Disruptive Gravity: gravitation as a quantizable spacetime bending force

Ramsès BOUNKEU SAFO

CentraleSupelec, 8-10 rue Joliot-Curie, 91190 Gif-sur-Yvette, FRANCE

e-mail: ramses.bounkeusafo@centralsupelec.fr

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Abstract

Gravity is the most problematic interaction of modern science. Our current understanding of gravitation as a spacetime curvature needs the introduction of both Dark Matter and Dark Energy accounting for 95% of the energy of the universe. Questioning the very foundations of gravity might be the key to understanding it better since its description changed over time. Newton described it as a force, Einstein described it as a spacetime curvature and this paper shows how gravity can be described as a force able to bend spacetime instead. Based on a physical interpretation of the Schwarzschild metric, this approach yields the same predictions as General Relativity such as Mercury's Perihelion Precession, Light Bending, Time Dilation and Gravitational Waves as well as predicted testable deviations from General Relativity. Applying this description of gravity to cosmology accounts for the accelerating expanding universe with no need for Dark Energy. Described as a spacetime bending force, gravity becomes quantizable as a force in a curved spacetime which is compatible with the Standard Model of particle physics. Therefore, one could adapt the Lagrangian of the Standard Model to this theory to achieve Quantum Gravity.

INTRODUCTION

Understanding gravity is one of the most important challenge of modern science. For a long time, General Relativity had no reason to be questioned since it was in line with the observations. That was until the observation of an unexpectedly high rotation speed of galaxies and then, more recently, the discovery of the accelerating expanding universe through observations of distant supernovae. Both are not explainable through General Relativity unless we hypothesize the existence of Dark Matter and Dark Energy respectively, accounting for 95% of the energy of the universe. Current research focuses on creating models to describe Dark Matter and Dark Energy instead of seriously questioning General Relativity. Thinking differently about gravitation might be the key to understanding it better.

Newton thought of it as a force, then Einstein theorized it as a spacetime curvature, but what if gravity could be described as a force able to bend spacetime instead? This paper shows that gravity can be consistently described as a spacetime bending force based on a physical principle inferred from the Schwarzschild metric. We show that, writing the Lagrangian of a force in a curved spacetime, we get equivalent equations of motion as General Relativity thanks to a physically acceptable hypothesis. From a simple homogeneous universe model, we then show that it is possible to explain the accelerating expanding universe with no Dark Energy. As a spacetime bending force, gravity becomes quantizable as a force in a curved spacetime analogous to electromagnetism.

In this paper, Greek letters range from 0 to 3 (representing spacetime) and Roman letters range from 1 to 3 (representing space). The metric signature is $(+ - --)$ and we use Einstein's summation convention. The Greek capital letter Φ is the gravitational potential.

I - A Spacetime Bending Force

Einstein's General Relativity states that a body moving through gravity is just following a straight path in a curved spacetime. This is described by the geodesic equations derived from a least action principle, with the following Lagrangian:

$$L_0 = -m_0 c \sqrt{g_{\mu\nu} \dot{x_\mu} \dot{x_\nu}}$$

where $g_{\mu\nu}$ is the metric of the curved spacetime and m_0 is the rest mass of the body. If gravity were a force, in a scalar theory, the Lagrangian would be of the form:

$$L_0' = -m_0 c \sqrt{\eta_{\mu\nu} \dot{x_\mu} \dot{x_\nu}} - m_0 \Phi$$

where $\eta_{\mu\nu}$ is Minkowsky's metric of a flat spacetime and Φ is the gravitational potential. We know this Lagrangian is not correct since it would lead to incorrect geodesic equations. So how could we get to the same geodesic equations as General Relativity taking into account spacetime curvature and a potential term? In a scalar theory, the Lagrangian of gravity as a force in a curved spacetime would be of the form:

$$L = -mc\sqrt{g_{\mu\nu}\dot{x_\mu}\dot{x_\nu}} - m\Phi$$

where m is the inertial mass. As such, we still wouldn't get the same geodesic equations as General Relativity. Is it possible to slightly change it in a physically acceptable way so it becomes equivalent to General Relativity's Lagrangian? Speed of light cannot be modified since Special Relativity laws wouldn't apply anymore. The only thing that could be changed is the inertial mass of the body. Let's then hypothesize that the inertial mass is relative such that:

$$m = \alpha(\Phi)m_0$$

where the rest mass m_0 is defined as the inertial mass in case of zero potential. So we have: $\alpha(0) = 1$. Inertial mass relativity is physically acceptable since we already consider that the relativistic mass of a body is relative depending on its speed.

