The Riemann Hypothesis Proof

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Abstract

I am using eta function because it extends the zeta function from Re(s) > 1 to the larger domain Re(s) > 0.
I am going to use eta function spiral and its behavior of convergent points on the complex plane to get two functions f(x) and g(x). Then I am going to show why and when those two functions are eqaul to zero the spiral is converging to zero as well. I will then make a new function h(x) that will show the non trival zeros only when its eqaul to zero. Then I am will a new function q(x) that will show if its eqauls for a given value of zeta(2a) then this is the case of a non trival zero. Then I am going to do convergence tests on the critical strip to show that there are no zeros on the strip other then the criticl line.

I am going to show that $\boxed{\frac{1}{k^{(a+ib)}} = \frac{\cos(b\cdot\ln k)}{k^a} - i\cdot\frac{\sin(b\cdot\ln k)}{k^a}}$ by using binomial theorem and exponential function

you can skip the next 4 pages if you like (Pages 2-4)

$$\boxed{e^{\theta} = \lim_{n\to\infty}\left(1+\frac{\theta}{n}\right)^n}$$ (exponential function)

$$\theta = -ib\cdot\ln k$$

$$e^{\theta} = e^{-ib\cdot\ln k} = k^{-ib}$$

$$\frac{1}{k^{(a+ib)}} = \frac{1}{k^a}\cdot k^{-ib} = \frac{1}{k^a}\cdot e^{\theta} = \frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1+\frac{\theta}{n}\right)^n = \frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n$$

$$\boxed{(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n}$$ (binomial theorem)

$$(x-iy)^n = x^n - i\binom{n}{1}x^{n-1}y^1 - \binom{n}{2}x^{n-2}y^2 + i\binom{n}{3}x^{n-3}y^3 + \binom{n}{4}x^{n-4}y^4$$
$$-i\binom{n}{5}x^{n-5}y^5 - \binom{n}{6}x^{n-6}y^6 + i\binom{n}{7}x^{n-7}y^7 + \binom{n}{8}x^{n-8}y^8 - \ldots \pm \binom{n}{n}x^0(-iy)^n$$

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[\begin{array}{l} x^n - i\binom{n}{1}x^{n-1}y^1 - \binom{n}{2}x^{n-2}y^2 + i\binom{n}{3}x^{n-3}y^3 + \binom{n}{4}x^{n-4}y^4 \\ -i\binom{n}{5}x^{n-5}y^5 - \binom{n}{6}x^{n-6}y^6 + i\binom{n}{7}x^{n-7}y^7 + \binom{n}{8}x^{n-8}y^8 - \ldots \end{array}\right]$$

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[x^n - \binom{n}{2}x^{n-2}y^2 + \binom{n}{4}x^{n-4}y^4 - \binom{n}{6}x^{n-6}y^6 + \binom{n}{8}x^{n-8}y^8 + \ldots\right]$$
$$+ i\cdot\lim_{n\to\infty}\left[-\binom{n}{1}x^{n-1}y^1 + \binom{n}{3}x^{n-3}y^3 - \binom{n}{5}x^{n-5}y^5 + \binom{n}{7}x^{n-7}y^7 + \ldots\right]$$

$$\boxed{\binom{n}{k} = \frac{n!}{(n-k)!k!}}$$ (binomial coefficient formula)

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[x^n - \frac{n!x^{n-2}y^2}{(n-2)!2!} + \frac{n!x^{n-4}y^4}{(n-4)!4!} - \frac{n!x^{n-6}y^6}{(n-6)!6!} + \frac{n!x^{n-8}y^8}{(n-8)!8!} + \ldots\right]$$
$$+ i\cdot\lim_{n\to\infty}\left[-\frac{n!x^{n-1}y^1}{(n-1)!1!} + \frac{n!x^{n-3}y^3}{(n-3)!3!} - \frac{n!x^{n-5}y^5}{(n-5)!5!} + \frac{n!x^{n-7}y^7}{(n-7)!7!} - \ldots\right]$$

lets replace $x = 1, y = \frac{b\cdot\ln k}{n}$

$$\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n=\lim_{n\to\infty}\left[1-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^2}{(n-2)!2!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^4}{(n-4)!4!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^6}{(n-6)!6!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^8}{(n-8)!8!}+\dots\right]$$
$$+i\lim_{n\to\infty}\left[-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^1}{(n-1)!1!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^3}{(n-3)!3!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^5}{(n-5)!5!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^7}{(n-7)!7!}-\dots\right]$$

now lets multiply by k^{-a}

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[1-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^2}{(n-2)!2!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^4}{(n-4)!4!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^6}{(n-6)!6!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^8}{(n-8)!8!}+\dots\right]$$
$$+i\cdot\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^1}{(n-1)!1!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^3}{(n-3)!3!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^5}{(n-5)!5!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^7}{(n-7)!7!}-\dots\right]$$

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[1-\frac{n!(b\cdot\ln k)^2}{n^2(n-2)!2!}+\frac{n!(b\cdot\ln k)^4}{n^4(n-4)!4!}-\frac{n!(b\cdot\ln k)^6}{n^6(n-6)!6!}+\frac{n!(b\cdot\ln k)^8}{n^8(n-8)!8!}+\dots\right]$$
$$+i\cdot\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[-\frac{n!(b\cdot\ln k)^1}{n^1(n-1)!1!}+\frac{n!(b\cdot\ln k)^3}{n^3(n-3)!3!}-\frac{n!(b\cdot\ln k)^5}{n^5(n-5)!5!}+\frac{n!(b\cdot\ln k)^7}{n^7(n-7)!7!}-\dots\right]$$

if $1\le m$ then

$$\lim_{n\to\infty}\frac{n!(b\cdot\ln k)^m}{n^m(n-m)!m!}=\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{n!}{n^m(n-m)!}=$$
$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{1}{n^m}\cdot\frac{1\cdot2\cdot3\cdot\dots\cdot(n-m)\cdot\dots\cdot(n-2)(n-1)n}{1\cdot2\cdot3\cdot\dots\cdot(n-m)}=$$
$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{(n-(m-1))\cdot\dots\cdot(n-2)(n-1)(n-0)}{n^m}=$$
$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{(n-(m-1))}{n}\cdot\dots\cdot\frac{(n-2)}{n}\cdot\frac{(n-1)}{n}\cdot\frac{(n)}{n}$$

when m is a number then $\boxed{\lim_{n\to\infty}\frac{n!(b\cdot\ln k)^m}{n^m(n-m)!m!}=\frac{(b\cdot\ln k)^m}{m!}}$

when m is an expression n then $\boxed{\lim_{n\to\infty}\frac{(n-(m-1))}{n}\cdot\dots\cdot\frac{(n-2)}{n}\cdot\frac{(n-1)}{n}\cdot\frac{(n)}{n}=0}$ making $\boxed{\lim_{n\to\infty}\frac{n!(b\cdot\ln k)^m}{n^m(n-m)!m!}=0}$

this will result binomial coefficients be equal to zero starting from “the middle of the binomial expration” resulting: $\frac{1}{k^{(a+ib)}} =$

$$\frac{1}{k^a}\cdot\left[1-\frac{(b\cdot\ln k)^2}{2!}+\frac{(b\cdot\ln k)^4}{4!}-\frac{(b\cdot\ln k)^6}{6!}+\frac{(b\cdot\ln k)^8}{8!}+\ldots\right]+i\cdot\frac{1}{k^a}\cdot\left[-\frac{(b\cdot\ln k)^1}{1!}+\frac{(b\cdot\ln k)^3}{3!}-\frac{(b\cdot\ln k)^5}{5!}+\frac{(b\cdot\ln k)^7}{7!}-\ldots\right]$$

$$\boxed{\cos(x)=\frac{1}{0!}-\frac{(x)^2}{2!}+\frac{(x)^4}{4!}-\frac{(x)^6}{6!}+\frac{(x)^8}{8!}-\ldots}$$

$$\boxed{\sin(x)=\frac{(x)^1}{1!}-\frac{(x)^3}{3!}+\frac{(x)^5}{5!}-\frac{(x)^7}{7!}-\ldots}$$

$$\cos(b\cdot\ln k)=\left[1-\frac{(b\cdot\ln k)^2}{2!}+\frac{(b\cdot\ln k)^4}{4!}-\frac{(b\cdot\ln k)^6}{6!}+\frac{(b\cdot\ln k)^8}{8!}-\ldots\right]$$

$$\sin(b\cdot\ln k)=\left[\frac{(b\cdot\ln k)^1}{1!}-\frac{(b\cdot\ln k)^3}{3!}+\frac{(b\cdot\ln k)^5}{5!}-\frac{(b\cdot\ln k)^7}{7!}-\ldots\right]$$

$$\boxed{\frac{1}{k^{(a+ib)}}=\frac{\cos(b\cdot\ln k)}{k^a}-i\cdot\frac{\sin(b\cdot\ln k)}{k^a}}$$

$$\boxed{\eta(a+ib)=\frac{1}{1^{(a+ib)}}-\frac{1}{2^{(a+ib)}}+\frac{1}{3^{(a+ib)}}-\frac{1}{4^{(a+ib)}}+\ldots}$$

$$+\frac{1}{1^{(a+ib)}}=\left[+\frac{\cos(b\cdot\ln 1)}{1^a}\right]+i\cdot\left[-\frac{\sin(b\cdot\ln 1)}{1^a}\right]$$

$$-\frac{1}{2^{(a+ib)}}=\left[-\frac{\cos(b\cdot\ln 2)}{2^a}\right]+i\cdot\left[+\frac{\sin(b\cdot\ln 2)}{2^a}\right]$$

$$+\frac{1}{3^{(a+ib)}}=\left[+\frac{\cos(b\cdot\ln 3)}{3^a}\right]+i\cdot\left[-\frac{\sin(b\cdot\ln 3)}{3^a}\right]$$

