymbol{v}$ and $\boldsymbol{u}$ be the velocities of the satellite in the outermost manifold and the innermost manifold, respectively. In terms of spherical polar coordinates **(*t*, *r*, *θ*, *ϕ*),** these have the components **(v^0, 0, 0, v^3)** and ($\boldsymbol{u^0}$**, 0, 0,** $\boldsymbol{u^3}$**),** and they satisfy the following geodesic equations.

$$\boldsymbol{dv^k/ds + \{ij,k\}^{\bar{\bar{s}}} v^i v^j = 0} \tag{34}$$

$$\boldsymbol{d'u^k/ds' + \{ij,k\}^{\underline{s}} u^i u^j = 0} \tag{35}$$

Symbols $\{\boldsymbol{ij,k}\}^{\bar{\bar{s}}}$ and $\{\boldsymbol{ij,k}\}^{\underline{s}}$ are the respective connection coefficients that correspond to the two metrics (33) and (E20) in §5.2.1 and §5.2.2. Parameters $\boldsymbol{s}$ and $\boldsymbol{s'}$ in equations **(34)** and **(35)** are the respective proper times. Since the motions are circular in both manifolds, only ***k* = 1** in **(34))** and **(35)** needs be considered and then these equations reduce to

$$\boldsymbol{s_v^2 = (rd\phi/dt)^2 = m/r} \tag{36}$$

$$\boldsymbol{s_u^2 = (rd'\phi/d't)^2 = r/(4M)} \tag{37}$$

where $\boldsymbol{r}$ is radial distance of the satellite, and $\boldsymbol{s_v}$ and $\boldsymbol{s_u}$ are the rotational speeds that correspond to the velocities $\boldsymbol{v}$ and $\boldsymbol{u}$ respectively.

Since the satellite motion is circular, $\boldsymbol{s_v}$ in **(36)** is the same as that given by the Newton's law of gravitation and $\boldsymbol{m/r}$ in **(36)** is the Newtonian gravitational potential at the radius $\boldsymbol{r}$. If the density of the mass distribution in the innermost manifold is $\boldsymbol{1/(8\pi Mr)}$ then $\boldsymbol{s_u}$ in **(37)** is also the same as that given by the Newton's law of gravitation and $\boldsymbol{r/(4M)}$ is the Newtonian gravitational potential at the radius $\boldsymbol{r}$. Accordingly, two Newtonian forces act radially inwards on the satellite and the magnitude of the speed $\boldsymbol{s}$ that corresponds to the resultant of these two forces is given by

$$|s| = \sqrt{m/r \ + r/4M} \tag{38}$$

According to **(38)**, the contribution from the innermost manifold is negligible if

$$m/r \gg r/4M \tag{39}$$

This in turn means

$$\underline{m}/\underline{r}^2 \gg 1 \tag{40}$$

where $\underline{m}$ is the normalised mass of the central body and $\underline{r}$ is the normalised radial distance of the satellite, normalisations being with respect to mass $\boldsymbol{M}$ and the radius $\boldsymbol{2M}$ respectively. For any reasonable estimate of $\boldsymbol{M}$, the value of $\underline{m}/\underline{r}^2$ at the radial distance of the planet Pluto in the solar system is of the order of $\mathbf{10^5}$. Therefore, the innermost manifold makes no appreciable contribution to the dynamics of the solar system.

5.4 Use of equation (37) to obtain the contribution made by the innermost manifold to the rotational speeds of stars in the M33 galaxy

The value of $\boldsymbol{M}$ used for the computation of speeds shown in Fig.2 was obtained using **(37)** at the point, **6.833 kpc** and **92.35 km/s** of M33 galaxy data **(ref.3).** This value of $\boldsymbol{M}$**,** which is the total dark matter mass in the physical world, is 7.47 **x** $\mathbf{10^{56}}$ **g.**

In Fig. 2 speeds due to dark matter obtained from observational data on the M33 galaxy are shown as a continuous curve. Speeds that **(37)** produces are shown as a dashed curve.

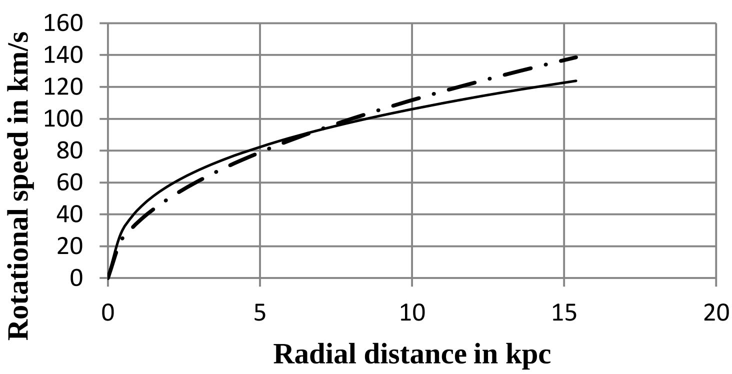

Fig. 2 The M33 galaxy

5.5. Gravitational lensing

According to **(38),** for circular motion, the total gravitational potential $\boldsymbol{\phi_r}$ at a radius $\boldsymbol{r}$, due to both the innermost manifold and the outermost manifold, is given by

$$\phi_r = m/r \ + r/4M \tag{41}$$

Accordingly, the angle of circular deviation of a ray of light due to both manifolds is given by

$$\boldsymbol{\alpha = 4(m/b \;\; + b/4M)} \tag{42}$$

where $\boldsymbol{b}$ is the impact parameter.