The Lagrangian becomes:

$$\boxed{L = -\alpha(\Phi)m_0c\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} - \alpha(\Phi)m_0\Phi} \quad (1)$$

For more clarity, let's also write: $\dot{s_0} = \sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$

We then have: $L = -\alpha(\Phi)m_0c\dot{s_0} - \alpha(\Phi)m_0\Phi$ [i]

The Lagrangian equation restricted to space variables is:

$$\frac{\partial L}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}^i} = 0 \quad \text{[ii]}$$

Since Φ doesn't depend explicitly on $\dot{x}^i$, we have:

$$-\frac{\partial \alpha(\Phi)m_0c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi)m_0\Phi}{\partial x^i} + \frac{d}{d\tau}\frac{\partial \alpha(\Phi)m_0c\dot{s_0}}{\partial \dot{x}^i} = 0 \quad \text{[iii]}$$

Leading to: $-\frac{\partial \alpha(\Phi)c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^i} + \frac{d}{d\tau}(\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial \dot{x^i}}) = 0$ [iv]

It comes:

$$-\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi)}{\partial x^i}c\dot{s_0} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} + \alpha(\Phi)\frac{d}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} = 0 \quad \text{[v]}$$

We see the Lagrangian equation of General Relativity in the first and last terms of the equation [v]. Let $L_0 = -m_0c\dot{s_0}$, it comes:

$$-\frac{\partial\alpha(\Phi)}{\partial x^i}c\dot{s_0} - \frac{\partial\alpha(\Phi)\Phi}{\partial x^i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial\dot{x}^i} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial\dot{x}^i})/m_0 = 0 \qquad \text{[vi]}$$

Parametrizing with the body's proper time, we have: $\dot{s_0} = c$. Thus:

$$-\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial\dot{x}^i} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial\dot{x}^i})/m_0 = 0 \qquad \text{[vii]}$$

Notations can be misleading. We cannot replace $\dot{s_0}$ by c in the expression $\frac{\partial c\dot{s_0}}{\partial\dot{x}^i}$ since it's a partial derivative. We have in fact:

$$\frac{\partial c\dot{s_0}}{\partial\dot{x}^i} = c\cdot\frac{\partial\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}}{\partial\dot{x}^i} = c\cdot\frac{2\cdot g_{\mu i}\dot{x}^\mu}{2\cdot\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}} = c\cdot\frac{2\cdot\dot{x}_i}{2\cdot c} = \dot{x}_i \qquad \text{[viii]}$$

Hence: $\frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial\dot{x}^i} = \frac{\partial\alpha(\Phi)}{\partial\Phi}\cdot\frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\cdot\dot{x}_i$ [ix]

And calculating $(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial\dot{x}^i})/m_0$ gives a known standard result of General Relativity [3][4][5][6]:

$$(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial\dot{x}^i})/m_0 = g_{\mu i}\ddot{x}^\mu + 1/2\cdot(-\partial^i g_{\mu\nu} + \partial^\mu g_{\nu i} + \partial^\nu g_{\mu i})\dot{x}^\mu\dot{x}^\nu \qquad \text{[x]}$$

Thus, after multiplying [vii] by g^{ik} (which is the inverse of the restriction of the metric to space), defining Christoffel symbols as:

$$\Gamma^k_{\mu\nu} = g^{ik}/2\cdot(-\partial^i g_{\mu\nu} + \partial^\mu g_{\nu i} + \partial^\nu g_{\mu i})\dot{x}^\mu\dot{x}^\nu$$

(as said in the introduction, Roman letters span from 1 to 3 whereas Greek letters span from 0 to 3) we get:

$$-g^{ik}\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} + \frac{\partial\alpha(\Phi)}{\partial\Phi}\frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^k + \alpha(\Phi)(\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu\dot{x}^\nu) = 0 \qquad \text{[xi]}$$

We see that, for it to give correct equations of motion in the Newtonian limit, we necessarily have:

$$\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} = 0 \qquad \text{[xii]}$$

It yields: $\alpha(\Phi) = (1 + \Phi/c^2)^{-1}$ [xiii]

Then: $\dfrac{\partial \alpha(\Phi)}{\partial \Phi} = -1/c^2 \cdot (1+\Phi/c^2)^{-2}$ [xiv]

Hence, recasting in [xi] we get:

$$-(1+\Phi/c^2)^{-2} \cdot \frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^k/c^2 + (1+\Phi/c^2)^{-1}(\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu \dot{x}^\nu) = 0 \qquad \text{[xv]}$$

After neglecting second order terms, it yields:

$$\boxed{\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^k/c^2} \ (2) \qquad \text{[xvi]}$$

These equations of motion look like the geodesic equations of General Relativity. For weak-fields and low speeds, we trivially get the Newtonian limit.

Hence, if the inertial mass is relative such that:

$$\boxed{m = (1+\Phi/c^2)^{-1} m_0} \ (3)$$

gravity described as a spacetime bending force instead of a spacetime curvature yields similar results. The small deviation from General Relativity induced by $\partial^\mu \Phi \dot{x}^\mu \dot{x}^k/c^2$ makes this theory testable.