$$-\frac{1}{4^{(a+ib)}}=\left[-\frac{\cos(b\cdot\ln 4)}{4^a}\right]+i\cdot\left[+\frac{\sin(b\cdot\ln 4)}{4^a}\right]$$

$$\eta(a+ib)=\frac{1}{1^{(a+ib)}}-\frac{1}{2^{(a+ib)}}+\frac{1}{3^{(a+ib)}}-\frac{1}{4^{(a+ib)}}+\ldots=$$

$$\left[\frac{\cos(b\ln 1)}{1^a}-\frac{\cos(b\ln 2)}{2^a}+\frac{\cos(b\ln 3)}{3^a}-\frac{\cos(b\ln 4)}{4^a}+\ldots\right]+$$

$$\left[-\frac{\sin(b\ln 1)}{1^a}+\frac{\sin(b\ln 2)}{2^a}-\frac{\sin(b\ln 3)}{3^a}+\frac{\sin(b\ln 4)}{4^a}+\ldots\right]\cdot i$$

another way (and much more easier way) to look at this as:

$$\eta(a+ib)=\left[\frac{1}{1^a}\cdot\cos(-b\ln 1)-\frac{1}{2^a}\cdot\cos(-b\ln 2)+\frac{1}{3^a}\cdot\cos(-b\ln 3)-\frac{1}{4^a}\cdot\cos(-b\ln 4)+....\right]$$
$$+\left[\frac{1}{1^a}\cdot\sin(-b\ln 1)-\frac{1}{2^a}\cdot\sin(-b\ln 2)+\frac{1}{3^a}\cdot\sin(-b\ln 3)-\frac{1}{4^a}\cdot\sin(-b\ln 4)+....\right]\cdot i$$

$$\vec{V}_k=\frac{1}{1^k} \qquad \theta_k=-b\ln k$$

$$\eta(a+ib)=\left[\vec{V}_1\cdot\cos(\theta_1)-\vec{V}_2\cdot\cos(\theta_2)+\vec{V}_3\cdot\cos(\theta_3)-\vec{V}_4\cdot\cos(\theta_4)+....\right]$$
$$+\left[\vec{V}_1\cdot\sin(\theta_1)-\vec{V}_2\cdot\sin(\theta_2)+\vec{V}_3\cdot\sin(\theta_3)-\vec{V}_4\cdot\sin(\theta_4)+....\right]\cdot i$$

moving on the xAxis $\vec{V}_1\cdot\cos(\theta_1)-\vec{V}_2\cdot\cos(\theta_2)+\vec{V}_3\cdot\cos(\theta_3)-\vec{V}_4\cdot\cos(\theta_4)+....$

moving on the yAxis $\vec{V}_1\cdot\sin(\theta_1)-\vec{V}_2\cdot\sin(\theta_2)+\vec{V}_3\cdot\sin(\theta_3)-\vec{V}_4\cdot\sin(\theta_4)+....$

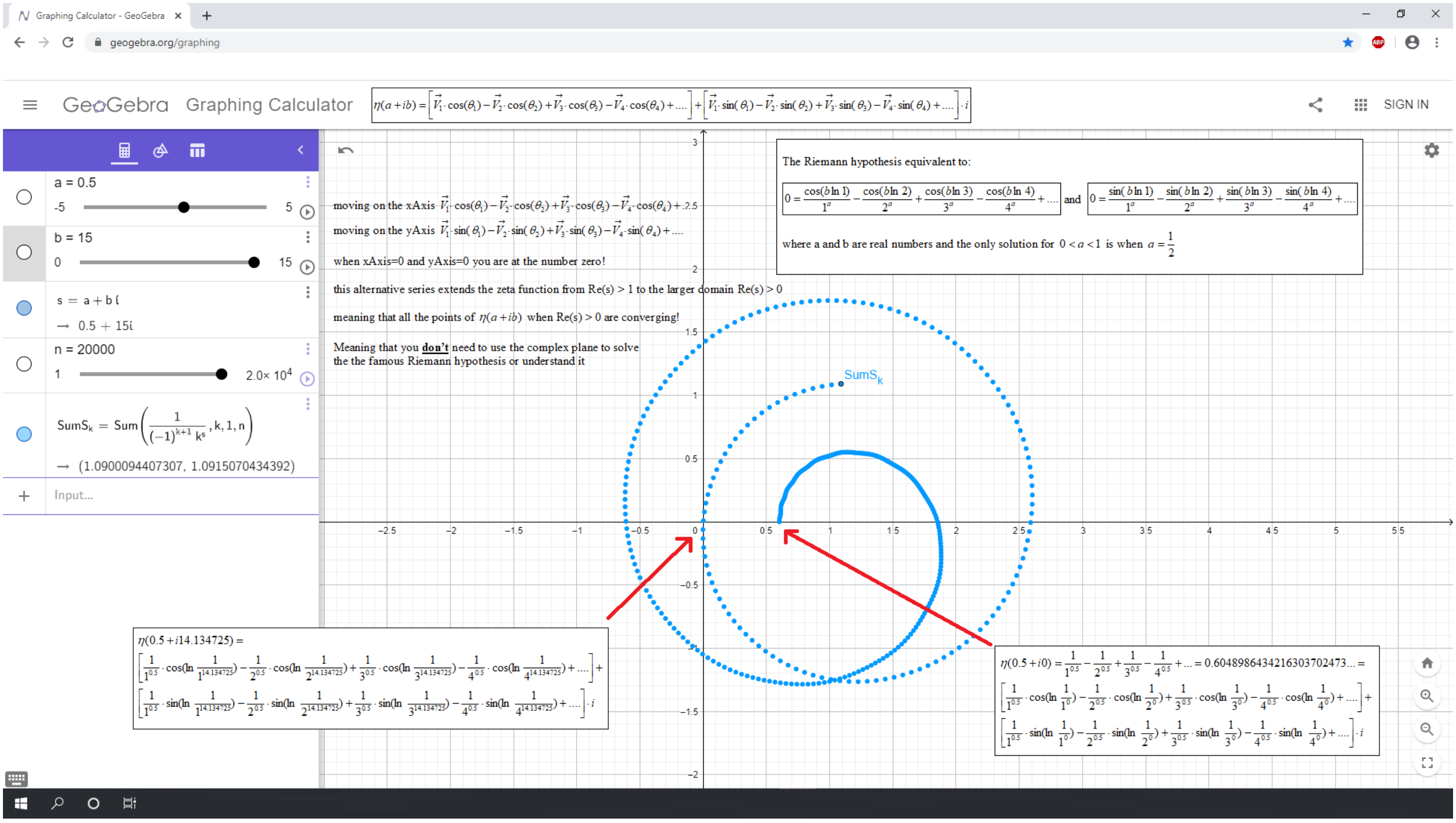
when xAxis=0 and yAxis=0 then $\eta(s)=0$ meaning that also $\xi(s)=\dfrac{\eta(s)}{(1-2^{1-s})}=0$

this helps extend the zeta function from Re(s) > 1 to the larger domain

The Riemann hypothesis equivalent to:

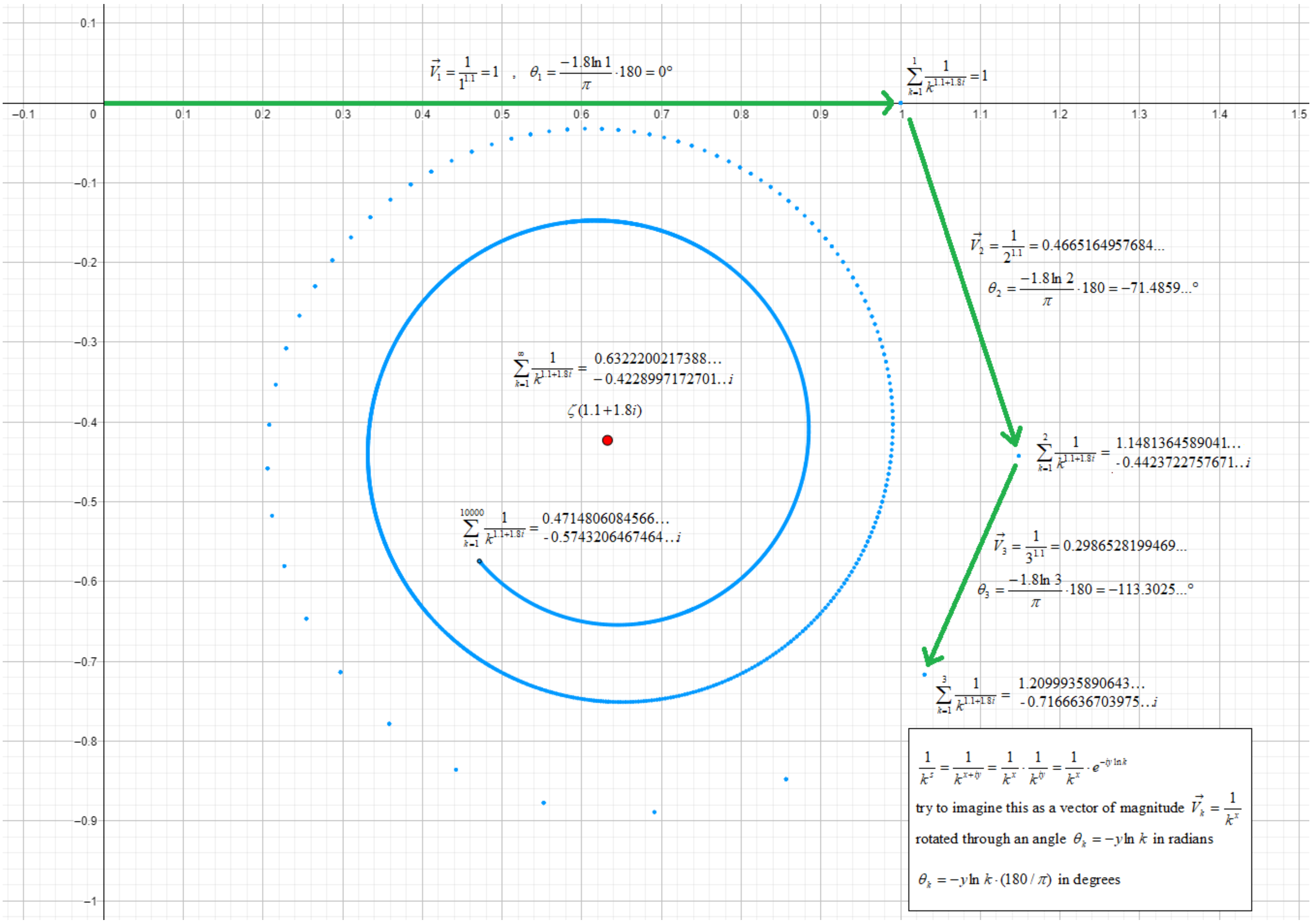
$$0=\frac{\cos(b\ln 1)}{1^a}-\frac{\cos(b\ln 2)}{2^a}+\frac{\cos(b\ln 3)}{3^a}-.... \quad \text{and} \quad 0=\frac{\sin(b\ln 1)}{1^a}-\frac{\sin(b\ln 2)}{2^a}+\frac{\sin(b\ln 3)}{3^a}-....$$