6. Application of ET to an atomic central body system

In **§5** ET was applied to a static, spherically symmetric cosmic central body system. Here an atomic central body system is examined using the Hydrogen atom as a composite structure that represents static spherically symmetric fields. This examination of the Hydrogen atom is only heuristic.

Following is the complete list of the fundamental particle equations.

$$\{R^k_{ijn} + C^k_{ijn}\}\overline{w}^i = 0 \tag{11}$$

$$\begin{aligned} R^k_{ijn} &= \{in,k\}^s_{,j} - \{ij,k\}^s_{,n} - \{mn,k\}^s\{ij,m\}^s \\ &+\{mj,k\}^s\{in,m\}^s \end{aligned} \tag{12}$$

$$\begin{aligned} C^k_{ijn} &= \{in,k\}^a_{;j} - \{ij,k\}^a_{;n} - \{mn,k\}^a\{ij,m\}^a \\ &+\{mj,k\}^a\{in,m\}^a \end{aligned} \tag{13}$$

$$\{ij,k\}^s = ½g^{lk}\left(\partial_j g_{il} + \partial_i g_{lj} - \partial_l g_{ji}\right) \tag{B4}$$

$$\{ij,k\}^a = ½h^{lk}\left(\partial_j h_{il} + \partial_i h_{lj} + \partial_l h_{ji}\right) \tag{B9}$$

$$(1 - \beta)g_{ij}\underline{v}^j = \pm h_{ij}\underline{u}^j \tag{17}$$

$$(1 + \beta)g_{ij}\underline{u}^j = \pm h_{ij}\underline{v}^j \tag{18}$$

$$g_{ij}\underline{v}^i\underline{u}^j = 0 \tag{19}$$

$$\breve{h}_{ij}\underline{v}^i\underline{u}^j = \mp\beta/2 \tag{20}$$

At the particle centre, the following equations hold.

$$\left(\overline{g}_{ij} \pm \overline{h}_{ij}\right)\underline{w}^i = 0, \quad \det(\overline{g} \pm \overline{h}) = 0 \tag{22}$$

Eliminating $\underline{u}$ between **(17)** and **(18)** we get

$$0 = \{-(1 - \beta^2)g_{ij} + h_{il}g^{lk}h_{kj} \;\}\,\underline{v}^j \tag{43}$$

For spherical symmetry, the symmetric and antisymmetric tensors $\boldsymbol{g}$ and $\boldsymbol{h}$, in terms of spherical polar coordinates (t, r, θ, ϕ), are as follows.

$$g = \begin{bmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ 0 & 0 & 0 & g_{33} \end{bmatrix} \tag{44}$$

$$\boldsymbol{h} = \begin{bmatrix} \mathbf{0} & \boldsymbol{h_{01}} & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{h_{01}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{h_{23}} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{h_{23}} & \mathbf{0} \end{bmatrix} \qquad \textbf{(45)}$$

Then $\boldsymbol{hg^{-1}h}$ within the curly brackets in **(43)** becomes

$$\boldsymbol{hg^{-1}h} = \begin{bmatrix} -\boldsymbol{h_{01}}^2/\boldsymbol{g_{11}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{h_{01}}^2/\boldsymbol{g_{00}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{h_{23}}^2/\boldsymbol{g_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{h_{23}}^2/\boldsymbol{g_{22}} \end{bmatrix} \qquad \textbf{(46)}$$

For the zero determinant of the tensor within the curly brackets in **(43)** we get

$$\mathbf{0} = (\mathbf{1} - \boldsymbol{\beta}^2)\boldsymbol{g_{00}} + \boldsymbol{h_{01}}^2/\boldsymbol{g_{11}} \qquad \textbf{(47)}$$
$$\mathbf{0} = (\mathbf{1} - \boldsymbol{\beta}^2)\boldsymbol{g_{11}} + \boldsymbol{h_{01}}^2/\boldsymbol{g_{00}} \qquad \textbf{(48)}$$

$$\mathbf{0} = (\mathbf{1} - \boldsymbol{\beta}^2)\boldsymbol{g_{22}} + \boldsymbol{h_{23}}^2/\boldsymbol{g_{33}} \qquad \textbf{(49)}$$
$$\mathbf{0} = (\mathbf{1} - \boldsymbol{\beta}^2)\boldsymbol{g_{33}} + \boldsymbol{h_{23}}^2/\boldsymbol{g_{22}} \qquad \textbf{(50)}$$

Due to spherical symmetry, only **(47)** and **(48)** are relevant in this case and from these we get

$$\boldsymbol{\beta}^2 = \mathbf{1} + \boldsymbol{h_{01}}^2/\boldsymbol{g_{00}g_{11}} \qquad \textbf{(51)}$$

The three tensor elements in **(51)** can be obtained by solving the field equation (11), or the simpler equation (14), provided the following conditions of $\boldsymbol{g}$ and $\boldsymbol{h}$ are met.

(***a***) $\boldsymbol{g}$ and $\boldsymbol{h}$ are static and spherically symmetric
(***b***) $\boldsymbol{g}$ and $\boldsymbol{h}$ are approximately due to the proton and the electron, respectively.
(***c***) $\boldsymbol{g}$ is that present in the spacetime that envelops the proton which is located at the centre of the coordinate system. Hence this $\boldsymbol{g}$ satisfies the 'empty' spacetime field equation ion GR. $\boldsymbol{h}$ is that due to the electron which occupies this 'empty' spacetime manifold.

These conditions are those that create a static spherically symmetric electron (manifold) of mass negligible in comparison to that of the proton present at the origin of the system of spherical polar coordinates. For these conditions equation (14) in which $\overline{\boldsymbol{w}}$ has only the time-component produces the following.