For more clarity, let's write: $m_0 \dfrac{\partial \Phi}{\partial x^\mu}\dot{x}^\mu \dot{x}^k/c^2 = (-\vec{F}\cdot\vec{v})\cdot\vec{v}/c^2$

where $\vec{F}$ is the gravitational force and $\vec{v}$ the speed of the body. We can interpret it as an anomalous thrust unexpected from General Relativity. In the case of Mercury, its speed around the Sun is $v = 47km/s$ so $v^2/c^2 = 2.5 \cdot 10^{-8}$ that makes it neglectable and hard to detect. Such an anomaly is expected to be measurable in the recently launched Parker Solar Probe if solar wind and radiation pressure can be neglected so close to the Sun. That would be a test of this theory.

In case of an orbital motion, we see that for a circular trajectory, this force is null. Thus, it can be neglected for low eccentricities yielding the same predictions of orbit precession as General Relativity, especially Mercury's perihelion precession.

However, the reader could demonstrate that the influence of this force over a revolution period is a resulting force parallel to the great axis and directed towards the aphelion of the trajectory that increases in magnitude with the eccentricity. Thus, it contributes to increasing the eccentricity of the trajectory over time. That might be the main reason why Mercury's eccentricity is high compared to other planets although tidal circularization would tend to make it null.

In this section, we described gravity with a scalar theory as an introduction. We need to extend it to a vectorial theory that would make it a special case. That is the aim of Section II and III.

II - Modifying Gravitoelectromagnetism

Describing gravity as a spacetime bending force yields equivalent equations of motion as General Relativity but has to account for predictions such as: Time Dilation, Light Bending, Shapiro Delay, Lens-Thirring and geodetic effects.

We know the Lens-Thirring and the geodetic effects are both well described by Gravitoelectromagnetism [2] which is a theory of gravity in a flat spacetime analogous to Maxwell's theory of electromagnetism. So including spacetime curvature in Gravitoelectromagnetism would still make those predictions.

Analogous to electromagnetism in General Relativity, we can consider gravity as some kind of gravitoelectromagnetism in a curved spacetime and see if it makes the same predictions as General Relativity. The Lagrangian of an electrically charged body in General Relativity is:

$$L = -mc\sqrt{g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} - q\dot{x}^{\mu}A_{\mu}$$

where A_μ is the electromagnetic four-vector potential and q the electric charge of the body. The idea is to consider a gravitational four-vector potential G_μ analogous to the electromagnetic four-vector potential A_μ and consider the following Lagrangian:

$$L = -m_{inertial}c\sqrt{g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} - m_{gravitational}\dot{x}^{\mu}G_{\mu}$$

where $m_{inertial}$ is the inertial mass of the body and $m_{gravitational}$ is its gravitational mass. For some reason that will become clear in Section III, we define the gravitational mass as:

$$m_{gravitational} = \gamma^{-1}m_{inertial}$$

where γ is defined as $\gamma^{-1} = \sqrt{g_{\mu\nu}\frac{dx^{\mu}}{dx^{0}}\frac{dx^{\nu}}{dx^{0}}}$ similar to Lorentz factor.

Same as Section I, we hypothesize that the inertial mass is relative such that:

$m_{inertial} = \alpha(\Phi) m_0$

where m_0 is the rest mass, defined as the inertial mass if the gravitational potential is null: $\alpha(0) = 1$. The Lagrangian becomes:

$$\boxed{L = -\alpha(\Phi) m_0 c \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} - \gamma^{-1} \alpha(\Phi) m_0 \dot{x}^\mu G_\mu} \quad (4)$$

How the Gravitational four-vector potential G_μ is calculated is not of relevance in this paper since gravity is not postulated to be Newtonian. It should then be subject to further studies. It depends on the type of gravitational potential. If Newtonian, it would be the exact analogous of electromagnetism in curved spacetime as we would just have to replace ϵ_0 by $-1/4\pi\mathcal{G}$ where $\mathcal{G}$ is Newton's constant.

In electromagnetism, the four-vector potential is of the form $A_\mu = (V/c, \vec{A})$ where V is the electrical potential and $\vec{A}$ is the potential vector . Even though G_μ remains to be calculated depending on the gravitational potential theory used (not necessarily Newtonian), we know that, analogously to electromagnetism it is of the form $G_\mu = (\Phi/c, \vec{G})$ where Φ is the gravitational potential.

