# Einstein's field equations geometrized completely

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## ABSTRACT

**Aim:** The possibility of the geometrization of the stress-energy-tensor and the problems associated with such an undertaking is reviewed again.
**Methods:** The usual tensor calculus rules were used.
**Results:** The stress-energy-tensor was geometrized. The stress-energy-tensor of the electromagnetic field was geometrized too. A method how to calculate the value of the anti-cosmological constant $\underline{\Lambda}$ is developed.
**Conclusion:** Finally, Einstein's field equations are geometrized completely .

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## Introduction

Gravity[1] as the dominant interaction at large length scales is an essential part of cosmology and is equally deeply interrelated with energy[2] time[3] and space. Einstein[4] introduced a new way of representing gravity by replacing the single gravitational potential and the associated field equation of Newton's theory. One of the basic features of Einstein's theory of general relativity (GTR) and equally that what distinguishes GTR sharply from all other competing physical theories, is the geometrization of a physical interaction which opened the theoretical possibility to understand the gravitational field as something like the manifestation of space - time curvature.Einstein's point of view was that the gravitational field can be described by using particular mathematical tools like a metric tensor $g_{\mu\nu}$. However, this need not imply that gravity is and has to be reduced to geometry in its own right. In point of fact, Einstein's stress-energy momentum tensor of GTR is a weak spot of his theory because this field is thus far devoid[5] of any geometrical significance. Various proposals for a unified field theory "**a generalization of the theory of the gravitational field**"[6] were influenced by the desired replacement of the stress-energy momentum tensor of matter by geometrical structures. In order to bring some order into the many different ways to include the electromagnetic field into a geometric setting, Einstein's theory of general relativity[7] can serve as a point of departure for this undertaking. However, I do not see any reason to assume that 'geometrization " and 'unification 'are incompatible. Still, both need not to be conceptually identical. A complete geometrization of Einstein's gravitational field equations could eventually end up at a unified field theory in the sense of Weyl and Eddington's classical field theory in which all fundamental interactions are described by objects of space-time geometry. Trying to answer these and related questions[8] was the subject of many publications and is of this paper too.

[1]NEWTON, Isaac: Philosophiae naturalis principia mathematica, Londini 1687.
[2]EINSTEIN, A.: Zur Elektrodynamik bewegter Körper, in: Annalen der Physik 10 (1905), 891–921.
[3]BARUKČIĆ, Ilija: The Equivalence of Time and Gravitational Field, in: Physics Procedia (2011), 56–62.
[4]EINSTEIN, Albert: Die Feldgleichungen der Gravitation, in: Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 844-847. (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie, in: Annalen der Physik (1916), 769–822; EINSTEIN, Albert: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie, in: Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 142-152. (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe, in: Proceedings of the National Academy of Sciences of the United States of America 3 (1932), 213–214.
[5]GOENNER, Hubert F. M.: On the History of Unified Field Theories, in: Living Reviews in Relativity 1 (2004).
[6]EINSTEIN, Albert: On the Generalized Theory of Gravitation, in: Scientific American (1950), 13–17.
[7]BARUKČIĆ, Ilija: The Geometrization of the Electromagnetic Field, in: Journal of Applied Mathematics and Physics 12 (2016), 2135–2171.
[8]EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932).

## Material and methods

The Royal Society of London and the Royal Astronomical Society announced at their joint meeting on the sixth of November 1919 that astronomical observations made by a special British team during the solar eclipse on May 29 provided the first empirical test of the validity of Einstein's general theory of relativity. In order to obtain a kind of a deeper knowledge of the foundations of nature and physics as such it seems therefore that the basic concepts should be in accordance with Einstein's general of relativity[9] from the beginning. In point of fact, attempts to extend general relativity's geometrization of gravitational force to non-gravitational interactions, in particular, to electromagnetism[10], were not in vain.

### Definitions

**Definition 3.1 (Anti tensor).** Let $a_{\mu\nu}$ denote a certain tensor. Let $b_{\mu\nu}$ denote another tensor. Let $E_{\mu\nu}$ denote a thrid tensor. Let the relationship $a_{\mu\nu} + b_{\mu\nu} \equiv E_{\mu\nu}$ be given. The anti tensor of a tensor, i. e. $a_{\mu\nu}$ is denoted in general as $\underline{a}_{\mu\nu}$ and defined as

$$\underline{a}_{\mu\nu} \equiv E_{\mu\nu} - a_{\mu\nu} \equiv b_{\mu\nu} \tag{1}$$

There are circumstances were an anti-tensor is identical with an anti-symmetrical tensor, but both are not identical as such.