(***a***) The Symmetric and Antisymmetric tensor fields are separate
(***b***)

$$\boldsymbol{g_{00}g_{11}} = -\mathbf{1} \qquad \textbf{(52)}$$

(***c***) The coordinate ***r*** in the case of the antisymmetric tensor field becomes a constant. Since, as a result, this ***r*** cannot vary continuously it varies discretely. Hence, the electron manifold exists as a series of discrete concentric spherical surfaces.
(***d***) The tensor element $\boldsymbol{h_{01}}$ becomes indeterminate

Intuitively, $\boldsymbol{h_{01}}$ becomes indeterminate due to the unnatural static state of the electromagnetic field. Now, using findings in Quantum Mechanics the following may be assumed**.**

$$\boldsymbol{\beta} = \pm\mathbf{1}/\boldsymbol{n}^2 \qquad \textbf{(53)}$$

where $\boldsymbol{n}$ is an integer. In this case equation **(51)** provides the corresponding $\boldsymbol{h_{01}}$.

When the unity in equation **(53)** is replaced by $\boldsymbol{\alpha}$, the fine structure constant, $\boldsymbol{n}$ in equation **(53)** is the Hydrogen radial quantum number. When $\boldsymbol{\alpha}$ =1 and $\boldsymbol{n}$ = 1, then $|\boldsymbol{\beta}| = 1$ and, as outlined in §1.2, for this condition pairing of $\boldsymbol{v}$ and $\boldsymbol{u}$ breaks down and the particle structure no longer exists. Therefore, the least value of $\boldsymbol{n}$ is 2, which is in accordance with Balmer, Lyman and Paschen series of Hydrogen energy levels. Note that according to **(53)** the particle centre at which $\boldsymbol{\beta}$ = 0 is reached only as $\boldsymbol{n}$ tends to infinity, which is indicative of the unusual nature of the particle centre.

7 Discussion and conclusions

The mathematical framework of ET resembles that of the fibre bundle. However, in this paper, the emphasis is on physics and not on mathematics and that was the way ET developed over a period of more than two decades.

According to ET the fundamental form of a particle of matter, or simply the particle, holds the key to the complete structure of the physical world. The particle is made up of the following three pair of complementary opposites.

time - space
translation - rotation
gravitation - electromagnetism

Translation and rotation unify at a special point, which is the particle centre. This state of unification, which is the exact opposite of the state of these motions in standard physics, results in a pair of null photon velocity vectors. Each of the remaining two pairs also unifies at the particle centre via this pair of null vectors. Due to these three unifications, the particle centre, which in ET is the counterpart of the standard point particle, is of an extraordinary character.

In standard physics, rotational and translational motions are separate and so are gravitation and electromagnetism. As mentioned in the Introduction to this paper, the dynamic interaction between members of each of these two separations (along with those between time and space) is what characterises the central region of the physical world. Therefore, the inescapable conclusion is that standard physics applies only in this region whilst ET applies throughout the physical world.

In ET strong force is of gravitational origin and since weak force is already linked to the electromagnetic force, in ET only a pair of balanced forces exist, which are gravitational and electromagnetic, respectively

According to Appendix F the spacetime curvature of the global 'dark matter' manifold has only a marginal effect on the observed Hubble expansion of the physical universe.

According to ET, an intrinsic physical reality may exist, which in itself is inaccessible to us. It consists of a wave-particle duality of which the wave aspect consists of a field of a pair of eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ that exists in a state perpetual nothingness and satisfies a set of field equations $\mathbf{F_1}$. The particle aspect of this duality consists of a pair of translational and rotational velocity vectors $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{u}}$ and it has a unique centre characterised by a null photon velocity vector which satisfies a set of equations $\mathbf{F_3}$. A pair of symmetric and antisymmetric tensor fields that characterises $\mathbf{F_1}$ and $\mathbf{F_3}$ may appear at first as arbitrary, and yet they may be simply autonomous by nature. The arbitrary perturbation of the motion of the pair of eigenvelocity vectors in §1.4 that nonetheless maintains their state of nothingness opens external access to the pair of symmetric and antisymmetric tensor fields in terms of a set of field equations $\mathbf{F_2}$. This access is the cause of the extrinsic physical reality, which lends itself to observation and experimentation.

Finally, consider the two fundamental branches of mathematics, algebra and geometry.

(a) Algebra: The primary element of the complete form of algebra is the complex number, which consists of a pair of any two numbers. A simple manipulation of the imaginary operator $\boldsymbol{i}$, which integrates this pair of numbers establishes that, on its own, it stands for the unification of the pair of opposites +1 and -1.

(b) Geometry: The primary elements of simple geometry may be considered as a diameter and its associated circle. The length of the diameter and the circumference of the circle become unified in the transcendental number π.

The above unifiers $\boldsymbol{i}$ and π themselves become unified in the following null fashion which is a generalisation of the Euler identity. In (54) the integer $\boldsymbol{n}$ is greater than 1.

$$\sum_{k=0}^{n-1} e^{2\pi i\frac{k}{n}} = 0 \qquad (54)$$

From the foregoing it follows that pair formation and an associated underlying unification of these pairs, is a feature shared by both mathematics and physics. The physical counterpart of the null equation (54) is the pair of null photon velocity vectors, at the particle centre, that unifies the two members of each of the three fundamental pairs of opposites, mentioned above.

8. Future Work

The complete structure of the Hydrogen atom.

9. Acknowledgements

I thank Dr. Edvige Corbelli [3] for promptly emailing to me the set of M33 data that I used for obtaining the curves in Fig .2.