In electromagnetism, the magnetic field is derived as the curl of $\vec{A}$. Analogously, defining the gravitational tensor as:

$$F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu = \begin{pmatrix} 0 & -\frac{1}{c}E^x_{\mathcal{G}} & -\frac{1}{c}E^y_{\mathcal{G}} & -\frac{1}{c}E^z_{\mathcal{G}} \\ \frac{1}{c}E^x_{\mathcal{G}} & 0 & B^z_{\mathcal{G}} & -B^y_{\mathcal{G}} \\ \frac{1}{c}E^y_{\mathcal{G}} & -B^z_{\mathcal{G}} & 0 & B^x_{\mathcal{G}} \\ \frac{1}{c}E^z_{\mathcal{G}} & B^y_{\mathcal{G}} & -B^x_{\mathcal{G}} & 0 \end{pmatrix}$$

provides a good description of the Lens-Thirring and the geodetic effects.

Another prediction of General Relativity is Gravitational Waves. It is not mentioned in the tests because it is in fact due to a gauge choice. Whereas viewing gravity as a spacetime bending force, gravitational waves would not be due to a gauge choice since G_μ is Lorentzian by definition. Indeed, Lorentz gauge induces a wave equation of the potential.

III - First Order Non-Relativistic Dynamics

As we said in the previous section, we consider the following Lagrangian:

$$L = -\alpha(\Phi) m_0 c \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} - \gamma^{-1} \alpha(\Phi) m_0 \dot{x}^\mu G_\mu \quad [i]$$

Let's demonstrate that this Lagrangian yields the special case scalar theory of Section I when the Lens-Thirring and the geodetic effects can be neglected in case of non-relativistic speeds and in weak-fields.

Let's first simplify the Lagrangian by neglecting second order terms. If the Lens-Thirring and the geodetic effects can be neglected, then cross-terms between space and time can be neglected. Parametrizing with the body's proper time, we have $c^2 = g_{00}(\dot{x}^0)^2 + g_{ij}\dot{x}^i\dot{x}^j$ which yields for non-relativistic fields:

$$\dot{x}^0 \cdot \sqrt{g_{00}} = c \cdot (1 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2) \qquad \text{[ii]}$$

Similarly, with $\gamma^{-1} = \sqrt{g_{\mu\nu}\frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0}}$, we have:

$$\gamma^{-1}/\sqrt{g_{00}} = \sqrt{g_{\mu\nu}/g_{00} \cdot \frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0}} = \sqrt{1 + g_{ij}/g_{00} \cdot \frac{dx^i}{dx^0}\frac{dx^j}{dx^0}} \qquad \text{[iii]}$$

Since $\frac{dx^0}{d\tau} = \dot{x}^0$ and for non-relativistic speeds $\dot{x}^0 \approx c$, neglecting second order terms it comes:

$$\gamma^{-1}/\sqrt{g_{00}} = \sqrt{1 + g_{ij}/g_{00} \cdot \dot{x}^i\dot{x}^j/(\dot{x}^0)^2} = 1 + 1/2 \cdot g_{ij}/g_{00} \cdot \dot{x}^i\dot{x}^j/c^2 \qquad \text{[iv]}$$

Since in weak-fields $1/g_{00} \approx 1 - 2\Phi/c^2$, neglecting second order terms yields:

$$\gamma^{-1}/\sqrt{g_{00}} = (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2) \qquad \text{[v]}$$

Multiplying [ii] and [v] we get:

$$\gamma^{-1}\dot{x}^0 = c \cdot (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - (1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2)^2) \qquad \text{[vi]}$$

Neglecting second order terms again it comes: $\gamma^{-1}\dot{x}^0 = c$ [vii]

The Lens-Thirring and the geodetic effects being neglected, we also have $G_0 = \Phi/c$ and $G_i = 0$ we get:

$$\dot{x}^\mu G_\mu = \dot{x}^0 G_0 = \dot{x}^0\Phi/c \qquad \text{[viii]}$$

Recasting [vii] yields: $\gamma^{-1}\dot{x}^\mu G_\mu = \gamma^{-1}\dot{x}^0\Phi/c = \Phi$ [ix]

Introducing Lorentz factor in the definition of the gravitational mass is convenient as it suppresses perturbative terms. Its physical meaning is quite intuitive though: the faster a body, the more massive it gets in term of relativistic mass, and the less the influence of a force on it. Taking this into account implies the introduction of Lorentz factor in the definition of the gravitational mass.

The Lagrangian [i] becomes:

$$L = -\alpha(\Phi)m_0 c\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} - \alpha(\Phi)m_0\Phi \qquad \text{[x]}$$

Which is the special case already studied in Section I.

The Lagrangian's variables are x^μ and $\dot{x}^\mu$ but parametrizing with the body's proper time, we have $c^2 = g_{00}(\dot{x}^0)^2 + g_{ij}\dot{x}^i\dot{x}^j$ which shows that the variables are not independent. We then have to choose a set of independent variables. Since space and time are disjoint by hypothesis, it is really convenient to choose x^i and $\dot{x}^i$ as a set of independent variables. This is why we restricted the Lagrangian equation to space variables in Section I.

