Solution to the advance of the perihelion of Mercury in Newtonian theory

Christian Corda

June 22, 2020

International Institute for Applicable Mathematics and Information Sciences, B. M. Birla Science Centre, Adarshnagar, Hyderabad 500063 (India). E-mail: *cordac.galilei@gmail.com*

Abstract

It is shown through three different approaches that, contrary to a long-standing conviction more than 160 years long, the orbit of Mercury behaves as required by Newton's equations with a very high precision if one correctly analyses the situation in the framework of the two-body problem without neglecting the mass of Mercury.

General relativity remains more precise than Newtonian physics, but the results in this paper show that Newtonian framework is more powerful than researchers and astronomers were thinking till now.

1 Introduction

Based on astronomical observations, in the early 1600s Kepler established that the orbit described by a planet in the solar system is an ellipse, with the Sun occupying one of its foci. Assuming that a planet is subject only to the gravitational attraction of the Sun, Kepler's result is easily obtained mathematically in Newton's theory. But the other planets also have a gravitational attraction on the planet in question. What is the effect of their presence? If one repeats the calculation taking into account this complication, one finds that the attraction exerted by all the other planets of the solar system on the planet in question induces an advance (a precession), orbit after orbit, of the perihelion (the point of maximum approach to the Sun of the orbit of the planet). The precession of the Earth's rotation axis also gives rise to the same effect. For example, Mercury's perihelion moves slightly at the speed of 5,600 arcseconds per century, in the same direction in which the planet rotates around the Sun. However, when the contribution of the Earth's precession is removed (5,025 arcseconds), that due to the attraction of the other planets, calculated according to Newtonian physics, is not able to correctly predict what happens in reality. The balance indeed misses 43 arcseconds. It is a general conviction, supported by centennial computations, that this deviation of Mercury's orbit from the observed precession cannot be achieved by Newtonian theory. This is the famous anomalous rate of precession of the perihelion of Mercury's orbit. It was originally recognized by the French Astronomer Urbain Le Verrier in 1859 as being an important astronomical problem [1]. Starting from 1843 [2], Le Verrier indeed reanalyzed various observations of the perihelion of Mercury's orbit from 1697 to 1848, by showing that the rate of the precession of the perihelion was not consistent with the previsions of Newtonian theory. This discrepancy by $38''$ arcseconds per tropical century, which has been corrected to $43''$ by the Canadian-American astronomer Simon Newcomb in 1882 [3], seemed till now impossible to be accounted through Newton's theory. Various ad hoc and unsuccessful solutions have been proposed, but such solutions introduced more problems instead. The most famous approach by 19th century astronomers was the attempting to explain this discrepancy through the perturbing effect of a planet, Vulcan, hitherto escaped observation, smaller than Mercury and closer than this to the Sun. However, the search for this planet turned out to be unfruitful. The solution of the problem is due to Albert Einstein through his magnificent general theory of relativity in 1916 [4]. Recent analyses due to the MESSENGER data plus the Cassini mission gave a value of about $42,98''$ to the general relativistic contribution to the precession of perihelion of Mercury per tropical century [5]. If one expresses the perihelion shift in radians per revolution (in this work, polar coordinates will be used), one gets instead the general relativistic value [6]

$$\triangle\varphi \simeq \frac{24\pi^3 a^2}{T_0^2 c^2 \left(1-e^2\right)}, \tag{1}$$

where a is the semi-major axis of the orbit, T_0 is Mercury's Newtonian orbital period, c is the speed of light, and e is the orbital eccentricity. Eq. (1) corresponds to a total angle swept per revolution by Mercury

$$\varphi \simeq \varphi_0 \left(1 + \frac{2\pi^2 a^2}{T_0^2 c^2 \left(1-e^2\right)}\right), \tag{2}$$

where $\varphi_0 = 2\pi$ is the unperturbed (i.e. in absence of precession) total angle swept by Mercury during a complete revolution around the Sun. Inserting the numerical values in Eq. (1), see for example [7–9], one gets the well known general relativistic value $\triangle\varphi \simeq 5.02 * 10^{-7}$ radians per revolution which corresponds to about $0,1$ arcseconds.