Without the generous encouragement and support of my wife Lalitha, this paper, which took more than two decades to complete, would not have been a possibility.

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Appendix A

Set of algebraic equations F_1 that the pair of eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ and its eigenvalue $\boldsymbol{\beta}$ satisfy

Consider two vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ that relate to each other in general as

$$\boldsymbol{b v^i = a^i_k u^k} \quad \textbf{(A1)}$$

$$\boldsymbol{b' u^i = a'^i_k v^k} \quad \textbf{(A2)}$$

where $\boldsymbol{a}$ and $\boldsymbol{a'}$ are tensors of type **(1, 1)** and $\boldsymbol{b}$ and $\boldsymbol{b'}$ are two scalar variables. Rewriting **(A1)** and **(A2)** with $\boldsymbol{v}$ and $\boldsymbol{u}$ in their coordinate form we get

$$\boldsymbol{b dx^i/d\mathfrak{s} = a^i_k d'x^k/d\mathfrak{s}'} \quad \textbf{(A3)}$$

$$\boldsymbol{b' d'x^i/d\mathfrak{s}' = a'^i_k dx^k/d\mathfrak{s}} \quad \textbf{(A4)}$$

where $\boldsymbol{x^0}$ is the time coordinate and $\boldsymbol{\mathfrak{s}}$ and $\boldsymbol{\mathfrak{s}'}$ are path parameters. Let $\boldsymbol{b}$, $\boldsymbol{b'}$ and the two path parameters be adjusted so that

$$\boldsymbol{b(d\mathfrak{s}'/d\mathfrak{s}) = b'(d\mathfrak{s}/d\mathfrak{s}') = \beta} \quad \textbf{(A5)}$$

where $\boldsymbol{\beta}$ is a scalar variable. Then **(A1)** and **(A2)** become

$$\boldsymbol{\beta v^i = a^i_k u^k} \quad \textbf{(A6)}$$

$$\boldsymbol{\beta u^i = a'^i_k v^k} \quad \textbf{(A7)}$$

where

$$\boldsymbol{v^i = dx^i/ds} \quad \textbf{(A8)}$$

$$\boldsymbol{u^i = d'x^i/ds} \quad \textbf{(A9)}$$

The path parameter $\boldsymbol{s}$ is the ET-equivalent of proper time in Relativity.

Equations **(A6)** and **(A7)** can be re-written as follows.

$$\boldsymbol{\beta g_{ij} v^j = (g_{ij} + h_{ij}) u^j} \quad \textbf{(A10)}$$

$$\boldsymbol{\beta g'_{ij} u^j = (g'_{ij} + h'_{ij}) v^j} \quad \textbf{(A11)}$$

where $\boldsymbol{g}$ and $\boldsymbol{g'}$ are symmetric and $\boldsymbol{h}$ and $\boldsymbol{h'}$ are antisymmetric. The condition that $\boldsymbol{v}$ and $\boldsymbol{u}$ are eigenvectors is $\boldsymbol{a'}$ in **(A7)** is the transpose of $\boldsymbol{a}$ in **(A6) (ref.1)**. This condition is satisfied if

$$\boldsymbol{g' = g} \quad \textbf{(A12)}$$

$$\boldsymbol{h' = -h} \quad \textbf{(A13)}$$

Re-writing **(A10)** and **(A11)** to include **(A12)** and **(A13)** we have

$$\boldsymbol{\beta g_{ij} v^j = (g_{ij} + h_{ij}) u^j} \quad \textbf{(A14)}$$

$$\boldsymbol{\beta g_{ij} u^j = (g_{ij} - h_{ij}) v^j} \quad \textbf{(A15)}$$

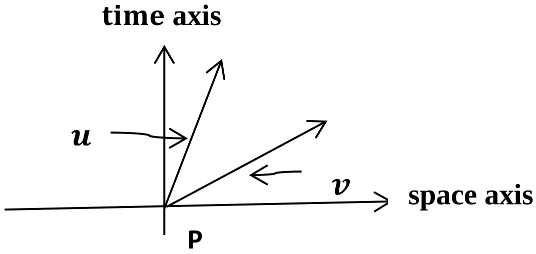

Fig. A1 Time and space axes coplanar with ($\boldsymbol{v}, \boldsymbol{u}$)

In Minkowski spacetime let the coordinate axes at the point of intersection of $\boldsymbol{v}$ and $\boldsymbol{u}$ be so configured that $\boldsymbol{v}$ and $\boldsymbol{u}$ are coplanar with the time-axis and a space-axis, as shown in Fig. A1.

In this case, **(A14)** and **(A15)** produce the following.

$$\sqrt{1-s^2}\begin{bmatrix} v^0 \\ v^1 \end{bmatrix} = \begin{bmatrix} 1 & s \\ s & 1 \end{bmatrix}\begin{bmatrix} u^0 \\ u^1 \end{bmatrix} \quad \textbf{(A16)}$$

$$\sqrt{1-s^2}\begin{bmatrix} u^0 \\ u^1 \end{bmatrix} = \begin{bmatrix} 1 & -s \\ -s & 1 \end{bmatrix}\begin{bmatrix} v^0 \\ v^1 \end{bmatrix} \quad \textbf{(A17)}$$

$$s = h_{01} \quad \textbf{(A18)}$$

According to these the two eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ are Lorentzian boosts of each other with a boost speed of $\boldsymbol{s}$**.**