This way we now have a vectorial theory of gravitation that yields equivalent equations of motion as General Relativity. We are then left with finding a way to derive the metric. Let's first have a look at the physical implication of inertial mass relativity.

IV - Physical Implications

The hypothesis of inertial mass relativity yields equivalent results as General Relativity in weak fields and non-relativistic speeds. This hypothesis has physical implications and interpretations as we will see.

Mathematically, a natural physical interpretation arises. Indeed, we can give a physical meaning to $E_\Phi = m_{inertial}c^2$ thanks to inertial mass relativity:

$$E_\Phi = m_0 c^2/(1 + \Phi/c^2)$$

Generalized to a relativistic body, we have:

$E_\Phi = \gamma mc^2/(1 + \Phi/c^2)$ where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is Lorentz factor [7].

Let's rewrite it as: $E_\Phi = \sqrt{m_0^2c^4 + p_0^2c^2}(1 + \Phi/c^2)$

Or rather, for brevity : $\boxed{E_\Phi = E_0/(1 + \Phi/c^2)}$ (5)

Applied to photons of energy $E_0 = h\nu_0$, with $E_\Phi = h\nu_\Phi$ we have:

$\nu_\Phi = \nu_0/(1 + \Phi/c^2)$

That looks a lot like General Relativity's formula of gravitational redshift. Thus we define E_Φ as the Apparent Energy of the body.

Writing it as $E_\Phi = E_0/\sqrt{g_{00}}$, it's as if the energy of a body could be redshifted. It's as if a body was also a wave which we know accurate since De Broglie's hypothesis of wave-particle duality.

Apparent Energy is nothing new. When a wave is Doppler-shifted for a moving observer, the shifted frequency is said to be apparent frequency. Analogously, the energy of a photon for a moving observer doesn't change, but since its frequency is Doppler-shifted, the change in energy is in fact Apparent Energy.

This physical meaning implies the time dilation factor be: $g_{00} = (1 + \Phi/c^2)^2$

This provides another testable deviation from General Relativity. Indeed in General Relativity we have:

$g_{00,schwarzschild} = 1 + 2\Phi/c^2$

The second order difference is $(\Phi/c^2)^2$. It's really small but measurable and can be tested.

V - Physical meaning of the Schwarzschild metric

Most tests of General Relativity are based on the Schwarzschild metric [1] below. Let's see if we can give a physical meaning to it.

$ds^2 = (1 + 2\Phi/c^2)c^2dt^2 - (1 + 2\Phi/c^2)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)$

First let's consider the following equivalent metric in weak-fields:

$ds^2 = (1 + \Phi/c^2)^2c^2dt^2 - (1 + \Phi/c^2)^{-2}dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)$

Space and time being disjoint, we can define the space metric:

$ds^2_{Space} = (1 + \Phi/c^2)^{-2}dr^2 + r^2(d\theta^2 + sin^2\theta d\psi^2)$

The volume element of a Riemannian manifold is the square root of the determinant of the metric in absolute value times the coordinate elements. For the Schwarzschild space metric it yields:

$$dV = \sqrt{(1 + \Phi/c^2)^{-2} \cdot r^2 \cdot r^2 sin^2\theta} \cdot drd\theta d\psi = (1 + \Phi/c^2)^{-1} \cdot r^2 |sin\theta| drd\theta d\psi$$

It comes:

$$(1 + \Phi/c^2) \cdot dV = r^2 |sin\theta| drd\theta d\psi$$

This doesn't depend on Φ which is an invariance principle. Let's multiply by ρc^2 where ρ is a hypothetical mass density of vacuum, we get:

$$(\rho c^2 + \rho\Phi) \cdot dV = \rho c^2 \cdot r^2 |sin\theta| drd\theta d\psi$$

In other words, analogous to the invariance of the speed of light, we have the following principle:

"The energy of vacuum is invariant".

It seems like the same way speed of light invariance induces time dilation, vacuum energy invariance induces space dilation. Just as the Strong Equivalence principle is a postulate of General Relativity, Vacuum Energy Invariance (VEI) can be taken as a postulate. We will see that it yields the Schwarzschild metric in weak-fields and therefore provides the same predictions as General Relativity.

In Section VI and VII, we derive the metric thanks to this principle.

VI - Metric Derivation (part I)

We showed in Section I, II and III that gravity can be coherently described as a spacetime bending force if the inertial mass is relative. We are left with how the metric can be derived such that the Schwarzschild metric is a particular case.

We naturally postulate that the metric $g_{\mu\nu}$ is of the form:

$$g = \begin{pmatrix} g_{00}(\Phi) & 0 \\ 0 & -g_s(\Phi) \end{pmatrix}$$

Indeed, in General Relativity, cross terms between space and time are responsible for the Lens-Thirring and the geodetic effects but since these are already accounted for by considering gravity as spacetime bending force, we can postulate that space and time curvature are disjoint.