Now, the precession of the perihelion of Mercury's orbit is calculated in the Newtonian framework. Three different approaches will be considered.

2 Approximation of circular orbit

One starts from the case in which Mercury's mass is considered negligible with respect to the mass of the Sun, i.e. one considers the planet as being a test mass immersed in the Newtonian gravitational field of the Sun. In addition, one considers Mercury's orbit as being circular instead of elliptical. Thus, the case under consideration here is the simplest one. One takes the origin of the frame of reference in the center of the Sun. By using the traditional Newtonian equations, in order to obtain the orbital period, one merely equals the gravitational force to the centripetal one as

$$\frac{GMm}{r_0^2} = \frac{mv_0^2}{r_0}, \tag{3}$$

where G is the gravitational constant, M is the solar mass, m the mass of Mercury, r_0 the orbit's radius and v_0 the velocity of rotation of the planet. Hence, v_0 is easily obtained as

$$v_0 = \left(\frac{GM}{r_0}\right)^{\frac{1}{2}}. \tag{4}$$

Then, the Newtonian orbital period is

$$T_0 = \frac{2\pi r_0}{v_0} = \frac{2\pi r_0^{\frac{3}{2}}}{(GM)^{\frac{1}{2}}}. \tag{5}$$

The corresponding angular velocity is

$$\omega_0 = \frac{2\pi}{T_0}. \tag{6}$$

Thus, in radians per revolution the angular distance that Mercury sweeps during the Newtonian orbital period T_0 is

$$\varphi_0 = \omega_0 T_0 = 2\pi. \tag{7}$$

Now one asks: what does it happen if one removes the approximation to consider Mercury's mass negligible with respect to the solar mass? One argues that a Newtonian observer set in the center of the Sun must replace Eq. (3) with

$$\frac{G\,(M+m)\,m}{r_0^2} = \frac{mv^2}{r_0}, \tag{8}$$

i.e. one must replace M with $M+m$ in Eq. (3). Let us clarify this point. The Newtonian law of universal gravitation can be written down in its general form for Mercury and the Sun as

$$\overrightarrow{F}_G = \frac{GMm}{r^2}\hat{u}_r, \tag{9}$$

where r is the distance between the Sun and Mercury and $\hat{u}_r$ is the unit vector in the radial direction. Thus, for an external inertial Newtonian observer, the Newtonian equations of motion for the Sun and Mercury are

$$Ma_s\hat{u}_r = \frac{GMm}{r^2}\hat{u}_r \Longrightarrow a_s\hat{u}_r = \frac{Gm}{r^2}\hat{u}_r \tag{10}$$

and

$$ma_m\hat{u}_r = -\frac{GMm}{r^2}\hat{u}_r \Longrightarrow a_m\hat{u}_r = -\frac{GM}{r^2}\hat{u}_r, \tag{11}$$

respectively, where a_s is the acceleration of the Sun and a_m is the acceleration of Mercury. Thus, the relative acceleration of Mercury with respect to the Sun is

$$a\hat{u}_r \equiv a_m\hat{u}_r - a_s\hat{u}_r = -\left(\frac{GM}{r^2} + \frac{Gm}{r^2}\right)\hat{u}_r = -\frac{G(M+m)}{r^2}\hat{u}_r. \tag{12}$$

Then, the total force acting on Mercury as it is seen by a Newtonian observer set in the center of the Sun is

$$F\hat{u}_r = -\frac{G(M+m)\,m}{r^2}\hat{u}_r, \tag{13}$$

which immediately justify Eq. (8) for a circular motion. From Eq. (8) one gets immediately the perturbed velocity of rotation of the planet as

$$v = \left[\frac{G(M+m)}{r_0}\right]^{\frac{1}{2}} \tag{14}$$

corresponding to a period

$$T = \frac{2\pi r_0}{v} = \frac{2\pi r_0^{\frac{3}{2}}}{\left[G(M+m)\right]^{\frac{1}{2}}}. \tag{15}$$