Appendix B

The mutual transport equations of $\boldsymbol{v}$ and $\boldsymbol{u}$

Let an arbitrary infinitesimal perturbation $\boldsymbol{\delta x^i}$**,** $\boldsymbol{i}$ = **0,…*n*-1,** be applied to the point of intersection of the pair of eigenvelocity vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ so that $\boldsymbol{\beta}$ is kept invariant. Then according equations **(5)** in **§1.2** we have

$$\delta\left(g_{ij}v^i u^j\right) = \delta\beta = 0 \quad \textbf{(B1)}$$

$$\delta\left(h_{ij}v^i u^j\right) = \delta\beta\left(1-\beta^2\right) = 0 \quad \textbf{(B2)}$$

On expanding **(B1)** we get

$$\left[g_{kn}\frac{d}{ds}\left\{\frac{d'x^n}{ds}\right\} + g_{kn}\frac{d'}{ds}\left\{\frac{dx^n}{ds}\right\} + 2g_{ko}\{ij,o\}^s\frac{d'x^i}{ds}\frac{dx^j}{ds}\right]\delta x^k = \beta_g \quad \textbf{(B3)}$$

where

$$\{ij,k\}^s = \frac{1}{2}g^{lk}\left(\frac{\partial g_{il}}{\partial x^j} + \frac{\partial g_{lj}}{\partial x^i} - \frac{\partial g_{ji}}{\partial x^l}\right) \quad \textbf{(B4)}$$

$$\beta_g = \frac{d}{ds}\left\{g_{ij}\delta x^i\frac{d'x^j}{ds}\right\} + \frac{d'}{ds}\left\{g_{ij}\frac{dx^i}{ds}\delta x^j\right\} \quad \textbf{(B5)}$$

Since $\boldsymbol{\delta x^i}$ is arbitrary, **(B3)** effectively represents $\boldsymbol{n}$ equations, which respectively correspond to the $\boldsymbol{n}$ elements of $\boldsymbol{\delta x}$**.**

Let $\boldsymbol{\delta x^l}$ be an element of $\boldsymbol{\delta x}$ with the remaining elements of $\boldsymbol{\delta x}$ set to zero. Let the scalars within the curly brackets in **(B5)** be denoted by $\boldsymbol{c^{'l}}$ and $\boldsymbol{c^l}$ as follows.

$$g_{lj}\delta x^l \frac{d'x^j}{ds} = c'^l \tag{B5a}$$

$$g_{lj}\delta x^l \frac{dx^j}{ds} = c^l \tag{B5b}$$

The summation on $\boldsymbol{l}$ is suspended in **(B5a)** and **(B5b).** Eliminating $\boldsymbol{\delta x^l}$ from **(B5a)** and **(B5b)** we get

$$(g_{lj}/c'^l)\frac{d'x^j}{ds} = (g_{lj}/c^l)\frac{dx^j}{ds} \tag{B5c}$$

In the $\boldsymbol{n}$ independent equations that correspond to $\boldsymbol{l}$ **=0,…*n*-1**, which (B5c) represents, the ratios between $\boldsymbol{c^{'l}}$ and $\boldsymbol{c^l}$ can be maintained constant using the set of $\boldsymbol{n}$ arbitrary elements of $\boldsymbol{g}$**.** Then because $\boldsymbol{x^l}$ is arbitrary**,** $\boldsymbol{c^{'l}}$ and $\boldsymbol{c^l}$ can be individually maintained constant. Therefore

$$\beta_g = 0 \tag{B6}$$

Then we have

$$\left[g_{kn}\frac{d}{ds}\left\{\frac{d'x^n}{ds}\right\} + g_{kn}\frac{d'}{ds}\left\{\frac{dx^n}{ds}\right\} + 2g_{ko}\{ij,o\}^s\frac{d'x^i}{ds}\frac{dx^j}{ds}\right] = 0 \tag{B7}$$

On expanding **(B2)** in the same way that **(B1)** was expanded, we get

$$\left[h_{kn}\frac{d}{ds}\left\{\frac{d'x^n}{ds}\right\} - h_{kn}\frac{d'}{ds}\left\{\frac{dx^n}{ds}\right\} + 2h_{ko}\{ij,o\}^a\frac{d'x^i}{ds}\frac{dx^j}{ds}\right]\delta x^k = \beta_h \tag{B8}$$

where

$$\{ij,k\}^a = \frac{1}{2}h^{lk}\left(\frac{\partial h_{il}}{\partial x^j} + \frac{\partial h_{lj}}{\partial x^i} + \frac{\partial h_{ji}}{\partial x^l}\right) \tag{B9}$$

$$\beta_h = \frac{d}{ds}\left\{h_{ij}\delta x^i\frac{d'x^j}{ds}\right\} + \frac{d'}{ds}\left\{h_{ij}\frac{dx^i}{ds}\delta x^j\right\} \tag{B10}$$

The reasoning that was used to obtain **(B6)** is applicable in this case also as the 1-form that $\boldsymbol{h}$ contains consists of $\boldsymbol{n}$ arbitrary elements. Thus, we get

$$\beta_h = 0 \tag{B11}$$

$$\left[h_{kn}\frac{d}{ds}\left\{\frac{d'x^n}{ds}\right\} - h_{kn}\frac{d'}{ds}\left\{\frac{dx^n}{ds}\right\} + 2h_{ko}\{ij,o\}^a\frac{d'x^i}{ds}\frac{dx^j}{ds}\right] = 0 \tag{B12}$$