where a and b are real numbers and the only solution for $0<a<1$ is when $a=\dfrac{1}{2}$



For me (as I see it) when I am looking at the zeta function I dont see (or use) the term “Assigned Value” or “Analytic Continuation” Instead I see “spirals” all around the grid.

The simplest way is to first look at the behavior of convergent points on the complex plane $\zeta(x+iy)=a+ib$ where $x>1$ The spiral swirls around inwards to an unique point which the series Converges



Same goes for the other way around!

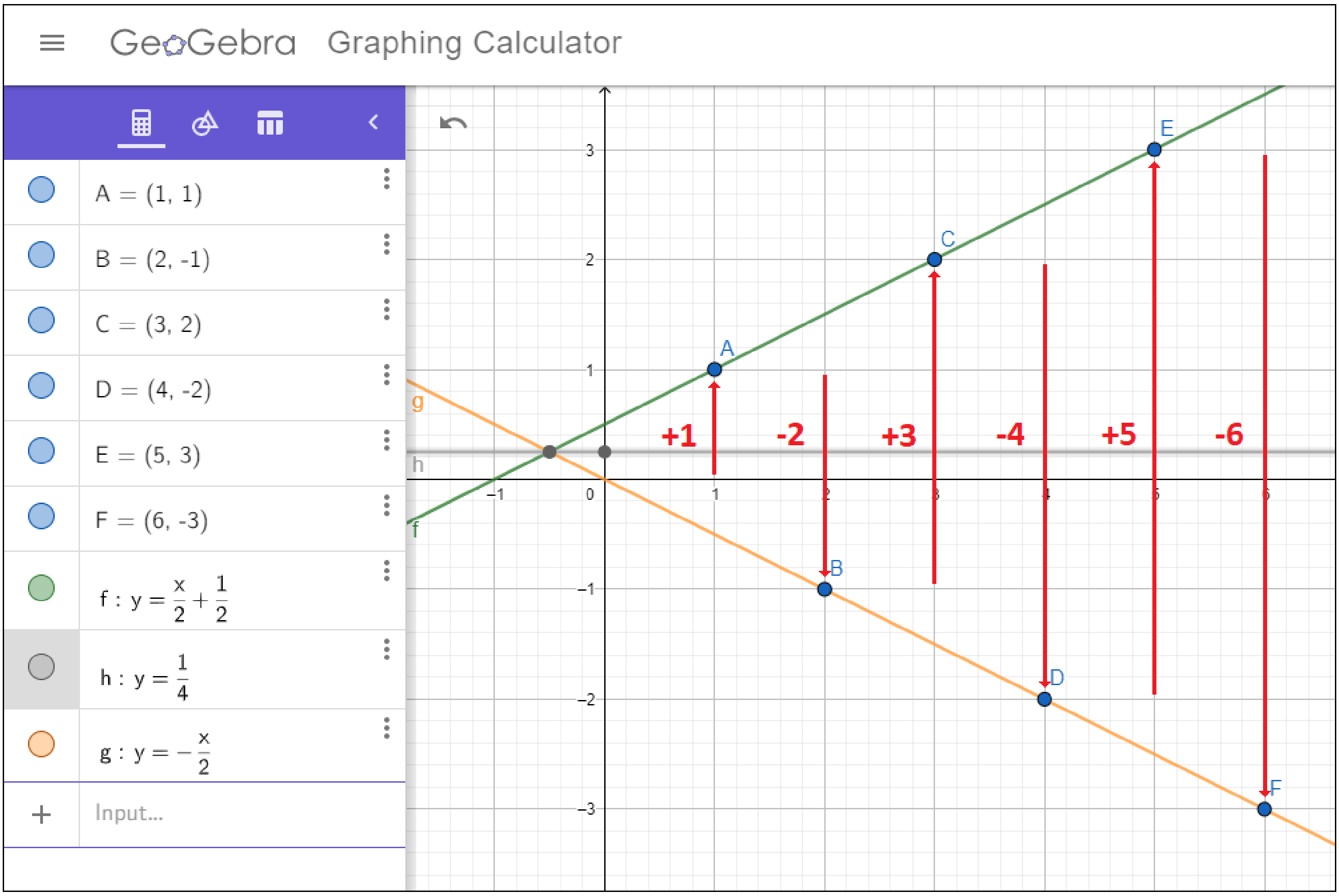
When I am looking at the Complex plane $\zeta(x+iy)=a+ib$ where $x<1$ and the behavior of divergent points

The spiral swirls around outwards but if you look closely you will notice that the spiral has a “center point” or an “origin” and that “origin” is the “Assigned Value” everyone is talking about.

when I first started to read about the zeta function I didn’t know what are those “Assigned Values” or “Analytic Continuation” and how and why people are trying to give a value for divergent series And why that specific value and not something else? I wanted an explanation other then “because the formula says so” and without going deeper into all the “Analytic Continuation stuff".

Those “origin points” did the trick!

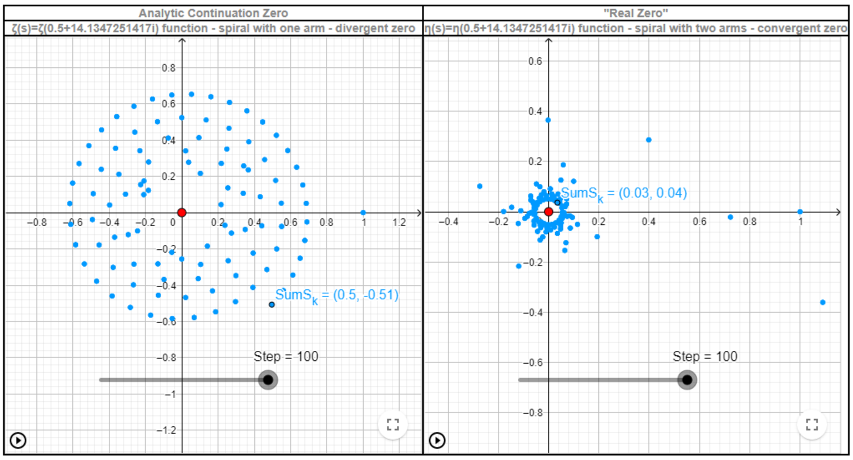
the simplest origin point to understend is $\eta(-1) = 1 - 2 + 3 - 4 + 5 - 6 + ...$



the (Assigned) value 1/4 is not the summation of $\eta(-1)$
it's simply represents the intersection points of the two lines
or as i like to describe it as the origin point of the spiral on the complex plane

for extra info make sure to check my article that i submitted to Vixra on
Dirichlet Eta Function Negative Integer Formula https://vixra.org/pdf/2005.0048v3.pdf

If you are assigning a value for a series that decreases to a specific value (case #1)
Then you can assigning a value for a series that increases from a specific value (case #2)