**Definition 3.2 (Einstein's field equations).** Let $R_{\mu\nu}$ denote the Ricci tensor[11] of 'Einstein's general theory of relativity'[12], a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let R denote the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let $\Lambda$ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "**anti cosmological constant**"[13]. Let $g_{\mu\nu}$ metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's curvature tensor. Let $\underline{G}_{\mu\nu}$ denote the "**anti tensor**"[14] of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote stress-energy tensor of energy. Let $\underline{E}_{\mu\nu}$ denote tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let $\gamma$ denote Newton's gravitational "**constant**"[15]. Let $\pi$ denote the number pi. Einstein's field equation, published by Albert Einstein[16] for the first time in 1915, and finally 1916[17] but later with the "**cosmological constant**"[18] term are determined as

$$R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv E_{\mu\nu} \tag{2}$$

The stress-energy tensor of energy $E_{\mu\nu}$ is determined in detail as follows.

$$\begin{aligned} E_{\mu\nu} &\equiv \left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \\ &\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\ &\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \end{aligned} \tag{3}$$

[9] EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916).
[10] BARUKČIĆ, I.: The Geometrization of the Electromagnetic Field (2016).
[11] RICCI, M. M. G. / LEVI-CIVITA, T.: Méthodes de calcul différentiel absolu et leurs applications, in: Mathematische Annalen 1 (1900), 125–201.
[12] EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916).
[13] BARUKČIĆ, Ilija: Anti Einstein – Refutation of Einstein's General Theory of Relativity, in: International Journal of Applied Physics and Mathematics 1 (2015), 18–28.
[14] Idem: Unified Field Theory, in: Journal of Applied Mathematics and Physics 08 (2016), 1379–1438.
[15] Idem: Anti Newton - Refutation Of The Constancy Of Newton's Gravitatkional Constant G: List of Abstracts, in: Quantum Theory: from Problems to Advances (QTPA 2014) : Växjö, Sweden, June 9-12, 2014 (2014), 63; idem: Anti Einstein – Refutation of Einstein's General Theory of Relativity (2015); idem: Newton's Gravitational Constant Big G Is Not a Constant, in: Journal of Modern Physics 06 (2016), 510–522; idem: Unified Field Theory (2016).
[16] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915).
[17] EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916).
[18] EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy, in: Bulletin of the American Mathematical Society 4 (1935), 223–230.

However, the left-hand side of the Einstein's field equations represents only part (Ricci curvature) of the geometric structure (Weyl curvature).

**Definition 3.3 (The tensor of non-energy).** Under conditions of Einstein's general[19] theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:

$$\underline{E}_{\mu\nu} \equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \equiv c_{\mu\nu} + d_{\mu\nu} \tag{4}$$

**Definition 3.4 (The anti Einstein's curvature tensor or the tensor or non-curvature ).** Under conditions of Einstein's general[20] theory of relativity, the tensor of non-curvature is defined/derived/determined as follows:

$$\underline{G}_{\mu\nu} \equiv R_{\mu\nu} - G_{\mu\nu} \equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)\right) \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu} \equiv b_{\mu\nu} + d_{\mu\nu} \tag{5}$$

**Definition 3.5 (The tensor $\mathbf{d}_{\mu\nu}$ (neither curvature nor momentum)).** Under conditions of Einstein's general[21] theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined as follows:

$$d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - b_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - c_{\mu\nu} \tag{6}$$

There may exist circumstances where this tensor indicates pure vacuum, the space devoid of any matter.

**Definition 3.6 (The tensor $\mathbf{c}_{\mu\nu}$).** Under conditions of Einstein's general[22] theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined as follows:

$$c_{\mu\nu} \equiv b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{7}$$

**Definition 3.7 (The tensor $\mathbf{b}_{\mu\nu}$).** The co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. Under conditions of Einstein's general[23] theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field[24] expressed more compactly and in a coordinate-independent is

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,c} \times F_{\nu\,d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{8}$$

where $F_{de}$ is called the (traceless) Faraday/electromagnetic/field strength tensor.

**Definition 3.8 (The stress-energy tensor of ordinary matter $\mathbf{a}_{\mu\nu}$).** Under conditions of Einstein's general[25] theory of relativity, the stress-energy tensor of ordinary matter $a_{\mu\nu}$ is defined/derived/determined as follows:

$$a_{\mu\nu} \equiv a_{\mu\nu} + b_{\mu\nu} - b_{\mu\nu} \equiv a_{\mu\nu} + c_{\mu\nu} - c_{\mu\nu} a_{\mu\nu} \equiv R_{\mu\nu} - b_{\mu\nu} - c_{\mu\nu} - d_{\mu\nu} \tag{9}$$

[19] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[20] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[21] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[22] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[23] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[24] HUGHSTON, L. P. / TOD, K. P.: An introduction to general relativity (London Mathematical Society student texts 5), Cambridge ; New York 1990, p. 38.