Equations **(B7)** and **(B12)** can be re-written as

$$\frac{d}{ds}\left\{\frac{d'x^k}{ds}\right\} + \frac{d'}{ds}\left\{\frac{dx^k}{ds}\right\} + 2\{ij,k\}^s\frac{d'x^i}{ds}\frac{dx^j}{ds} = 0 \tag{B13}$$

$$\frac{d}{ds}\left\{\frac{d'x^k}{ds}\right\} - \frac{d'}{ds}\left\{\frac{dx^k}{ds}\right\} + 2\{ij,k\}^a \frac{d'x^i}{ds}\frac{dx^j}{ds} = 0 \tag{B14}$$

Adding **(B13)** and **(B14)** and subtracting **(B14)** from **(B13)** we get the following equations that determine the mutual transport of $\boldsymbol{v}$ and $\boldsymbol{u}$.

$$\frac{d}{ds}\left\{\frac{d'x^k}{ds}\right\} + \{ij,k\}\frac{d'x^i}{ds}\frac{dx^j}{ds} = 0 \tag{B15}$$

$$\frac{d'}{ds}\left\{\frac{dx^k}{ds}\right\} + \{ij,k\}\frac{dx^i}{ds}\frac{d'x^j}{ds} = 0 \tag{B16}$$

where

$$\{ij,k\} = \{ij,k\}^s + \{ij,k\}^a \tag{B17}$$

Appendix C

A simple generalisation of Maxwell's electromagnetic equations

In ET, the antisymmetric tensor $\boldsymbol{h}$ is the sum of a 2-form and the exterior derivative of a 1-form $\boldsymbol{p}$. This $\boldsymbol{h}$ can be used to generalise the following Maxwell's electromagnetic equations

$$F_{ij} = \partial_j p_i - \partial_i p_j \tag{C1}$$

$$\partial_j\{\sqrt{-g}\,g^{im}g^{jn}F_{mn}\} = \sqrt{-g}J^i \tag{C2}$$

where $\boldsymbol{J}$ is the charge-current density vector. If the 2-form component of $\boldsymbol{h}$, say $\underline{\boldsymbol{h}}$, is assumed to be related to $\boldsymbol{J}$ as

$$\partial_j\{\sqrt{-g}\,g^{im}g^{jn}\underline{h}_{mn}\} = -\sqrt{-g}J^i \tag{C3}$$

then in **ET** the above Maxwell's equations become generalised as

$$\partial_j\{\sqrt{-g}\,g^{im}g^{jn}h_{mn}\} = 0 \tag{C4}$$

Appendix D

A simple insight into Planck-Einstein Relation

In 2-dimensional Minkowski spacetime, equation

$$\left(\overline{g}_{ij} \pm \overline{h}_{ij}\right)\underline{w}^i = 0, \quad \det(\overline{g} \pm \overline{h}) = 0 \tag{22}$$

reduces to just the following.

$$dt/\,d\not{s} = dx/d\not{s} \tag{D1}$$

where $\boldsymbol{dt}$ and $\boldsymbol{dx}$ are the time and space increments of the photon travel and $\boldsymbol{d\not{s}}$ is path parameter increment yet to be established owing to the null character of $\underline{\boldsymbol{w}}$. Now $\boldsymbol{dt/ds}$ is the dimensionless photon energy $\boldsymbol{E'}$. Therefore, **(D1)** becomes

$$E' = dx/d\not{s} \tag{D2}$$

Since this energy $\boldsymbol{E'}$ is dimensionless it has to be multiplied by a fundamental energy constant to obtain $\boldsymbol{E}$ in the usual units of energy. This constant is Planck energy $(\hbar c/G)^{1/2}c^2$ where $\hbar$ is Planck's reduced constant, $\boldsymbol{c}$ is speed of light and $\boldsymbol{G}$ is gravitation constant. Then **(D2)** becomes

$$\boldsymbol{E} = (\hbar c/G)^{1/2} c^2 (dx/d\not{s}) \qquad \textbf{(D3)}$$

Now **(22)** is the result of unification of translational and rotational motions of light. This unification in this 2-dimensional case connects $\boldsymbol{dx}$ with an incremental hyperbolic angle $\boldsymbol{d\xi}$ using Planck distance as follows.

$$\boldsymbol{dx} = (\boldsymbol{Planck\ distance})\boldsymbol{d\xi} \qquad \textbf{(D4)}$$

Since Planck distance is $(\hbar G/c^3)^{1/2}$, **(D3)** becomes

$$\boldsymbol{E} = (\hbar c)(d\xi/d\not{s}) \qquad \textbf{(D5)}$$

If the normal angular velocity is the same as $\boldsymbol{d\xi}/(\boldsymbol{d\not{s}}/\boldsymbol{c})$ then that would define the path parameter $\not{s}$ and **(D5)** becomes the same as Planck-Einstein relation**.**

$$\boldsymbol{E} = h\boldsymbol{\nu} \qquad \textbf{(D6)}$$

where $\boldsymbol{\nu}$ is normal frequency.