Other then those two cases there is one more

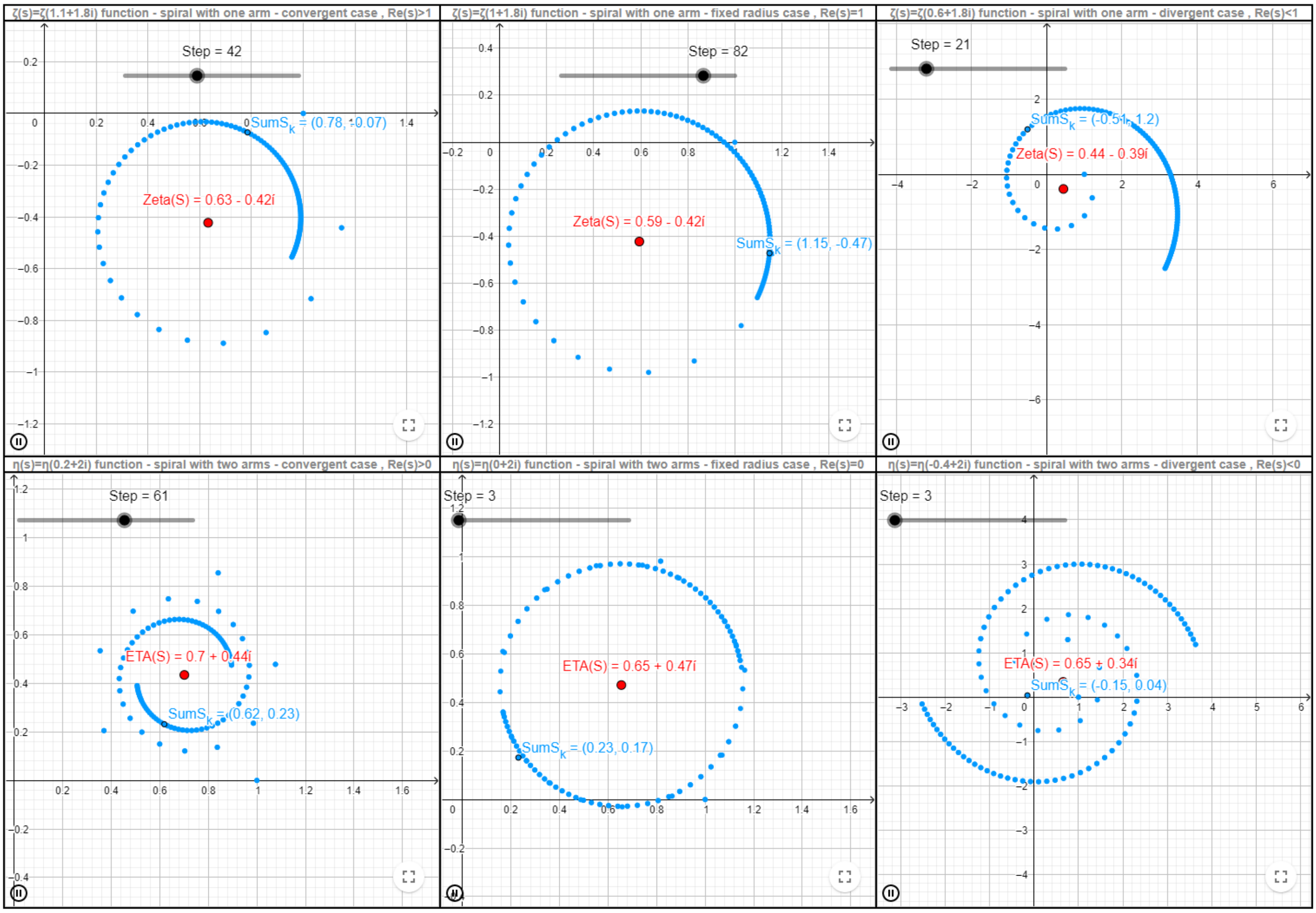
This is when the spiral at some point start to spin around a specific value with a “fixed radius” those cases appears at the zeta function $\zeta(s)=\zeta(x+iy)=a+ib$ when $x=1$ and the radius will be $1/y$ meaning that this is a divergent series with a “fixed radius”

This was a small intro for the eta function spirals

Its true that the zeta function spirals have 3 cases but they are all spirals with **one arm**
Now at the eta function the spirals have **two arms** (that is because of the +/- swapping) with the same 3 cases

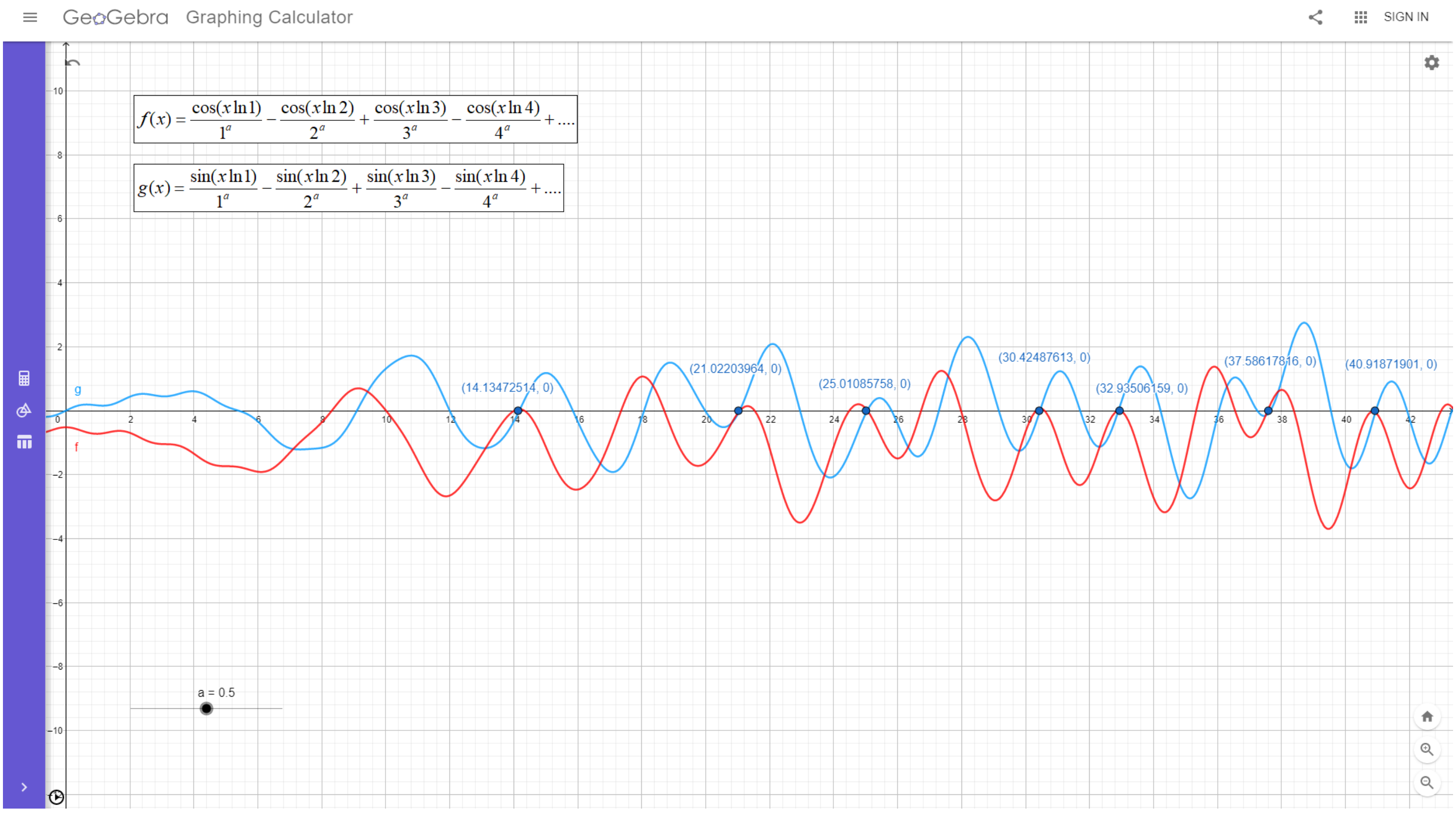
By the way the “fixed radius” appears at the eta function $\eta(s)=\eta(x+iy)=a+ib$ when $x=0$

If you like to know more I am providing further details at http://myzeta.125mb.com

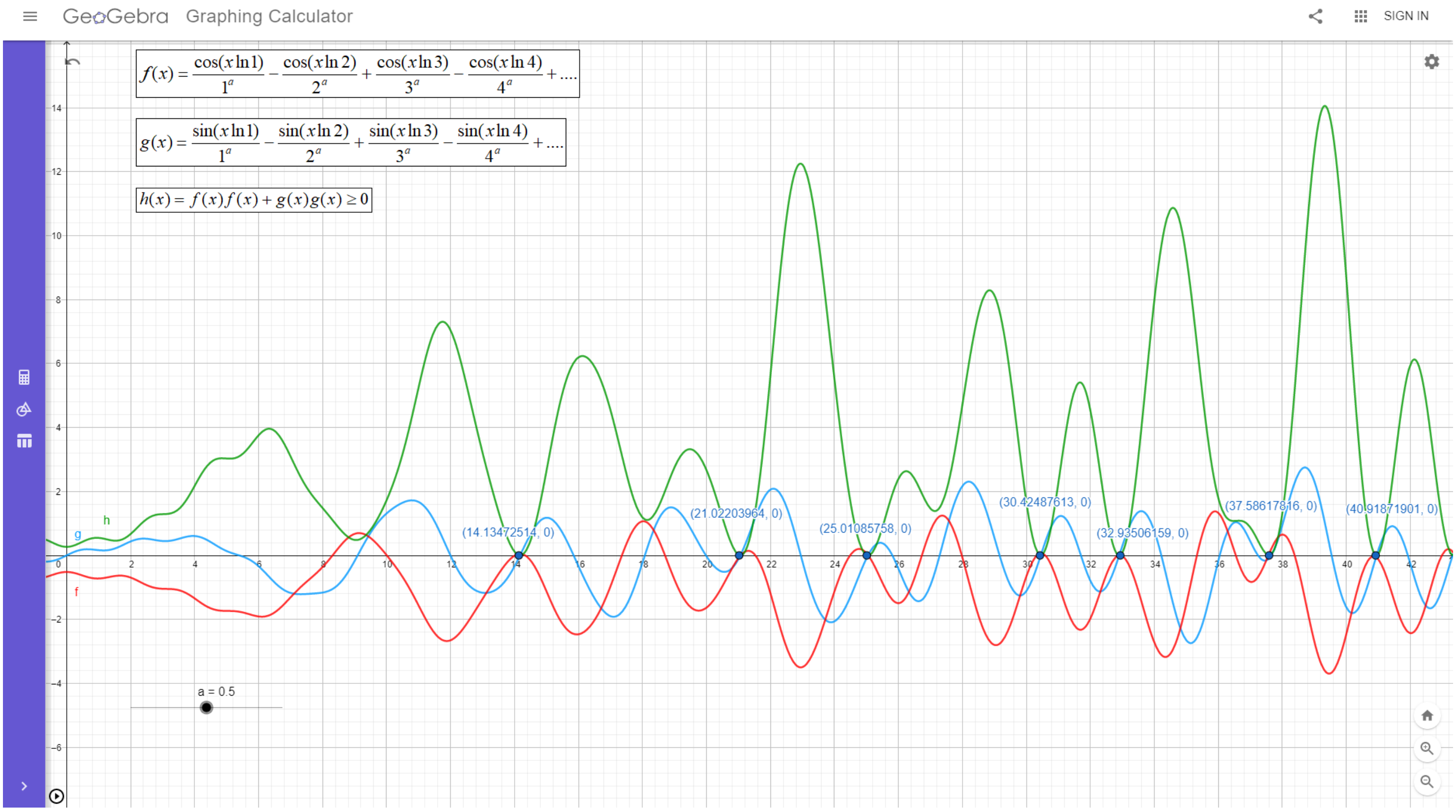


$$f(x)=\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-....$$

$$g(x)=\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-....$$



I am going to make a new function $h(x)$ that will include both cases and will be 0 only when both functions $f(x)$ and $g(x)$ are 0 as well. The simplest way is to have $h(x)=f(x)f(x)+g(x)g(x)$ where $h(x)\geq 0$ and $h(x)=0$ only when you have those non-trival zeros