But it is also

$$(M+m)^{-\frac{1}{2}} = M^{-\frac{1}{2}}\left(1+\frac{m}{M}\right)^{-\frac{1}{2}}, \tag{16}$$

which, inserted in Eq. (15), gives

$$T = \frac{2\pi r_0^{\frac{3}{2}}\left(1+\frac{m}{M}\right)^{-\frac{1}{2}}}{\left[G(M)\right]^{\frac{1}{2}}} = T_0\left(1+\frac{m}{M}\right)^{-\frac{1}{2}}. \tag{17}$$

Then, the corresponding perturbed angular velocity is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{T_0}\left(1+\frac{m}{M}\right)^{\frac{1}{2}} = \omega_0\left(1+\frac{m}{M}\right)^{\frac{1}{2}}. \tag{18}$$

Hence, the angle that Mercury sweeps during the period T_0 is

$$\varphi = \omega T_0 = 2\pi\left(1+\frac{m}{M}\right)^{\frac{1}{2}} \simeq 2\pi\left(1+\frac{m}{2M}\right), \tag{19}$$

in radians per revolution, where in the last step the first-order approximation in $\frac{m}{M}$ has been used, that is $\left(1+\frac{m}{M}\right)^{\frac{1}{2}} \simeq 1+\frac{m}{2M}$, because it is $m \ll M$. Therefore, in each complete revolution around the Sun, Mercury sweeps an angle larger than the unperturbed angle (7) and the difference, in radians per revolution, is

$$\triangle\varphi = \varphi - \varphi_0 \simeq \frac{\pi m}{M}. \tag{20}$$

The NASA official data give $m \simeq 3.3 * 10^{23} Kg$ [8] and $M = 1,99 * 10^{30} Kg$ [7]. Thus, one gets $\triangle\varphi \simeq 5.21 * 10^{-7}$radians per revolution which corresponds to about $0,107$ arcseconds. On the other hand, the Mercury/Earth ratio of the tropical orbit periods is 0.241[9]. Thus, one gets $44.39''$ per tropical century. This is a remarkable result that shows that, despite the above calculation has been made in the approximation of circular orbit, the correct value of the contribution of Newtonian theory to the precession of perihelion for Mercury per tropical century well approximates the value of about $42,98''$ per tropical century of general relativity [5] and the well known observational value of $43''$ per tropical century.

The physical interpretation of this nice result is that it is Mercury's back reaction, in terms of Newton's third law of motion (*to every action there is always opposed an equal reaction*), see Eqs. from (9) to (13), that generates the advance of the perihelion of Mercury in Newtonian framework.

3 Mercury's orbit as harmonic oscillator

Following [10], one recalls that each central attractive force can produce a circular orbit that should not necessarily be closed. It is closed if the radial oscillation period is a rational multiple of the orbit period. Now, let $F_c(r)$ be the total central force. Mercury's equation of motion in the radial direction is given by [10]

$$F_c(r) = m\left(\ddot{r} - \dot{\theta}^2 r\right), \tag{21}$$

where, again, r is the distance between the Sun and Mercury for an observer in the center of the Sun. The last term in Eq. (21) can be physically interpreted as a force centrifuge. Since the angular momentum J is a constant of motion, one has that

$$J = mr^2\dot{\theta}. \tag{22}$$

Solving for $\dot{\theta}$ and substituting in Eq. (21), one gets

$$F_c(r) = m\left(\ddot{r} - \frac{J^2}{m^2 r^3}\right). \tag{23}$$

In the case of a circular orbit of radius r_0, $\ddot{r} = 0$ and Eq. (23) reduces to

$$F_c(r_0) = -\frac{J^2}{m r_0^3}. \tag{24}$$

If Mercury is now slightly perturbed in the plane of its orbit and perpendicularly to its initial trajectory, it will oscillate around r_0 [10]. Then, one introduces $x = r - r_0$ and expresses the radial equation of motion in terms of x. Therefore [10]

$$\begin{aligned} F_c\left(x + r_0\right) &= m\ddot{x} - \tfrac{J^2}{m(x+r_0)^3} \\ &= m\ddot{x} - \tfrac{J^2}{m r_0^3\left(1+\frac{x}{r_0}\right)^3}. \end{aligned} \tag{25}$$