Appendix E

Solution of (25m), 26m) and (24nm) in **§2.4**, **for static spherically symmetric space**

Equations **(25m), (26m)** and their auxiliary equations are as follows.

$$\underline{\boldsymbol{R}}_{ij} + \underline{\boldsymbol{C}}^{s}_{ij} = \boldsymbol{b}\underline{\boldsymbol{g}}_{ij} \qquad \textbf{(25m)}$$

$$\underline{\boldsymbol{C}}^{a}_{ij} = \pm\boldsymbol{b}\underline{\boldsymbol{h}}_{ij} \qquad \textbf{(26m)}$$

$$\det\left(\underline{\boldsymbol{g}} \pm \underline{\boldsymbol{h}}\right) = \boldsymbol{0} \qquad \textbf{(24nm)}$$

where

$$\underline{\boldsymbol{R}}_{ij} = \{\boldsymbol{ik},\boldsymbol{k}\}^{s}_{,j} - \{\boldsymbol{ij},\boldsymbol{k}\}^{s}_{,k} - \{\boldsymbol{mk},\boldsymbol{k}\}^{s}\{\boldsymbol{ij},\boldsymbol{m}\}^{s} + \{\boldsymbol{mj},\boldsymbol{k}\}^{s}\{\boldsymbol{ik},\boldsymbol{m}\}^{s} \qquad \textbf{(27)}$$

$$\underline{\boldsymbol{C}}^{s}_{ij} = \{\boldsymbol{ik},\boldsymbol{k}\}^{a}_{;j} + \{\boldsymbol{mj},\boldsymbol{k}\}^{a}\{\boldsymbol{ik},\boldsymbol{m}\}^{a} \qquad \textbf{(28)}$$

$$\underline{\boldsymbol{C}}^{a}_{ij} = -\{\boldsymbol{mk},\boldsymbol{k}\}^{a}\{\boldsymbol{ij},\boldsymbol{m}\}^{a} - \{\boldsymbol{ij},\boldsymbol{k}\}^{a}_{,k} \qquad \textbf{(29)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{s} = ½\underline{\boldsymbol{g}}^{lk}\left(\partial_j\underline{\boldsymbol{g}}_{il} + \partial_i\underline{\boldsymbol{g}}_{lj} - \partial_l\underline{\boldsymbol{g}}_{ji}\right) \qquad \textbf{(30)}$$

$$\{\boldsymbol{ij},\boldsymbol{k}\}^{a} = ½\underline{\boldsymbol{h}}^{lk}\left(\partial_j\underline{\boldsymbol{h}}_{il} + \partial_i\underline{\boldsymbol{h}}_{lj} + \partial_l\underline{\boldsymbol{h}}_{ji}\right) \qquad \textbf{(31)}$$

For static spherically symmetric conditions $\underline{\boldsymbol{g}}$ and $\underline{\boldsymbol{h}}$ in spherical polar coordinates $(\boldsymbol{t}, \boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$ have the following forms.

$$\underline{\boldsymbol{g}}_{ij} = \boldsymbol{diag}\left\{e^{\nu}, -e^{\lambda}, -r^2, -r^2 sin^2\theta\right\} \qquad \textbf{(E1)}$$

$$\underline{\boldsymbol{h}}_{ij} = \{\,\underline{\boldsymbol{h}}_{01} = -\,\underline{\boldsymbol{h}}_{10} = -\,e^{\alpha},\ \underline{\boldsymbol{h}}_{23} = -\,\underline{\boldsymbol{h}}_{32} = sin\theta e^{\rho}\} \qquad \textbf{(E2)}$$

Parameters $\boldsymbol{\nu}$, $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\rho}$ are functions of $\boldsymbol{r}$ only and all other elements of $\underline{\boldsymbol{h}}$ are zero. The elements of $\{\boldsymbol{ij},\boldsymbol{k}\}^{s}$ that correspond to $\underline{\boldsymbol{g}}$ in **(E1)** are well established in the literature and they can be found on page 84 of ref. 2. The non-zero elements of $\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{a}}$ that correspond to $\underline{\boldsymbol{h}}$ in **(E2)** are as follows.

$$\{\mathbf{12},\mathbf{2}\}^{\boldsymbol{a}} = -\boldsymbol{\rho}'/\mathbf{2} \tag{E3}$$

$$\{\mathbf{13},\mathbf{3}\}^{\boldsymbol{a}} = -\,\boldsymbol{\rho}'/\mathbf{2} \tag{E4}$$

$$\{\mathbf{21},\mathbf{2}\}^{\boldsymbol{a}} = +\boldsymbol{\rho}'/\mathbf{2} \tag{E5}$$

$$\{\mathbf{23},\mathbf{0}\}^{\boldsymbol{a}} = +\boldsymbol{sin\theta e^{\rho-\alpha}\rho}'/\mathbf{2} \tag{E6}$$

$$\{\mathbf{31},\mathbf{3}\}^{\boldsymbol{a}} = +\boldsymbol{\rho}'/\mathbf{2} \tag{E7}$$

$$\{\mathbf{32},\mathbf{0}\}^{\boldsymbol{a}} = -\boldsymbol{sin\theta e^{\rho-\alpha}\rho}'/\mathbf{2} \tag{E8}$$

The accent,**′**, on a symbol denotes differentiation with respect to $\boldsymbol{r}$. On substituting these elements of $\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{a}}$ in the expression for $\underline{\boldsymbol{C}}_{\boldsymbol{ij}}^{\boldsymbol{a}}$ in **(29)**, we get

$$\underline{\boldsymbol{C}}_{\boldsymbol{ij}}^{\boldsymbol{a}} = \mathbf{0} \tag{E9}$$

Hence, it follows that the parameter $\underline{\boldsymbol{b}}$ in **(25m**) and **(26m)** is zero. On substituting the above elements of $\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{a}}$ in the expression for $\underline{\boldsymbol{C}}_{\boldsymbol{ij}}^{\boldsymbol{s}}$ in **(28)**, we get the following.