[25] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

or equally as

$$a_{\mu\nu} \equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) - b_{\mu\nu} \equiv R_{\mu\nu} - \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) + d_{\mu\nu} \tag{10}$$

or

$$a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu \mathrm{c}} \times F_{\nu \mathrm{d}} \times g^{\mathrm{cd}}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{11}$$

**Definition 3.9 (The Ricci tensor $\mathbf{R}_{\mu\nu}$).** Under conditions of Einstein's general[26] theory of relativity, the Ricci tensor is defined/derived/determined as follows:

$$R_{\mu\nu} \equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) + \left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)\right) \tag{12}$$

**Definition 3.10 (The Ricci scalar R).** Under conditions of Einstein's general[27] theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda $\Lambda$ and anti-lamda[28] $\underline{\Lambda}$ as

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \tag{13}$$

A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general it is

$$R \times g_{\mu\nu} \equiv \left(\Lambda \times g_{\mu\nu}\right) + \left(\underline{\Lambda} \times g_{\mu\nu}\right) \tag{14}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Table 1 provides an overview of the definitions of the four basic[29] fields of nature.

| | | Curvature | | |
|---|---|---|---|---|
| | | YES | NO | |
| Momentum | YES | $a_{\mu\nu}$ | $b_{\mu\nu}$ | $E_{\mu\nu}$ |
| | NO | $c_{\mu\nu}$ | $d_{\mu\nu}$ | $\underline{E}_{\mu\nu}$ |
| | | $G_{\mu\nu}$ | $\underline{G}_{\mu\nu}$ | $R_{\mu\nu}$ |

**Table 1.** Einstein field equations and the four basic fields of nature

**Definition 3.11 (The inverse metric tensor $\mathbf{g}^{\mu\nu}$ and the metric tensor $\mathbf{g}_{\mu\nu}$).** Under conditions of Einstein's general[30] theory of relativity, it is[31]:

$$g_{\mu\nu} \times g^{\mu\nu} \equiv +4 \tag{15}$$

[26] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[27] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[28] BARUKČIĆ, I.: Anti Einstein – Refutation of Einstein's General Theory of Relativity (2015).

[29] Idem: Unified Field Theory (2016); idem: The Geometrization of the Electromagnetic Field (2016).

[30] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[31] Idem: Die Grundlage der allgemeinen Relativitätstheorie (1916), p. 796.

or

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{16}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other.

Einstein's point of view is that "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"[32]

**Definition 3.12 (Laue's scalar T).** Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar[33] (criticised[34] by Einstein) as the contraction of the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Under conditions of Einstein's general[35] theory of relativity, it is

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{17}$$

where $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter"[36]. In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense."[37]

**Definition 3.13 (Index raising).** For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[38] raises each index. In simple words, it is

$$F^{\left(\begin{smallmatrix} 1 & 3 \\ \mu & c \end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix} 1 & 2 \\ \mu & \nu \end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix} 3 & 4 \\ c & d \end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix} \nu & d \\ 2 & 4 \end{smallmatrix}\right)} \tag{18}$$

or more professionally

$$F^{\mu\,\mathrm{c}} \equiv g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\,\mathrm{d}} \tag{19}$$

[32] EINSTEIN, Albert: The meaning of relativity. Four lectures delivered at Princeton University, May, 1921, Princeton 1923, p. 88.

[33] LAUE, M.: Zur Dynamik der Relativitätstheorie, in: Annalen der Physik 8 (1911), 524–542.

[34] EINSTEIN, Albert / GROSSMANN, Marcel: Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation: Physikalischer Teil von Albert Einstein. Mathematischer Teil von Marcel Grossmann, Leipzig 1913.

[35] EINSTEIN, A.: Die Feldgleichungen der Gravitation (1915); EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932); EINSTEIN, A.: Elementary Derivation of the Equivalence of Mass and Energy (1935).

[36] EINSTEIN, A.: The meaning of relativity. Four lectures delivered at Princeton University, May, 1921 (1923), p. 88.

[37] Ibid., p. 93.

[38] KAY, David C.: Schaum's outline of theory and problems of tensor calculus (Schaum's outline series. Schaum's outline series in mathematics), New York 1988.