Since $\frac{x}{r_0} \ll 1$, one can use series expansion for the term in parentheses, considering only the first order terms in $\frac{x}{r_0}$. Expanding the member on the left in Taylor series around the point $r = r_0$ one gets [10]

$$F_c(r_0) + F_c'(r_0) = m\ddot{x} - \frac{J^2}{mr_0^3}\left(1 - \frac{3x}{r_0}\right). \tag{26}$$

Inserting Eq. (24) in Eq. (26) one obtains [10]

$$\ddot{x} + m^{-1}\left[-\frac{3F_c(r_0)}{r_0} - F_c'(r_0)\right]x = 0 \tag{27}$$

One notes that this equation describes a simple harmonic oscillator if the term in parentheses is positive [10]. If this term was negative, there would be an exponential solution and the orbit would not be stable [10]. Thus, for stable orbits, the period of oscillation around $r = r_0$ is [10]

$$T_0 = 2\pi\left(\frac{m}{-\frac{3F_c(r_0)}{r_0} - F_c'(r_0)}\right)^{\frac{1}{2}}. \tag{28}$$

One defines the apse angle $\frac{\varphi_0}{2}$ as the angle swept by the radial vector between two consecutive points of the orbit where the radial vector itself takes on an extremal value [10]. The time that Mercury needs to travel this angle is $\frac{T_0}{2}$. Since the orbit can be considered approximately circular and being therefore constant r and equal to r_0, one solves Eq. (22) for $\dot{\theta}$ and finds [10]

$$\frac{\varphi_0}{2} = \frac{T_0}{2}\dot{\theta} = \pi\left(\frac{m}{-\frac{3F_c(r_0)}{r_0} - F_c'(r_0)}\right)^{\frac{1}{2}}\frac{J}{mr_0^2}. \tag{29}$$

Furthermore, observing Eq. (24), one notes that the last term of Eq. (29) can be rewritten as [10]

$$\frac{J}{mr_0^2} = \left(-\frac{F_c(r_0)}{mr_0}\right)^{\frac{1}{2}}. \tag{30}$$

Then, one gets [10]

$$\varphi_0 = 2\pi\left[3 + \frac{F_c'(r_0)}{F_c(r_0)}\right]^{-\frac{1}{2}}, \tag{31}$$

and, by setting $F_c = F_G$ in Eq. (31), where F_G is the Newtonian gravitational force given by Eq. (9), one finds $\varphi_0 = 2\pi$, which is exactly Eq. (7).

But, again, in the computation in this Section Mercury's mass has been considered negligible with respect to the mass of the Sun. A good way to take into account the presence of Mercury's mass is to work in the framework of the two-body problem. The two-body problem studies the dynamics of a system consisting of two massive objects (the Sun having mass M and Mercury having mass m in the present case) subjected to a central force. Central force is defined as a force that only depends by the modulus of the difference of the vectors position of the two objects and which is directed along the junction of the two bodies. The expression of this kind of force is well known:

$$\overrightarrow{F} = F\left(|r_m - r_M|\right) \frac{\overrightarrow{r}_m - \overrightarrow{r}_M}{|\overrightarrow{r}_m - \overrightarrow{r}_M|}, \tag{32}$$

where r_m and r_M are the positions of the two objects of mass m and M respectively, that are subject to the central force of Eq. (32) in an inertial reference system. One introduces the variables relative position, r, and position of the center of mass, R. In this way, it is always possible to approach to the general two-body problem with two independent problems through the following change of variables:

$$\begin{aligned} \overrightarrow{R} &= \tfrac{m\overrightarrow{r}_m + M\overrightarrow{r}_M}{M+m} \\ \overrightarrow{r} &= \overrightarrow{r}_m - \overrightarrow{r}_M. \end{aligned} \tag{33}$$