$$\underline{\boldsymbol{C}}_{\mathbf{00}}^{\boldsymbol{s}} = \boldsymbol{e^{\nu-\lambda}\nu}'(\boldsymbol{\rho}')/\mathbf{2} \tag{E10}$$

$$\underline{\boldsymbol{C}}_{\mathbf{11}}^{\boldsymbol{s}} = -(\boldsymbol{\rho}'') + \lambda'(\boldsymbol{\rho}')/\mathbf{2} - (\boldsymbol{\rho}')^{\mathbf{2}}/\mathbf{2} \tag{E11}$$

$$\underline{\boldsymbol{C}}_{\mathbf{22}}^{\boldsymbol{s}} = -\boldsymbol{re^{-\lambda}}(\boldsymbol{\rho}') \tag{E12}$$

$$\underline{\boldsymbol{C}}_{\mathbf{33}}^{\boldsymbol{s}} = -\boldsymbol{rsin^{2}\theta e^{-\lambda}}(\boldsymbol{\rho}') \tag{E13}$$

Elements $\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{s}}$ of the tensor $\underline{\boldsymbol{R}}_{\boldsymbol{ij}}$ in **(27)** can be found on page **85** of ref. 2 where $\boldsymbol{R}$ has been denoted as $\boldsymbol{G}$. On substituting in **(25m)** these elements $\{\boldsymbol{ij},\boldsymbol{k}\}^{\boldsymbol{s}}$ and the above elements of**,** $\underline{\boldsymbol{C}}_{\boldsymbol{ij}}^{\boldsymbol{s}}$ we get

$$\boldsymbol{e^{\nu-\lambda}}\left(-\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\nu}'' + \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{\lambda}'\boldsymbol{\nu}' - \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{\nu}'^{\mathbf{2}} - \frac{\boldsymbol{\nu}'}{\boldsymbol{r}}\right) + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{e^{\nu-\lambda}\nu}'\boldsymbol{\rho}' = \mathbf{0} \tag{E14}$$

$$\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\nu}'' - \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{\lambda}'\boldsymbol{\nu}' + \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{\nu}'^{\mathbf{2}} - \frac{\boldsymbol{\lambda}'}{\boldsymbol{r}} - (\boldsymbol{\rho}'') + \frac{1}{2}\boldsymbol{\lambda}'(\boldsymbol{\rho}') - \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{\rho}')^{\mathbf{2}} = \mathbf{0} \tag{E15}$$

$$\boldsymbol{e^{-\lambda}}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{r}(\boldsymbol{\nu}' - \boldsymbol{\lambda}')\right) - \mathbf{1} - \boldsymbol{re^{-\lambda}}(\boldsymbol{\rho}') = \mathbf{0} \tag{E16}$$

The parameters $\boldsymbol{\nu}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ that satisfy equations **(E14)** to **(E16)** are as follows.

$$\boldsymbol{\rho}' = \mathbf{2}/\boldsymbol{r} \tag{E17}$$

$$e^{\nu} = -(1 - r/(2M)) \quad \textbf{(E18)}$$

$$e^{\lambda} = -1/(1 - r/(2M)) \quad \textbf{(E19)}$$

M in **(E18)** and **(E19)** is a constant of integration. Applying the condition **(24nm)** to the parameters in **(E17)** to **(E19)**, $\underline{g}$ and $\underline{h}$ in **(E1)** and **(E2),** become

$$\underline{g}_{ij} = diag\{-\left(1 - \frac{r}{2M}\right), 1/\left(1 - \frac{r}{2M}\right), -r^2, -r^2 sin^2\theta\} \quad \textbf{(E20)}$$

$$\underline{h}_{ij} = \{\ \underline{h}_{01} = -\underline{h}_{10} = -1, \ \underline{h}_{23} = -\underline{h}_{32} = kr^2 sin\theta\} \quad \textbf{(E21)}$$

where k is a constant which may be set to unity.

Appendix F

Gravitational redshift due to spacetime curvature of the innermost manifold

The metric tensor field of the innermost manifold, obtained in Appendix E, for static spherically symmetric space is the following.

$$\underline{g}_{ij} = diag\{-\left(1 - \frac{r}{2M}\right), 1/\left(1 - \frac{r}{2M}\right), -r^2, -r^2 sin^2\theta\} \quad \textbf{(E20)}$$

Let a photon be emitted at a distance r from the origin O of the system of spherical polar coordinates **(*t, r, θ, ϕ*),** and let its frequency at this point of emission be ν_e. On reaching O let the photon frequency become ν_r. These two frequencies relate to each other as (**ref.4)**

$$\frac{\nu_e}{\nu_r} = (1 - r/(2M))^{-1/2} \quad \textbf{(F1)}$$

This frequency ratio in terms of a recessional speed s, is given by

$$\frac{\nu_e}{\nu_r} = \sqrt{\frac{1+s}{1-s}} \quad \textbf{(F2)}$$

Combining **(F1)** and **(F2)**, we get

$$s = \frac{r/(2M)}{2 - r/(2M)} \quad \textbf{(F3)}$$

For comparing with Hubble's law, let **(F3)** be re-written as $s = \underline{H}r$, where

$$\underline{H} = \frac{1}{4M - r} \quad \textbf{(F4)}$$

According to **(F4)** the maximum value of ***H*** is **1/(2*M*)** that occurs at ***r* = 2*M*.** Now ***M*** has been estimated in **§6.4** as **7.47 x 10^{56} g** which works out to **1.799x10^{07}** kpc. Hence the maximum value of ***H*** **(=1/(2*M*))** is **8.34 $kms^{-1}Mpc^{-1}$.** This value of ***H*** is considerably less than the present value of the Hubble's constant, **73.8 $kms^{-1}Mpc^{-1}$.** Therefore, the gravitational curvature of the innermost manifold has only a small effect on the observed Hubble expansion of the physical universe.