The following figure 1 may illustrate Einstein's field equations from a *possible* probabilistic point of view.

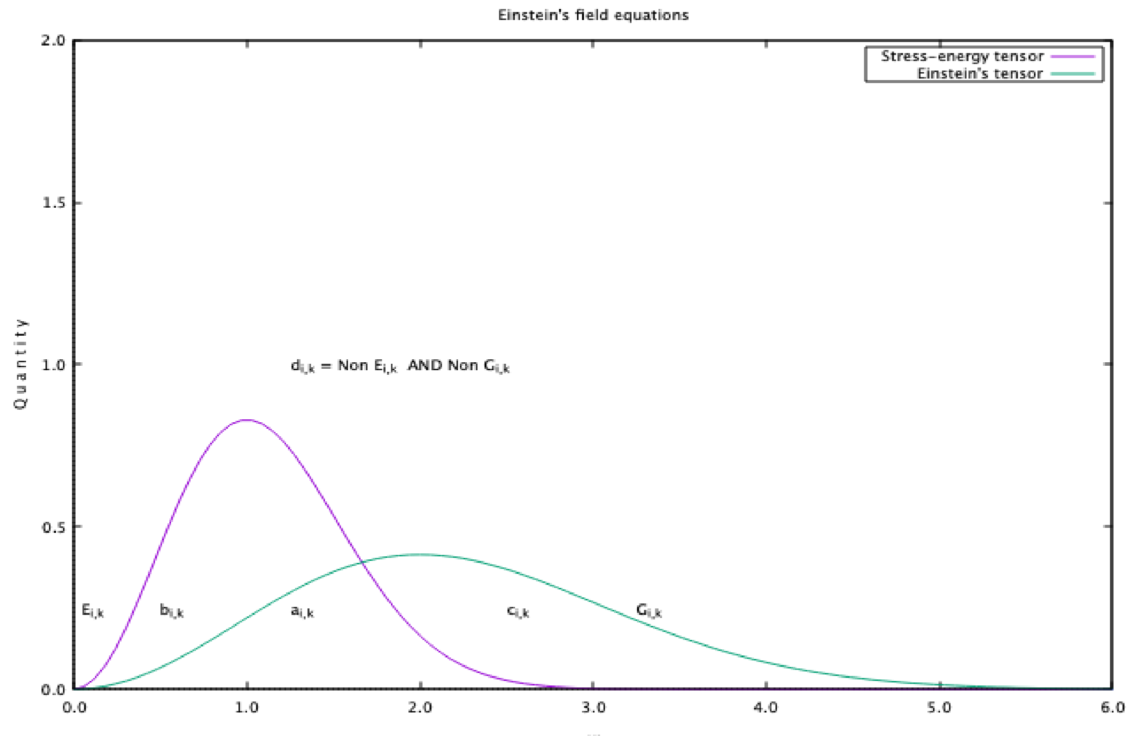


**Figure 1.** Einstein's field equation and probability theory

## 3.1 Axioms

## 3.2 Axioms in general

Axioms[39] and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms[40] too. Einstein himself brings it again to the point.[41]

> "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**."[42]

Einstein's previous position now been translated into English: *The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction.* It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from the same as a main logical foundation of any 'theory'.

> "**Grundgesetz (Axiome)**
> und
> **Folgerungen**
> zusammen bilden das was man
> **eine 'Theorie'**
> nennt. "[43]

Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited **the law of excluded middle** and **the law of contradiction** as examples of axioms. However, **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally as the most simple axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716):

> "**Chaque chose est ce qu'elle est.**
> **Et dans autant d'exemples qu'on voudra**
> **A est A, B est B**. "[44]

or **A = A**, **B = B** or **+1 = +1**. In this context, **lex contradictionis**, the negative of lex identitatis, or **+0 = +1** is of no minor importance too.

#### *3.2.1 Axiom I. Lex identitatis*

To say that +1 is identical to +1 is to say that both are the same.

**AXIOM 1. LEX IDENTITATIS.**

$$\left(g_{\mu\nu} \times g^{\mu\nu}\right) - 3 \equiv +1 \tag{20}$$

or

$$+1 \equiv +1 \tag{21}$$

[39] HILBERT, David: Axiomatisches Denken, in: Mathematische Annalen 1 (1917), 405–415.
[40] EASWARAN, Kenny: The Role of Axioms in Mathematics, in: Erkenntnis 3 (2008), 381–391.
[41] see EINSTEIN, Albert: Induktion and Deduktion in der Physik, in: Berliner Tageblatt and Handelszeitung (1919), Suppl. 4, hier p. 17.