With this change of variables the positions of Mercury and the Sun can be written as:

$$\begin{aligned} \overrightarrow{r}_m &= \overrightarrow{R} + \tfrac{M}{M+m}\overrightarrow{r} \\ \overrightarrow{r}_M &= \overrightarrow{R} - \tfrac{m}{M+m}\overrightarrow{r}. \end{aligned} \tag{34}$$

One also defines $M_T \equiv M + m$ and $\mu \equiv \frac{Mm}{M+m}$ as the total mass and the reduced mass of the system, respectively. It is well known that the problem of the dynamics of two bodies of masses m and M interacting through one force that depends only on mutual distance is reduced to the problem of a single body of reduced mass μ that moves in space under the action of a central field. In other words, in order to have a more precise description of the Sun-Mercury system one makes the replacement $m \to \mu$ in Eqs. from (21) to (31). In particular, Eqs. (29) and (30) now read

$$\frac{\varphi}{2} = \frac{T}{2}\dot{\theta} = \pi \left(\frac{\mu}{-\frac{3F_c(r_0)}{r_0} - F_c'\left(r_0\right)} \right)^{\frac{1}{2}} \frac{J}{\mu r_0^2}. \tag{35}$$

and

$$\frac{J}{\mu r_0^2} = \left(-\frac{F_c\left(r_0\right)}{\mu r_0} \right)^{\frac{1}{2}}, \tag{36}$$

respectively. To first order in $\frac{m}{M}$ the reduced mass can be rewritten as

$$\begin{aligned} \mu &= \left(\tfrac{M+m}{Mm}\right)^{-1} = \left(\tfrac{1}{m} + \tfrac{1}{M}\right)^{-1} \\ &= m\left(1 + \tfrac{m}{M}\right)^{-1} \simeq m\left(1 - \tfrac{m}{M}\right). \end{aligned} \tag{37}$$

Thus, Eq. (35) becomes

$$\begin{aligned}
\frac{\varphi}{2} = \frac{T}{2}\dot{\theta} &\simeq \pi\left(\frac{m\left(1-\frac{m}{M}\right)}{-\frac{3F_c(r_0)}{r_0}-F_c'(r_0)}\right)^{\frac{1}{2}}\frac{J}{m\left(1-\frac{m}{M}\right)r_0^2} \\
&\simeq \pi\left(1+\frac{m}{2M}\right)\left(\frac{m}{-\frac{3F_c(r_0)}{r_0}-F_c'(r_0)}\right)^{\frac{1}{2}}\frac{J}{mr_0^2}.
\end{aligned} \tag{38}$$

Then, one gets

$$\varphi = 2\pi\left(1+\frac{m}{2M}\right)\left[3+\frac{F_c'(r_0)}{F_c(r_0)}\right]^{-\frac{1}{2}}, \tag{39}$$

where now it is

$$\begin{aligned}
F_c &= \frac{GM_T\mu}{r^2} \\
&= \frac{G(M+m)\left(\frac{Mm}{M+m}\right)}{r^2} = F_G
\end{aligned} \tag{40}$$

and one finds

$$\varphi = 2\pi\left(1+\frac{m}{2M}\right), \tag{41}$$

which is the same result of Eq. (19).