$$f(x) = \frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots.$$

$f(x)\,f(x) = ?$

$$\begin{aligned}
&+\frac{\cos(x\ln 1)}{1^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots\right)\\
&-\frac{\cos(x\ln 2)}{2^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots\right)\\
&+\frac{\cos(x\ln 3)}{3^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots\right)\\
&-\frac{\cos(x\ln 4)}{4^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots\right)
\end{aligned}$$

$$\begin{aligned}
&+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 4)}{4^a} + \ldots\\
&-\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a} + \frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a} - \frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 3)}{3^a} + \frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 4)}{4^a} - \ldots\\
&+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 4)}{4^a} + \ldots\\
&-\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a} + \frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a} - \frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a} + \frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a} - \ldots
\end{aligned}$$

$$\begin{aligned}
&+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a}\\
&-2\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a} + \frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a}\\
&+2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a} - 2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a}\\
&-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a} + 2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a} - 2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a} + \frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a} - \ldots
\end{aligned}$$

$$g(x)=\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+....$$

$$g(x)g(x)=?$$

$$\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&-\frac{\sin(x\ln 2)}{2^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&+\frac{\sin(x\ln 3)}{3^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&-\frac{\sin(x\ln 4)}{4^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)
\end{aligned}$$

$$\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 4)}{4^a}+...\\
&-\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}-\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...\\
&+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 4)}{4^a}+...\\
&-\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...
\end{aligned}$$

$$\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}\\
&-2\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}\\
&+2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}\\
&-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...
\end{aligned}$$

$h(x) = f(x) f(x) + g(x) g(x) = ?$

now lets combine the two functions

$$+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a}$$
$$-2\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a}+\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a}$$
$$+2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a}-2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a}$$
$$-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a}+2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a}-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a}+\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a}-\ldots$$

$$+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}$$
$$-2\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}$$
$$+2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}$$
$$-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-\ldots$$

$$\boxed{\cos(a-b)=\cos(a)\cos(b)+\sin(a)\sin(b)}$$

now lets merge the two functions

$$+\frac{\cos(x\ln 1-x\ln 1)}{1^a\cdot 1^a}$$
$$-2\frac{\cos(x\ln 2-x\ln 1)}{2^a\cdot 1^a}+\frac{\cos(x\ln 2-x\ln 2)}{2^a\cdot 2^a}$$
$$+2\frac{\cos(x\ln 3-x\ln 1)}{3^a\cdot 1^a}-2\frac{\cos(x\ln 3-x\ln 2)}{3^a\cdot 2^a}+\frac{\cos(x\ln 3-x\ln 3)}{3^a\cdot 3^a}$$
$$-2\frac{\cos(x\ln 4-x\ln 1)}{4^a\cdot 1^a}+2\frac{\cos(x\ln 4-x\ln 2)}{4^a\cdot 2^a}-2\frac{\cos(x\ln 4-x\ln 3)}{4^a\cdot 3^a}+\frac{\cos(x\ln 4-x\ln 4)}{4^a\cdot 4^a}$$

$$+\frac{\cos(x\ln 1/1)}{1^a\cdot 1^a}$$
$$-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a}+\frac{\cos(x\ln 2/2)}{2^a\cdot 2^a}$$
$$+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a}-2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a}+\frac{\cos(x\ln 3/3)}{3^a\cdot 3^a}$$
$$-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a}+2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a}-2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a}+\frac{\cos(x\ln 4/4)}{4^a\cdot 4^a}$$

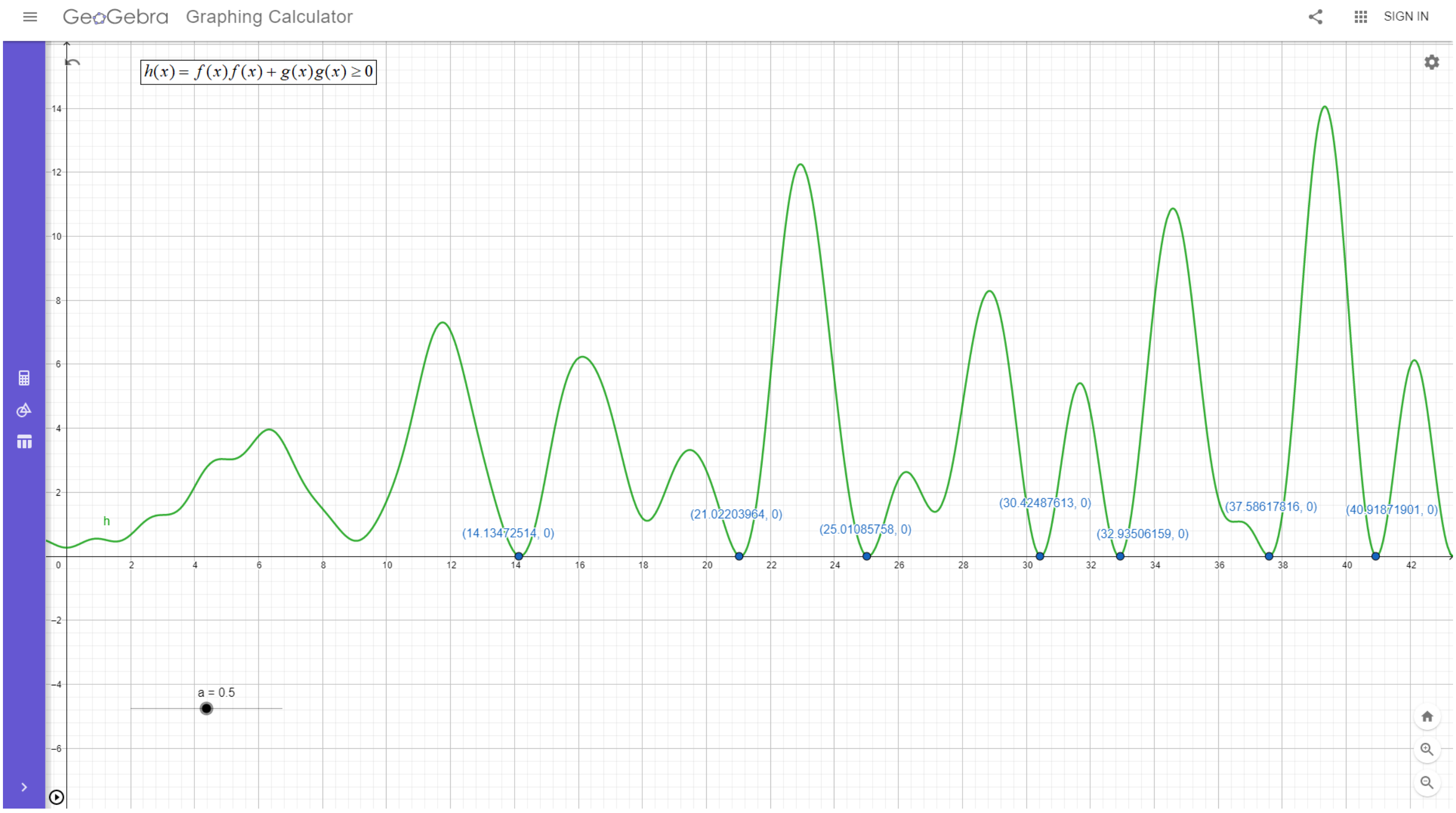
$$
\begin{array}{llll}
+\frac{1}{1^a\cdot 1^a} & & & \\
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & +\frac{1}{2^a\cdot 2^a} & & \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & +\frac{1}{3^a\cdot 3^a} & \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} & +\frac{1}{4^a\cdot 4^a}
\end{array}
$$

$$
\begin{array}{llll}
+\frac{1}{1^a\cdot 1^a} & & & \\
+\frac{1}{2^a\cdot 2^a} & -2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & & \\
+\frac{1}{3^a\cdot 3^a} & +2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & \\
+\frac{1}{4^a\cdot 4^a} & -2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a}
\end{array}
$$

$$
\zeta(2a)+\boxed{\begin{array}{llll}
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & & & \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & & \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} & \\
+2\frac{\cos(x\ln 5/1)}{5^a\cdot 1^a} & -2\frac{\cos(x\ln 5/2)}{5^a\cdot 2^a} & +2\frac{\cos(x\ln 5/3)}{5^a\cdot 3^a} & -2\frac{\cos(x\ln 5/4)}{5^a\cdot 4^a}
\end{array}}
$$

$$
\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}=\boxed{\begin{array}{llll}
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & & & \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & & \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} & \\
+2\frac{\cos(x\ln 5/1)}{5^a\cdot 1^a} & -2\frac{\cos(x\ln 5/2)}{5^a\cdot 2^a} & +2\frac{\cos(x\ln 5/3)}{5^a\cdot 3^a} & -2\frac{\cos(x\ln 5/4)}{5^a\cdot 4^a} \\
-\ldots & & & \\
+\ldots & & & \\
-\ldots & & &
\end{array}}
$$

$$
h(x)=f(x)f(x)+g(x)g(x)=\zeta(2a)+\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}
$$