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and be equally different from itself? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

#### *3.2.2 Axiom II. Lex contradictionis*

**AXIOM 2. LEX CONTRADICTIONIS.**

$$\left(g_{\mu\nu} \times g^{\mu\nu} - 3\right) - \left(g_{\mu\nu} \times g^{\mu\nu} - 3\right) \equiv +1 \tag{22}$$

or

$$+0 \equiv +1 \tag{23}$$

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (**a path is a straight line** from the standpoint of a co-moving observer at a certain point in space-time) **and** the other of itself, its own opposition (**the same path is not a straight line**, the same path is curved, from the standpoint of a stationary observer **at a certain point in space-time**)[45]. We may simply deny the existence of objective or of any other contradictions. Furthermore, even if it remains especially according to Einstein's special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is justified, Einstein's special theory of relativity insist that contradictions are objective and real. That this is so highlights the fact that from the standpoint of a co-moving observer, under certain circumstances, **a path is a straight line** and nothing else. However, under the same circumstances of special theory of relativity where the relative velocity v $> 0$, from the standpoint of a stationary observer **the same path is a not a straight line, the path is curved**. The justified question is, why should and how can an identical be a contradictory too?

#### *3.2.3 Axiom III. Lex negationis*

**AXIOM 3. LEX NEGATIONIS.**

$$\neg(+0) = (+1) \tag{24}$$

where $\neg$ denotes the (natural/logical) process of negation.

[45] BARUKČIĆ, Ilija: Aristotle's law of contradiction and Einstein's special theory of relativity, in: Journal of Drug Delivery and Therapeutics 2 (2019), 125–143.

## Results

**Theorem 3.1** (**The geometrical form of the stress-energy tensor of the electromagnetic field $\mathbf{b}_{\mu\nu}$**).

Within the frame of Einstein's theory of general[46] relativity the geometrization of the electromagnetic fields has been left behind as an unsolved problem. Many different trials proposed its own way to extend the geometry of general relativity that would, so it seemed, serve as a geometrization of the electromagnetic field as well. However, the conceptual differences between the geometrized gravitational field and the classical Maxwellian theory of the electromagnetic filed were so far insurmountable.
**Claim.**
In general, the completely geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is given by

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\text{ c}} \times F^{\mu\text{ c}}\right) - \left(F_{\text{de}} \times F^{\text{de}}\right)\right) \times g_{\mu\nu} \tag{25}$$

**Proof By Modus Ponens.**
**If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{26}$$

is true, **then** the following conclusion

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\text{ c}} \times F^{\mu\text{ c}}\right) - \left(F_{\text{de}} \times F^{\text{de}}\right)\right) \times g_{\mu\nu} \tag{27}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{28}$$

is true. Multiplying this premise by the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$, we obtain

$$(+1) \times b_{\mu\nu} \equiv (+1) \times b_{\mu\nu} \tag{29}$$

or

$$b_{\mu\nu} \equiv b_{\mu\nu} \tag{30}$$

Rearranging equation according to the definition 3.7 it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\text{ c}} \times F_{\nu\text{ d}} \times g^{\text{cd}}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\text{de}} \times F^{\text{de}}\right)\right)\right) \tag{31}$$

Rearranging equation before again it is

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{4}{4} \times \left(F_{\mu\text{ c}} \times F_{\nu\text{ d}} \times g^{\text{cd}}\right)\right) - \left(\left(\frac{1}{4} \times F_{\text{de}} \times F^{\text{de}}\right) \times g_{\mu\nu}\right)\right) \tag{32}$$

Rearranging the equation before, we obtain

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{4 \times \left(F_{\mu\text{ c}} \times F_{\nu\text{ d}} \times g^{\text{cd}}\right)}{4}\right) - \left(\left(\frac{F_{\text{de}} \times F^{\text{de}}}{4}\right) \times g_{\mu\nu}\right)\right) \tag{33}$$

According to definition 3.11 this equation simplifies as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{\left(g_{\mu\nu} \times g^{\mu\nu}\right) \times \left(F_{\mu\text{ c}} \times F_{\nu\text{ d}} \times g^{\text{cd}}\right)}{4}\right) - \left(\left(\frac{F_{\text{de}} \times F^{\text{de}}}{4}\right) \times g_{\mu\nu}\right)\right) \tag{34}$$

[46] Einstein, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916).