4 Weak deviation from third Kepler's law

In order to work again in the framework of the two-body problem, one starts by replacing $m \to \mu$ in Eq. (22), obtaining

$$J = \mu r^2\dot{\theta} = 2\mu\dot{A}_0, \tag{42}$$

where A is the area swept by $\vec{r}$ during the orbital motion. Thus, one obtains

$$J = 2\mu\frac{dA}{dt} \quad and \quad dt = 2\mu\frac{dA}{J}. \tag{43}$$

Then, by integration over a period, one obtains

$$T = 2\mu\frac{A}{J}. \tag{44}$$

Recalling that the generic expression for the area of a conic is given by

$$A = \pi a^2\left(1-e\right)^{\frac{1}{2}}, \tag{45}$$

where a and e are the semi-major axis and the eccentricity of the ellipse, respectively, one substitutes for (44) and gets

$$T = 2\pi\mu\frac{a^2\left(1-e\right)^{\frac{1}{2}}}{J}. \tag{46}$$

Also remembering that it is

$$\frac{J^2}{\mu k} = a\,(1-e)\,, \tag{47}$$

one obtains

$$J = [\mu a k\,(1-e)]^{\frac{1}{2}}\,. \tag{48}$$

Then, by inserting Eq. (48) in Eq. (46) and by using a bit of algebra, one gets

$$T = 2\pi\left(\frac{a^3\mu}{k}\right)^{\frac{1}{2}}. \tag{49}$$

As it is $k = GMm$ for the gravitational system of Mercury and the Sun, Eq. (49) becomes

$$T = 2\pi\left(\frac{a^3}{GM_T}\right)^{\frac{1}{2}}, \tag{50}$$

where $M_T = M + m$ is the total mass of the system. Hence, from Eq. (50) one easily obtains

$$\begin{aligned} \frac{a^3}{T^2} &= \frac{GM_T}{4\pi^2} \\ &= \frac{G(M+m)}{4\pi^2} = \frac{GM}{4\pi^2}\left(1+\frac{m}{M}\right), \end{aligned} \tag{51}$$

and one immediately sees that Kepler's third law, that is "*the ratio between T^2 and a^3 is constant for each planet in the solar system, depending only on the mass of the Sun and not from that of the planet*", i.e.

$$\frac{a_0^3}{T_0^2} = \frac{GM}{4\pi^2}, \tag{52}$$

is strictly correct only in the approximation $m \ll M$, when the mass of the planet is considered negligible with respect to the solar mass. a_0 and T_0 in Eq. (52) are the unperturbed semi-major axis and the unperturbed period of revolution of the ellipse, respectively. Therefore, if one considers the mass of the planet as being not negligible with respect to the solar mass, Eq. (51) shows that there is a weak deviation from Kepler's third law in Newtonian gravitation. Combining Eqs. (52) and (51) one obtains

$$\begin{aligned} \frac{a^3}{T^2} &= \frac{a_0^3}{T_0^2}\left(1+\frac{m}{M}\right) \\ \Longrightarrow \frac{a^3}{a_0^3} &= \frac{T^2}{T_0^2}\left(1+\frac{m}{M}\right). \end{aligned} \tag{53}$$

On the other hand, if one wants that the variation of the angle merely makes the ellipse precess [11], that means that the shape and area of the ellipse remain unchanged during the advance of the perihelion, one must set $a = a_0$. Then, inserting this in Eq. (53) one immediately gets

$$T = \frac{T_0}{\left(1+\frac{m}{M}\right)^{\frac{1}{2}}}, \tag{54}$$

which is exactly the result of Eq. (17) that was obtained in Section 2 in the approximation of circular orbity. One also easily checks that Eq. (54) is consistent with Eq. (38) in Section 3 too. Thus, the corresponding perturbed angular velocity is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{T_0}\left(1+\frac{m}{M}\right)^{\frac{1}{2}} = \omega_0\left(1+\frac{m}{M}\right)^{\frac{1}{2}}. \tag{55}$$

Hence, the angle that Mercury sweeps during the period T_0 is

$$\varphi = \omega T_0 = 2\pi\left(1+\frac{m}{M}\right)^{\frac{1}{2}} \simeq 2\pi\left(1+\frac{m}{2M}\right), \tag{56}$$

in radians per revolution, where in the last step the first-order approximation in $\frac{m}{M}$ has been used exactly like in previous Sections. The result of Eq. (56) is the same as that of Eqs. (19) and (41), but the analysis in this Section is more precise because it has been performed in the framework of the two-body problem and considering the exact elliptical orbit of Mercury.