We then consider that space and time are independently dilated by VEI.

Let's derive both $det(g_s)$ and g_{00} thanks to VEI principle.

At a given point in time t, in a volume element $dx_1dx_2dx_3$, under zero gravity (flat space) with vacuum energy density $\mathcal{E}_0$, we have:

$$dE_0 = \mathcal{E}_0 dx_1dx_2dx_3$$

and under Φ-gravity potential, we have:

$$dE_\Phi = \mathcal{E}_0(1+\Phi/c^2)\sqrt{det(g_s)}dx_1dx_2dx_3$$

Applying VEI, we have: $dE_0 = dE_\Phi$.

It comes:

$$\boxed{det(g_s) = (1+\Phi/c^2)^{-2}} \quad (6)$$

Let's apply VEI in time domain to have a more rigorous way to find g_{00}.

The reasoning is a bit similar to the one for the derivation of the gravitational redshift. We reason in terms of observational events.

Let E_0 be the total vacuum energy and N be the number of observational events.

The total vacuum energy by time unit for an observer under a global 0-potential is:

$$P_0 = \frac{d(NE_0)}{dt}$$

The total vacuum energy by time unit for the same observer under a global Φ-potential is:

$$P_\Phi = \frac{d(NE_0(1+\Phi/c^2))}{d\tau}$$

Applying VEI, we have: $P_0 = P_\Phi$

It comes: $E_0dNd\tau = E_0(1+\Phi/c^2)dNdt$

With $d\tau^2 = g_{00}dt^2$ it eventually comes:

$$\boxed{g_{00} = (1 + \Phi/c^2)^2} \quad (7)$$

The equation of motion [xvii] of Section I, for non-relativistic speeds becomes:

$$\ddot{x}^k + \Gamma^k_{00}\dot{x}^0\dot{x}^0 = 0$$

In weak-fields, standard result of linearized General Relativity yields :

$$\ddot{x}^k = -1/2 \cdot \partial_k h_{00} c^2$$

where $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is the perturbation of the metric.

From VEI we have $h_{00} = 2\Phi/c^2$ which yields Newton's law [8].

VII - Metric Derivation (part 2)

We still don't fully know g_s. Any g_s formula predicting a correct Light Deflection and reproducing the Schwarzschild metric for the Sun's mass distribution works to account for every experimental tests.

Considering gravity as a spacetime bending force would give us a space metric g_s different from General Relativity. It doesn't change anything to the Newtonian limit since in that case only g_{00} is relevant for the equations of motion. The idea is to aggregate the contributions of every mass of the distribution to the space deformation. In case of a compact spherical distribution, far from the sphere, space dilation would be purely radial just as in the Schwarzschild metric whereas it wouldn't be the case close to the mass distribution. A non-radial space dilation is a testable prediction of this theory.

Space deformations induced by a single punctual mass must be radial for trivial physical reasons. Then in a local orthonormal basis $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ where $\vec{e_r}$ is radial, space metric is $-g_{s,ruv}$ of the form:

$$g_{s,ruv} = \begin{pmatrix} \beta^{-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I + (\beta^{-2} - 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Applying VEI yields: $\beta = 1 + \Phi/c^2$.

Let M^T be the change of basis orthonormal matrix from $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ to $(\vec{e_1}, \vec{e_2}, \vec{e_3})$. So with $\vec{e_r} = r_i\vec{e_i}$, $\vec{e_u} = u_i\vec{e_i}$ and $\vec{e_v} = v_i\vec{e_i}$, changing coordinates we have:

$$g_s = M^T g_{s,ruv} M \text{ with } M^T = \begin{pmatrix} r_1 & u_1 & v_1 \\ r_2 & u_2 & v_2 \\ r_3 & u_3 & v_3 \end{pmatrix}$$

Since $M^T M = I$, it comes: $g_s = I + (\beta^{-2} - 1) M^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} M$

Eventually:

$$g_s = I + (\beta^{-2} - 1) \begin{pmatrix} r_1^2 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2^2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3^2 \end{pmatrix} \text{ or } g_{s,ij} = \delta_{ij} + (\beta^{-2} - 1) r_i r_j$$

In weak-fields, this is equivalent to the Schwarzschild metric written in Cartesian coordinates. This doesn't depend on the choice of $\vec{e_u}$ and $\vec{e_v}$. For a mass distribution, the unit vector pointing from a massive point towards a local point in space is the same as the radial vector $\vec{e_r}$ so we can aggregate their influence thanks to the above formula.