or as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi} \times \left( \left( \frac{\left(g_{\mu\nu} \times g^{\mu\nu}\right) \times \left(F_{\mu\ \mathrm{c}} \times F_{\nu\ \mathrm{d}} \times g^{\mathrm{cd}}\right)}{4} \right) - \left( \left( \frac{F_{\mathrm{de}} \times F^{\mathrm{de}}}{4} \right) \times g_{\mu\nu} \right) \right) \tag{35}$$

and equally as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi} \times \left( \left( \left( \frac{g^{\mu\nu} \times \left(F_{\mu\ \mathrm{c}} \times F_{\nu\ \mathrm{d}} \times g^{\mathrm{cd}}\right)}{4} \right) \times g_{\mu\nu} \right) - \left( \left( \frac{F_{\mathrm{de}} \times F^{\mathrm{de}}}{4} \right) \times g_{\mu\nu} \right) \right) \tag{36}$$

A further simplification of the relationship before yields the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ determined by the metric tensor of general relativity $\mathrm{g}_{\mu\nu}$ as

$$b_{\mu\nu} \equiv \frac{1}{4\times\pi} \times \left( \frac{1}{4} \times \left( \left(F_{\mu\ \mathrm{c}} \times g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\ \mathrm{d}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \right) \times g_{\mu\nu} \tag{37}$$

However, the term $\left(\left(F_{\mu\ \mathrm{c}} \times g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\ \mathrm{d}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)$ of the equation before can be simplified further. For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[47] raises each index. In other words, it is according to definition 3.13 it is in general $F^{\left(\begin{smallmatrix} 1 & 3 \\ \mu & c \end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix} 1 & 2 \\ \mu & \nu \end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix} 3 & 4 \\ c & d \end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix} \nu & d \\ 2 & 4 \end{smallmatrix}\right)}$ or more professionally $F^{\mu\ \mathrm{c}} \equiv g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\ \mathrm{d}}$ which simplifies the term above as

$$\left( \frac{1}{4} \times \left( \left(F_{\mu\ \mathrm{c}} \times F^{\mu\ \mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \right) \tag{38}$$

or as

$$\frac{1}{4} \times \left( \left(F_{\mu\ \mathrm{c}} \times F^{\mu\ \mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \tag{39}$$

This relationship simplifies the geometrical form of the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ further as

$$b_{\mu\nu} \equiv \frac{1}{4\times 4\times\pi} \times \left( \left(F_{\mu\ \mathrm{c}} \times F^{\mu\ \mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \times g_{\mu\nu} \tag{40}$$

The stress-energy momentum tensor of the electromagnetic field is geometrized completely, our conclusion is true.
**QUOD ERAT DEMONSTRANDUM.**

*Remark.* Whatever the geometrical form of the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ may be, the same tensor determines the tensor $\mathrm{d}_{\mu\nu}$ as $d_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) - b_{\mu\nu}$ (see definition 3.5). Based on the result of the theorem 3.1, the tensor $\mathrm{d}_{\mu\nu}$ is determined as

$$d_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) - \frac{1}{4\times 4\times\pi} \times \left( \left(F_{\mu\ \mathrm{c}} \times F^{\mu\ \mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \times g_{\mu\nu} \tag{41}$$

or as

$$d_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) - \frac{1}{4\times 4\times\pi} \times \left( \left(F_{\mu\ \mathrm{c}} \times F^{\mu\ \mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right) \right) \right) \times g_{\mu\nu} \tag{42}$$

[47] KAY, D. C.: Schaum's outline of theory and problems of tensor calculus (1988).

This relationship can be simplified as

$$d_{\mu\nu} \equiv \left( \left( \frac{2 \times R \times 4 \times \pi}{2 \times 2 \times 4 \times \pi} \right) - \frac{1}{4 \times 4 \times \pi} \times \left( \left( F_{\mu\ c} \times F^{\mu\ c} \right) - \left( F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) \times g_{\mu\nu} \tag{43}$$

or as

$$d_{\mu\nu} \equiv \left( \left( \frac{(4 \times 2 \times \pi \times R) - 1}{4 \times 4 \times \pi} \right) \times \left( \left( F_{\mu\ c} \times F^{\mu\ c} \right) - \left( F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) \times g_{\mu\nu} \tag{44}$$

However, there are circumstances were even this relationship can be simplified further. Let us assume in the following that the tensor $d_{\mu\nu}$ is determined by the relationship $d_{\mu\nu} \equiv - \Lambda \times g_{\mu\nu}$. **Under these conditions**, it is

$$d_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) - b_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) - \left( \Lambda \times g_{\mu\nu} \right) - c_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} \tag{45}$$

and

$$b_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + \Lambda \times g_{\mu\nu} \tag{46}$$