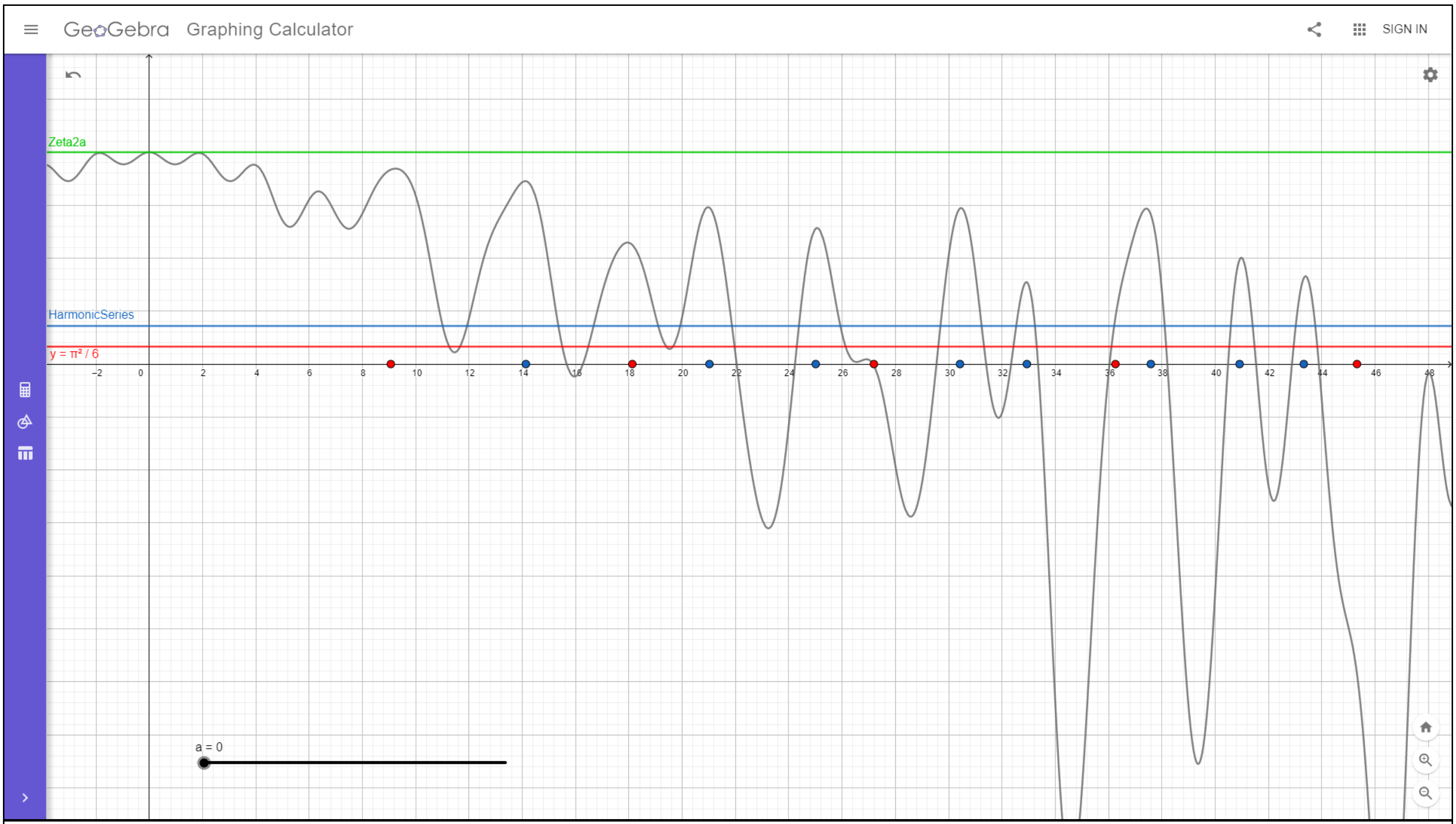


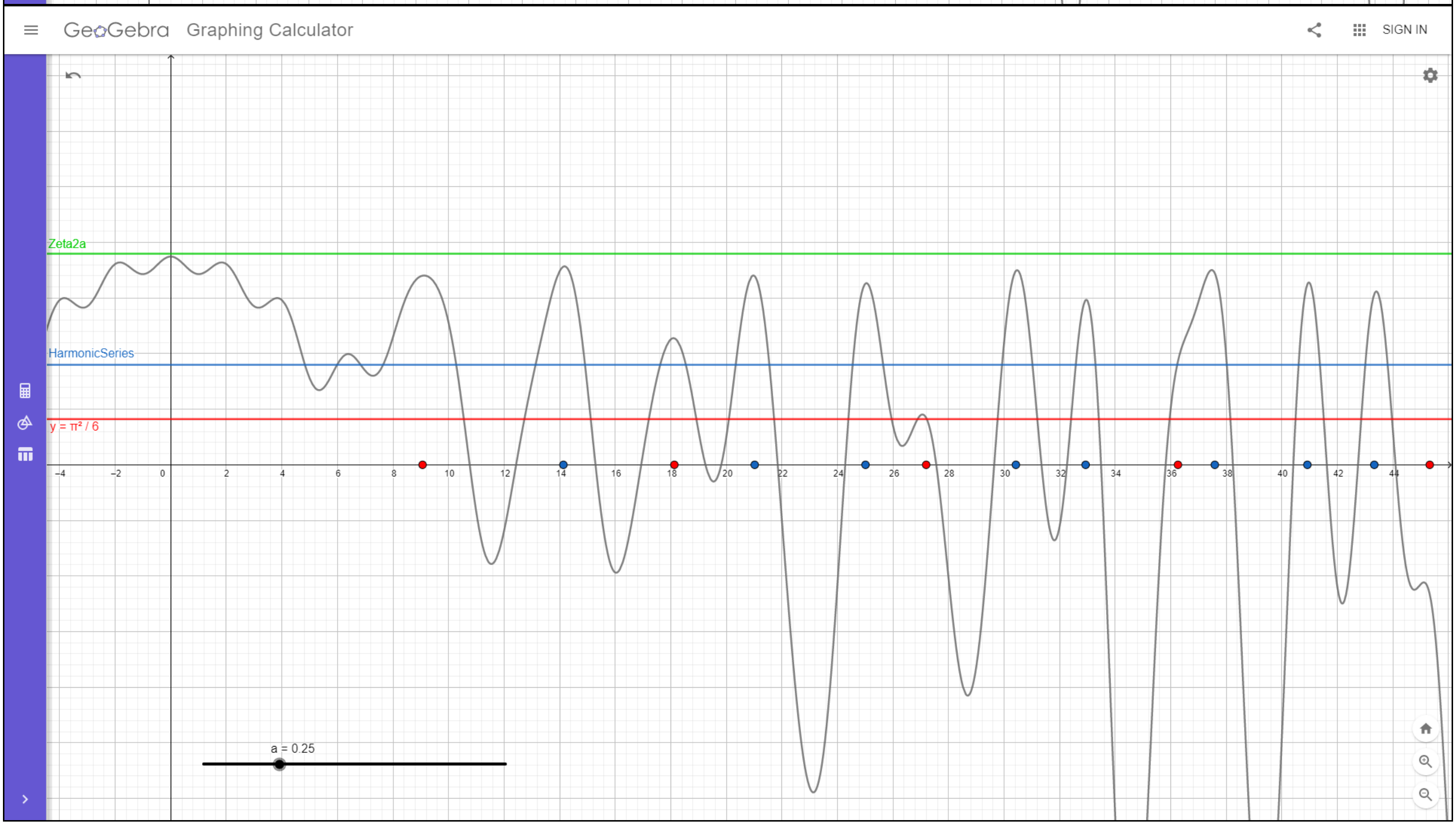
$$0 \le h(x) = \zeta(2a) + \sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}$$