5 Conclusion remarks

It has been shown through three different approaches that, contrary to a long-standing conviction longer than 150 years, the orbit of Mercury behaves as required by Newton's equations with a very high precision if one correctly analyses the situation in the framework of the two-body problem without neglecting the mass of Mercury. The results obtained are remarkable. The real value predicted by Newtonian theory concerning the advance of the perihelion of Mercury is of $44.39''$ per tropical century that well approximates the value of about $42,98''$ per tropical century of general relativity and the well known observational value of $43''$ per tropical century. Thus, the real difference between Einstein's and Newton's previsions concerning the advance of the perihelion of Mercury is not of about $43''$ as astronomers and researchers were thinking for more than 100 years. Instead, such difference is only of $1.41''$ per tropical century. The physical interpretation of this remarkable result is that it is Mercury's back reaction, in terms of Newton's third law of motion, that generates the advance of the perihelion of Mercury in Newtonian framework. General relativity remains more precise than Newtonian theory regarding the precession of Mercury's perihelion, but the difference is very little. Another important point is that general relativity achieves a very precise value for the advance of the perihelion of Mercury considering the planet as being a test mass immersed in the general relativistic gravitational field of the Sun. Instead, in order to gain power of predictability, Newtonian theory must consider Mercury's mass as being not negligible. Thus, surely the results in this paper are not against the great power of predictability of Einstein's theory. They instead endorse the issue that Newtonian theory is more powerful than researchers and astronomers were thinking till now! One also recalls Einstein's opinion on Newton's reseach work [12]: *"Enough of this! Newton, forgive me; you found the only way which, in your age, was just about possible for a man of highest thought - and creative power. The concepts, which* *you created, are even today still guiding our thinking in physics, although we now know that they will have to be replaced by others farther removed from the sphere of immediate experience if we aim at a more profound understanding of relationships.*"

In order to finalize this paper, one argues that it is not too much surprising that Newtonian theory can give such a very precise value for the advance of the perihelion of Mercury. In fact, it is a well known issue that general relativistic corrections are strictly necessary in physics only in the cases of relativistic velocities and/or strong gravitational fields. But, on one hand, the velocity of Mercury in its revolution around the Sun is never relativistic. On the other hand, there is no presence of strong gravitational fields, because the distance between Mercury and the Sun is always much longer than the Sun's gravitational radius.

Thus, the present paper shows that Newtonian theory still has big surprises for scientists!

Aknowledgements

The Author thanks Domenico Annunziata for stimulating discussions concerning the possibility to solve the problem of the perihelion of Mercury's orbit is a Newtonian framework.

References

[1] U. Le Verrier, Comptes Rendus de l'Académie des sciences **49**, 379 (1859, in French).

[2] U. Le Verrier, Comptes Rendus de l'Académie des Sciences **16**, 1054 (1843, in French).

[3] S. Newcomb, United States. Nautical Almanac Office. Astronomical paper 1(6), 363 (1882).

[4] A. Einstein, Annalen der Physik **49(7)**, 769 (1916).

[5] R. S. Park et. al., The Astronomical Journal. **153 (3)**, 121 (2017).

[6] A. H. Dediu, L. Magdalena and C. Martìn-Vide, *Theory and Practice of Natural Computing: Fourth International Conference, TPNC 2015, Mieres, Spain, December 15-16, 2015. Proceedings* (2015).

[7] D. R. Williams, https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html (2018).

[8] D. R. Williams, https://nssdc.gsfc.nasa.gov/planetary/factsheet/ (2019).

[9] D. R. Williams, https://nssdc.gsfc.nasa.gov/planetary/factsheet/planet_table_ratio.html (2019).

[10] M. P. Price and W. F. Rush, American Journal of Physics **47(6)**, 531 (1979).

[11] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Feeman and Co., 1973).

[12] A. Einstein, *Autobiographical Notes*, Open Court; Centennial edition (1999).