Equally it is

$$c_{\mu\nu} \equiv \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) \tag{47}$$

Following Tonnelat's[48] spirit gives us the unified field of gravitation and electromagnetism as "a theory joining the gravitational and the electromagnetic field into one single hyperfield whose equations represent the conditions imposed on the geometrical structure of the universe"[49] Under the conditions above, the unified field of gravitation and electromagnetism is given by

$$b_{\mu\nu} + c_{\mu\nu} \equiv \left( R \times g_{\mu\nu} \right) + \left( \Lambda \times g_{\mu\nu} \right) \tag{48}$$

while the stress-energy tensor of ordinary matter $a_{\mu\nu}$ is determined as

$$a_{\mu\nu} \equiv \left( \left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right) - b_{\mu\nu} \equiv G_{\mu\nu} + \left( \Lambda \times g_{\mu\nu} \right) - b_{\mu\nu} \equiv R_{\mu\nu} - \left( R \times g_{\mu\nu} \right) + 0 \tag{49}$$

**Under conditions** were $d_{\mu\nu} \equiv - \Lambda \times g_{\mu\nu}$, the following table 2 may provide an overview.

| | | Curvature: YES | Curvature: NO | |
|---|---|---|---|---|
| Momentum | YES | $R_{\mu\nu} - \left( R \times g_{\mu\nu} \right)$ | $\left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + \Lambda \times g_{\mu\nu}$ | $\left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) \times g_{\mu\nu}$ |
| | NO | $\left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right)$ | $-\Lambda \times g_{\mu\nu}$ | $\left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) - \Lambda \times g_{\mu\nu}$ |
| | | $G_{\mu\nu}$ | $\left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right)$ | $R_{\mu\nu}$ |

**Table 2.** Einstein field equations and the four basic fields of nature in detail

[48] TONNELAT, Marie-Antoinette: La théorie du champ unifié d'Einstein et quelques-uns de ses développements, Paris (France) 1955.
[49] Ibid.

**Theorem 3.2** (**THE GEOMETRICAL FORM OF THE STRESS-ENERGY TENSOR $\mathbf{T}_{\mu\nu}$**).

The starting point of Einstein's theory of general relativity is that gravity as such is a property of space-time geometry. Consequently, Einstein published a geometric theory of gravitation[50] while Einstein's initial hope to construct a purely geometric theory of gravitation in which even the sources of gravitation themselves would be of geometric origin has still not been fulfilled. Einstein's field equations have a source term, the stress-energy tensor of matter, radiation and vacuum et cetera, which is of order two and is still devoid of any geometry and free of any geometrical significance.

**CLAIM.**

In general, the completely geometrical form of the stress-energy momentum tensor of Einstein's theory of general relativity is given by

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\left(\frac{2\times \pi\times \gamma}{c^4}\right)\times T\right)\times g_{\mu\nu} \tag{50}$$

**PROOF BY MODUS PONENS.**

**If** the premise of modus ponens

$$\underbrace{+1=+1}_{(Premise)} \tag{51}$$

is true, **then** the following conclusion

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\left(\frac{2\times \pi\times \gamma}{c^4}\right)\times T\right)\times g_{\mu\nu} \tag{52}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1)=(+1) \tag{53}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1)\times\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv (+1)\times\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \tag{54}$$

or

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \tag{55}$$

and equally

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\frac{2\times \pi\times \gamma}{c^4}\right)\times 4\times T_{\mu\nu} \tag{56}$$

According to definition 3.11 this equation simplifies as

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\frac{2\times \pi\times \gamma}{c^4}\right)\times \left(g_{\mu\nu}\times g^{\mu\nu}\right)\times T_{\mu\nu} \tag{57}$$

or as

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\frac{2\times \pi\times \gamma}{c^4}\right)\times g_{\mu\nu}\times \left(g^{\mu\nu}\times T_{\mu\nu}\right) \tag{58}$$

According to the definition of Laue's scalar (definition 3.12) it is equally

$$\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv \left(\frac{2\times \pi\times \gamma}{c^4}\right)\times g_{\mu\nu}\times T \tag{59}$$

[50] EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916).