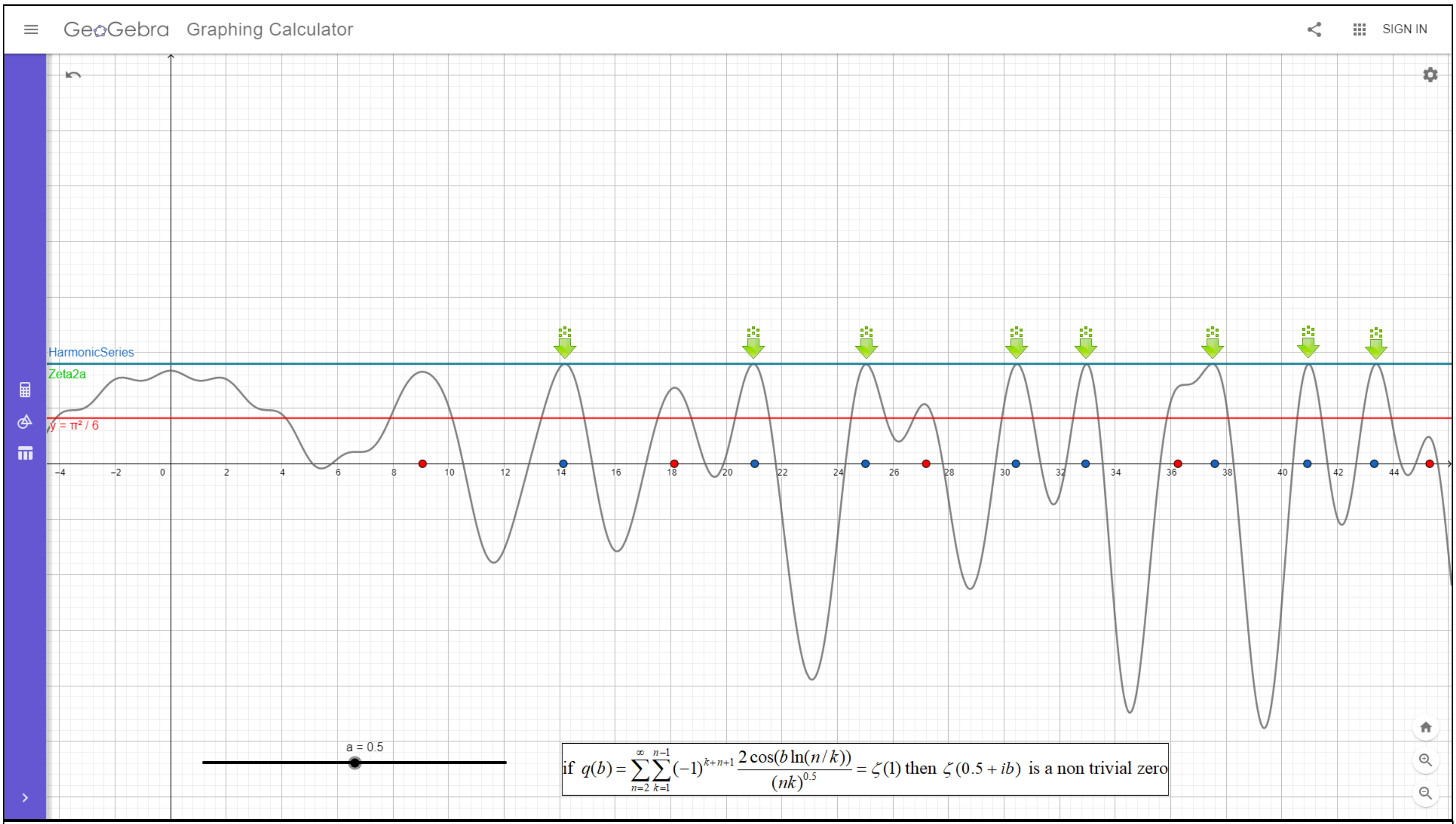
$$-\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a} \le \zeta(2a)$$

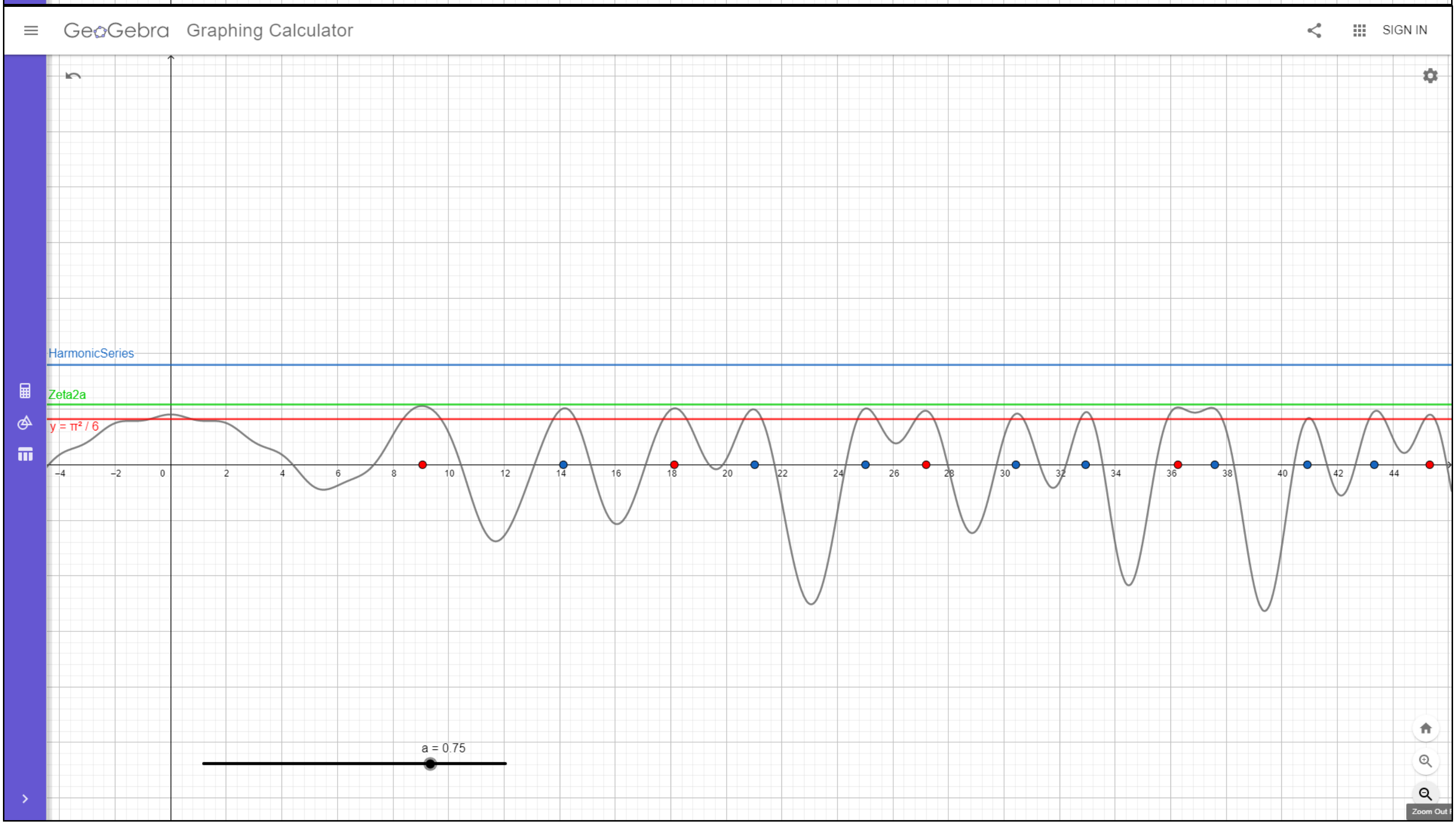
$$q(x) = \sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^a} \le \zeta(2a)$$

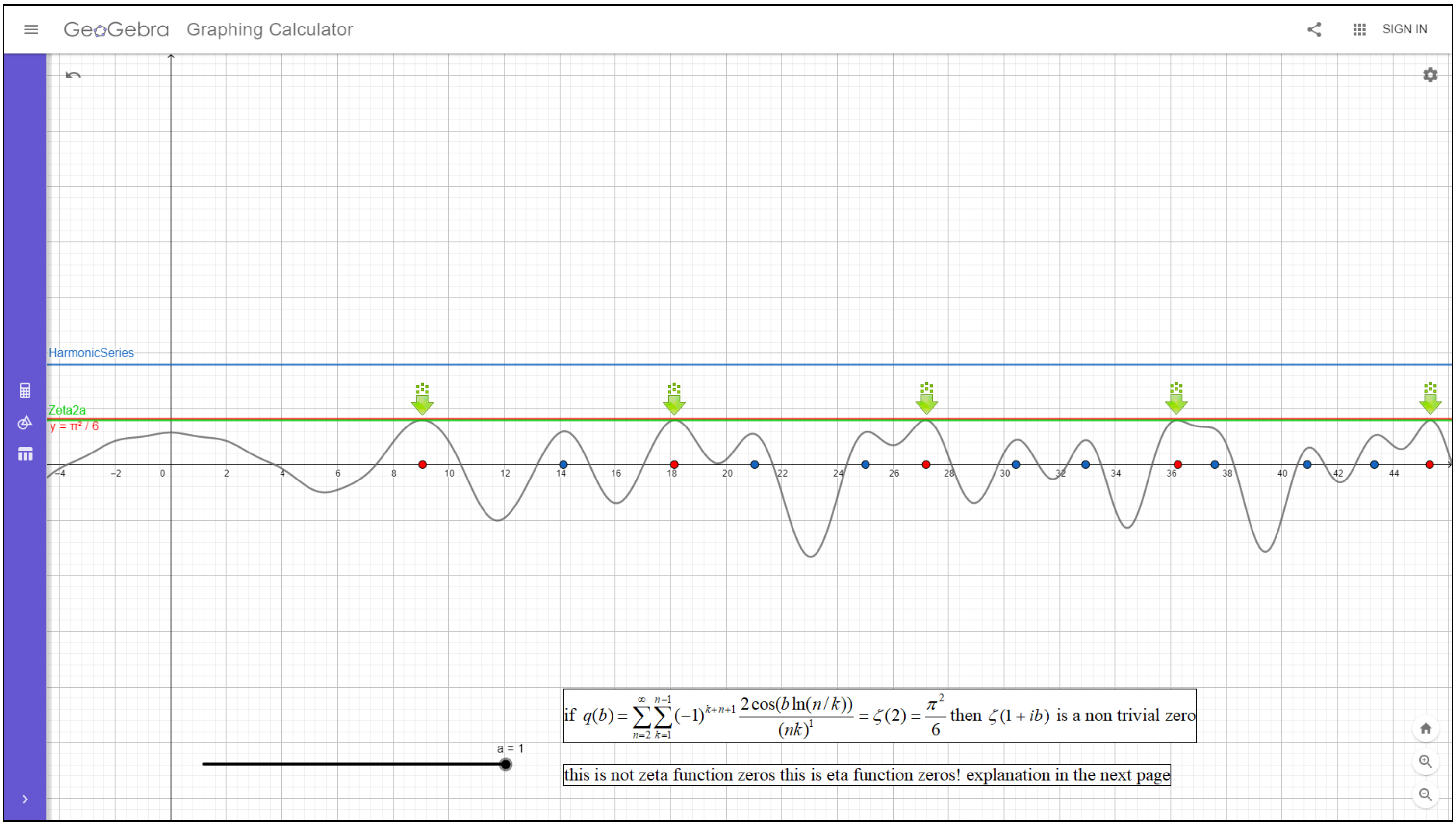
When $q(b) = \sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^a} = \zeta(2a)$ then $\zeta(a+ib)$ is a non trivial zero(because $h(b)=0$)