Indeed, for an infinitely small potential $d\Phi$, we have $\beta^{-2} - 1 = -2d\Phi/c^2$ and the metric becomes, when integrating over every infinitely small potential:

$$g_{s,ij} = \delta_{ij} + \lambda \cdot \textstyle\int -2 r_i r_j d\Phi/c^2 \text{ with } \lambda \text{ such that } det(g_s) = (1 + \Phi/c^2)^{-2}$$

Space being curved there might not be a unique choice of r_i. Therefore we introduce the potential angular distribution $\phi(\vec{\sigma})$, where $\vec{\sigma}$ is the observed direction. Leading to the following metric equation:

$$\boxed{g_{s,ij} = \delta_{ij} + \lambda \cdot \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma} \quad (8)$$

With: $$\boxed{B_{ij} = \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma} \quad (9)$$

We have: $g_{s,ij} = \delta_{ij} + \lambda B_{ij}$

In fact, for any 3x3 matricial function f such that $f(P^{-1}MP) = P^{-1} f(M) P$ and $f(M) = I + M$ if M is small, $g_s = f(\lambda B)$ would also be valid. For physical reasons, rather than summing the infinitely small perturbations, we should multiply the metrics induced by each infinitely small perturbations. That would yield:

$$\boxed{g_s = e^{\lambda B}} \quad (10)$$

Deriving λ is then straightforward since B being symmetric, it is diagonal in a certain basis, and $e^{\lambda B}$ would be a diagonal matrix in such a basis whose determinant is the exponential of the sum of its eigenvalues. The sum of the eigenvalues being the trace of λB, we have:

$$det(e^{\lambda B}) = e^{Tr(\lambda B)}$$

Applying VEI principle we then have : $e^{\lambda Tr(B)} = (1 + \Phi/c^2)^{-2}$

Hence : $\boxed{\lambda = -2 \cdot ln(1 + \Phi/c^2)/Tr(B)}$ (11)

So in the weak-fields limit we have: $\boxed{g_{s,ij} = \delta_{ij} - 2\Phi/c^2 \cdot B_{ij}/B_{kk}}$ (12)

Applying it to a punctual mass, space deformation being radial, in spherical coordinates we trivially obtain a modified Schwarzschild metric:

$$\boxed{ds^2 = (1 + \Phi/c^2)^2 c^2 dt^2 - (1 + \Phi/c^2)^{-2} dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)} \quad (13)$$

So this predicts Mercury's Perihelion Precession and Light Deflection by the Sun since its mass is concentrated in its core. But in case of a homogenous spherical mass distribution like the Earth, the radial dilation would be smaller than the one predicted by the Schwarzschild metric because the deformation is fairly distributed according to the influence of every part of the mass distribution, inducing an azimuthal space dilation not predicted by General Relativity. This could be measured through interferometry and provides another test.

VIII - Summary

Gravity as a spacetime bending force can be summarized by the following equations:

$$\Phi_0 = \Phi$$

$$L = -\alpha m_0 c \sqrt{g_{\mu\nu} \dot{x^\mu} \dot{x^\nu}} - \gamma^{-1} \alpha m_0 \dot{x^\mu} G_\mu$$

$$\alpha = (1 + \Phi_0/c^2)^{-1}$$

$$\gamma^{-1} = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dx^0} \frac{dx^\nu}{dx^0}}$$

$$g = \begin{pmatrix} (1+\Phi_0/c^2)^2 & 0 \\ 0 & -e^{\lambda B} \end{pmatrix}$$

$$B_{ij} = \int -2\phi_0(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma})r_j(\vec{\sigma})d\sigma$$

$$\lambda = -2 \cdot ln(1+\Phi/c^2)/Tr(B)$$

This can be easily adapted to any violation of the Weak Equivalence principle by separating vacuum gravitational potential from the bodies' gravitational potential: $\Phi_0 \neq \Phi$

IX - Universe Expansion

The Cosmological Redshift can as well be interpreted as due to an expanding universe if we postulate that the universe is homogeneous and isotropic and has a beginning. Indeed, if gravity is a force, gravitational potential propagates at the speed of light. The older the universe, the more propagated the gravitational potential and the greater space dilation would be.

Let's see how global vacuum gravitational potential evolves in a homogeneous and isotropic universe from its creation. The potential is induced by the mass in a cT radius sphere where T is the age of the universe. The gravitational potential is:

$$\Phi = \int_0^{cT} \phi(r)\rho \cdot 4\pi r^2 dr$$

Taking space dilation into account and conservation of matter, we have:

$$\rho = \rho_0 \cdot (1+\Phi/c^2)^{-1}$$

And with the variable change $t = r/c$ we have:

$$\Phi = 4\pi\rho_0 c^3 \cdot \int_0^T \phi(ct)(1+\Phi/c^2)^{-1} t^2 dt$$

Hence the following gravitational potential differential equation:

$$d\Phi/dT = 4\pi\rho_0 c^3 \cdot \phi(cT)(1+\Phi/c^2)^{-1} T^2$$

Separating variables, we get:

$$\Phi + \Phi^2/2c^2 = 4\pi\rho_0 c^3 \cdot \int_0^T \phi(ct)t^2 dt$$

Hence the solution:

$$\boxed{1 + \Phi/c^2 = \sqrt{1 + 8c\pi\rho_0 \cdot \int_0^T \phi(ct)t^2 dt}} \quad (14)$$

The age T is the time elapsed from the point of view of an observer in a null gravitational potential, as if he was shielded from gravity.

Since the universe is homogeneous, VEI implies that the scale factor is $a = (1 + \Phi/c^2)^{-1/3}$ so recasting the solution yields:

$$\boxed{a(T) = (1 + 8c\pi\rho_0 \cdot \int_0^T \phi(ct)t^2 dt)^{-1/6}} \quad (15)$$

To be able to compare this model with Friedmann-Lemaitre-Robertson-Walker models, we need to express the dilation factor with a time equivalent to comoving observers. The time T_c of a comoving observer satisfies:

$dT_c = \sqrt{g_{00}}dT = (1 + \Phi(T)/c^2)dT$

It comes: $\boxed{T_c = \int_0^T (1 + 8c\pi\rho_0 \cdot \int_0^t \phi(c\tau)\tau^2 d\tau)^{1/2} dt}$ (16)

Intuitively, the dilation factor has a positive acceleration because it is a division by a quantity that seems to near zero. In fact, the above equations shows that the absolute time T can have a finite limit value when the comoving time T_c tends to infinity. That depends on the gravitational potential theory used. Let's do the calculation for a Newtonian potential $\phi(r) = -\mathcal{G}/r$. We have:

$T_c = \int_0^T (1 - 4\pi\mathcal{G}\rho_0 t^2)^{1/2} dt$

And: $a(T) = (1 - 4\pi\mathcal{G}\rho_0 T^2)^{-1/6}$

From this simple Newtonian model, we see the scalar factor has a positive acceleration. The potential is not necessarily Newtonian, but we see that an accelerating expanding universe would be more expected than a non-accelerating universe, especially for non-Newtonian potentials such that $\mathcal{G}/r \cdot \phi(r)^{-1} = o(1)$. This model doesn't require Dark Energy to explain such acceleration.

X - Quantizing Gravity

Describing gravity as a force in a curved spacetime, we now have a coherent way to blend gravity into the quantum realm. What follows is based on Fock's equation [9] as a curved spacetime version of Dirac equation:

$$[i\gamma^{\mu}(\partial_{\mu} - \Gamma_{\mu} - ieA_{\mu}) - m] \cdot \psi = 0$$

Were γ_{μ} are the generalized gamma matrices defining the covariant Clifford algebra [10]: $\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 2g_{\mu\nu}$

were $g_{\mu\nu}$ is the spacetime metric, whose signature is $(+ - --)$, Γ_{μ} is the spinorial affine connection and A_{μ} is the electromagnetic four-vector potential.

In order to take into account gravity, we just write $m = m_0(1+\Phi_0/c^2)^{-1}$ and we take into account the gravitational four-vector potential G_{μ}. We get:

$$\boxed{[i\gamma^{\mu}(\partial_{\mu} - \Gamma_{\mu} - ieA_{\mu} - im_0(1+\Phi_0/c^2)^{-1}G_{\mu}) - m_0(1+\Phi_0/c^2)^{-1}] \cdot \psi = 0} \quad (17)$$

CONCLUSION

Gravity can be consistently described as a spacetime bending force based on an invariance principle inferred from the Schwarzschild metric. Analogous to the speed of light invariance which implies time dilation through speed, Vacuum Energy Invariance implies space dilation through gravitational potential. Writing the Lagrangian of a force in a curved spacetime, we get equivalent equations of motion as General Relativity if the inertial mass is relative depending on the gravitational potential. This is a physically acceptable hypothesis since the relativistic mass of a body is already relative depending on its speed.

This theory not only yields the same classical predictions as General Relativity such as Mercury's Perihelion Precession, Time Dilation or Light Bending but is also testable through many predicted deviations such as: an anomalous thrust, a time dilation second order correction and a non-radial space dilation described in Section I, IV and VII respectively.

This approach of gravitation is compatible with non-Newtonian gravitational potentials and violations of the weak equivalence principle. From there, many models can be developed to fit the available cosmological data. The reader can then complete this theory with a suitable description of the gravitational four-vector potential.

More than that, gravity as a force in a curved spacetime analogous to electromagnetism is renormalizable. Therefore, it is compatible with the Standard Model of particle physics. The reader could adapt the Standard Model's Lagrangian to this theory to achieve Quantum Gravity.

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