GeoGebra Graphing Calculator
SIGN IN
Zeta2a
HarmonicSeries
y = π² / 6
a = 0


GeoGebra Graphing Calculator
SIGN IN
Zeta2a
HarmonicSeries
y = π² / 6
a = 0.25


GeoGebra Graphing Calculator
SIGN IN
HarmonicSeries
Zeta2a
y = π² / 6
a = 0.5
if $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^{0.5}}=\zeta(1)$ then $\zeta(0.5+ib)$ is a non trivial zero


GeoGebra Graphing Calculator
SIGN IN
HarmonicSeries
Zeta2a
y = π² / 6
a = 0.75



(For $1<a$ there are no non trivial zeros this is a known fact so I am not showing why)

i used eta function summation to get $h(x)$ and because $\boxed{\left(1-\frac{2}{2^s}\right)\zeta(s)=\eta(s)}$ then for $b=\frac{2\pi t}{\ln 2}$ when $a=1$ we get

$$q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(2)$$

$$\boxed{\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos\left(2\pi t\frac{\ln(n/k)}{\ln 2}\right)}{(nk)^1}=\frac{\pi^2}{6}}$$

side note:

if $\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(1)$ then there were zeros on the $\zeta(1)$ line

but $\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(2)=\frac{\pi^2}{6}<\zeta(1)$ so no zeros on the $\zeta(1)$ line ☺

Critical Strip

When $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^{a}}=\zeta(2a)$ then $\zeta(a+ib)$ is a non trivial zero

Case #1

for the range $0.5<a<1$ we can multiply by $\left(1-\frac{2}{2^{2a}}\right)\neq 0$

$$\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a) \qquad \Rightarrow \qquad \left(1-\frac{2}{2^{2a}}\right)\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^{a}}=\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a)$$

the right side $\eta(2a)$ is **convergence absolutely** in the range $0.5<a<1$

meaning the function $f(x)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\left(1-\frac{2}{2^{2a}}\right)\frac{2\cos(x\ln(n/k))}{(nk)^{a}}$ has a sup value of

which is a fixed value (a real number!)

and because of that the function (theoretically) can have values of x that will result $f(x)=0$

Case #2

for the range $0<a<0.5$ we can multiply by $\left(1-\frac{2}{2^{2a}}\right)\neq 0$

$$\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a) \qquad \Rightarrow \qquad \left(1-\frac{2}{2^{2a}}\right)\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^{a}}=\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a)$$

the right side $\eta(2a)$ **converges conditionally** in the range $0<a<0.5$ (I will show that in the next page)

meaning the function $f(x)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\left(1-\frac{2}{2^{2a}}\right)\frac{2\cos(x\ln(n/k))}{(nk)^{a}}$ has no (fixed) sup value!

The sup value should have been $\eta(2a)$ but this is not a fixed value in the range $0<a<0.5$ and because of that the function changing all the time as n gets biger and biger making the values of x to changed on the cos function summation.

the x value cant diverge when $n\to\infty$ it need to be a fixed value!

That is why there are no zeros in the range of $0<a<0.5$

p-Series test:

$$\sum_{k=1}^{\infty}\frac{1}{k^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+...$$

$\sum_{k=1}^{\infty}\frac{1}{k^p}$ converges if $p>1$, and diverges if $p\le 1$.

Alternating series test:

$$a_k=\frac{1}{k^{2n}}$$

$$\sum_{k=1}^{\infty}(-1)^{k+1}a_k=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{2n}}=\frac{1}{1^{2n}}-\frac{1}{2^{2n}}+\frac{1}{3^{2n}}-\frac{1}{4^{2n}}+...$$

$|a_k|=\left|\frac{1}{k^{2n}}\right|$ decreases monotonically

$$\lim_{k\to\infty}a_k=\lim_{k\to\infty}\frac{1}{k^{2n}}=0$$

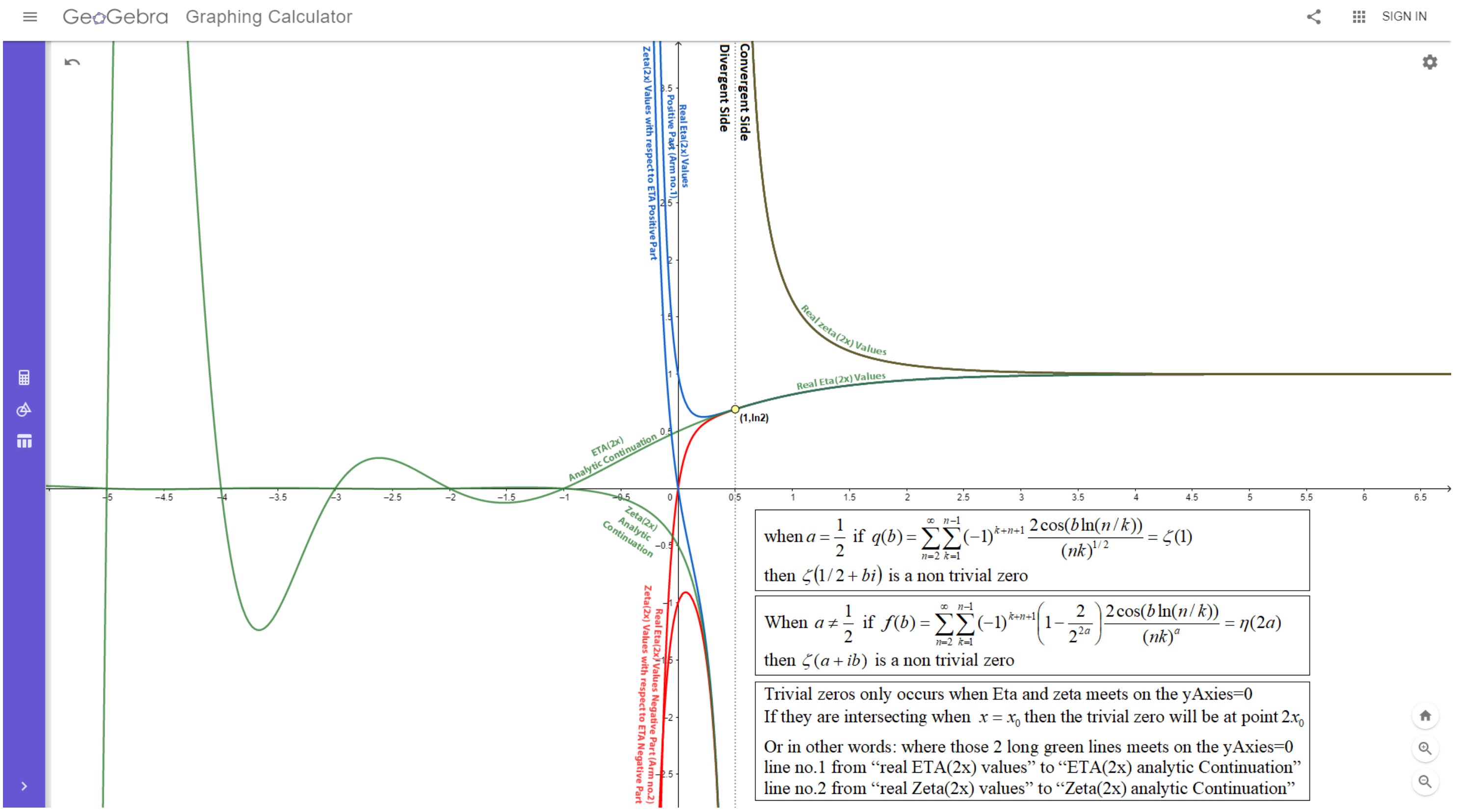
the alternating series converges

Absolute convergence test:

$$\sum_{k=1}^{\infty}\left|(-1)^{k+1}a_k\right|=\sum_{k=1}^{\infty}\left|\frac{(-1)^{k+1}}{k^{2n}}\right|=\sum_{k=1}^{\infty}\left|\frac{1}{k^{2n}}\right|=\frac{1}{1^{2n}}+\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\frac{1}{4^{2n}}+...$$

when $2n\le 1$ (by p-Series test) the series diverges

$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^{2n}}=\frac{1}{1^{2n}}-\frac{1}{2^{2n}}+\frac{1}{3^{2n}}-\frac{1}{4^{2n}}+...$ when $n\le 0.5$ the series converges Conditionally!



(This time I am using something that already been proven) (Functional equation)

$$\zeta(s)=2^{s}\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

Because in the range $0<a<0.5$ the function has no zeros
that means that in the range $0.5<a<1$ there are no zeros as well!

<u>Case #3</u> (This part I fixed sorry about older versions I simply didn’t removed that part from older draft)

when $a=0.5$ the function $q(x)=\zeta(1)$ is devergent

$$\lim_{M\to\infty}\sum_{n=2}^{M}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x_0\ln(n/k))}{(nk)^{1/2}}=\lim_{M\to\infty}\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{M}\right]=\lim_{M\to\infty}\sum_{n=1}^{M}\frac{1}{k}=\zeta(1)$$

and we already know there are infintly many zeroes on the critical line