The desired geometrical representation of the stress-energy momentum tensor of Einstein's general theory of relativity follows as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) \times g_{\mu\nu} \tag{60}$$

In other words, our conclusion is true.
**QUOD ERAT DEMONSTRANDUM.**

*Remark.* From the geometrical point of view the stress–energy momentum tensor $T_{\mu\nu}$ is more or less identical with the metric, enriched only by view constants and a scalar as

$$\left(\frac{\gamma \times 2 \times \pi \times T}{c^4}\right) \times g_{\mu\nu} \tag{61}$$

Using geometrized units where $\gamma = c = 1$, the geometrical form of the stress–energy momentum tensor $T_{\mu\nu}$ can be rewritten as

$$(2 \times \pi \times T) \times g_{\mu\nu} \tag{62}$$

However, describing the fundamental stress–energy momentum tensor $T_{\mu\nu}$, the source term of the gravitational field in Einstein's general theory of relativity, as an inherent geometrical structure, as being determined and dependent on the metric field $g_{\mu\nu}$ is associated with several and far reaching consequences. The properties of energy, momentum, mass, stress et cetera need no longer to be seen as intrinsic properties of matter. Theoretically, the properties which material systems posses could be determined in virtue of their relation to space-time structures too. The question could arise whether the energy tensor $T_{\mu\nu}$ at the end could be in different aspects less fundamental than the metric field $g_{\mu\nu}$ itself. Is and why is matter more fundamental[51] than space-time? In contrast to such a position, is the assumption justified that **without** the space-time structure encoded in the metric **no** energy tensor? To bring it to the point, can space-time (and its geometric structure) exist without matter and if yes, what kind of existence could this be? Einstein's starting point was to derive space-time structure from the properties of material systems. In contrast to this position, theorem 3.2 allow us to see that, on the contrary, the energy tensor depend on the metric field and is completely determined by the metric field. Consequently, the matter fields themselves are derivable from the structure of space-time or the very definition of an energy tensor is determined by space-time structures too. Thus far, the question is not answered definitely, which came first, either space-time structure or energy tensor.So it is reasonable to ask, is the energy-momentum tensor of matter only dependent on the structure of space-time or even determined by the structure of space-time or both or none? In other words, granddaddies **either chicken or the egg** dilemma is asking for an innovative and a comprehensive solution and may end up in an Anti-Machian theory. However, this leads us at this point too far afield.

[51] LEHMKUHL, D.: Mass-Energy-Momentum: Only there Because of Spacetime?, in: The British Journal for the Philosophy of Science 3 (2011), 453–488; LEHMKUHL, Dennis: Why Einstein did not believe that general relativity geometrizes gravity, in: Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics (2014), 316–326.

**Theorem 3.3** (**EINSTEIN'S FIELD EQUATION'S COMPLETELY GEOMETRIZED**).

Now, we can derive a completely geometrical form of Einstein's field equation's.

**CLAIM.**

In general, the completely geometrical form of Einstein's field equation's[52] is given by

$$R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) \times g_{\mu\nu} \tag{63}$$

**PROOF BY MODUS PONENS.**

**If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{64}$$

is true, **then** the following conclusion

$$R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) \times g_{\mu\nu} \tag{65}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{66}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv (+1) \times \left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \tag{67}$$

or

$$\left( \frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times 4 \times T_{\mu\nu} \tag{68}$$

Rearranging equation according to the definition 3.2 it is

$$R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times 4 \times T_{\mu\nu} \tag{69}$$

According to theorem 3.2 the equation can be simplified. Consequently, the conclusion is true that

$$R_{\mu\nu} - \left( \left( \frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) \times g_{\mu\nu} \tag{70}$$

**QUOD ERAT DEMONSTRANDUM.**

[52] EINSTEIN, A.: Die Grundlage der allgemeinen Relativitätstheorie (1916); EINSTEIN, A.: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917); EINSTEIN, A. / DE SITTER, W.: On the Relation between the Expansion and the Mean Density of the Universe (1932).

**Theorem 3.4** (**EINSTEIN'S COSMOLOGICAL CONSTANT $\Lambda$**).

An even more severe violation of our trust into physics is created by the cosmological constant $\Lambda$, which specifies as the overall vacuum energy density. Depending on the specific assumptions made, the physical value[53] of the cosmological constant $\Lambda$ is found to be very contradictory. Now, we can calculate the value of the cosmological constant $\Lambda$ very precisely.

**CLAIM.**
In general, the value of the cosmological constant $\Lambda$ is given by

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{4}\right) \tag{71}$$

**PROOF BY MODUS PONENS.**
**If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{72}$$

is true, **then** the following conclusion

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{4}\right) \tag{73}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{74}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{75}$$

and equally

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \tag{76}$$

According to definition 3.11 this equation simplifies as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times \left(g_{\mu\nu} \times g^{\mu\nu}\right) \times T_{\mu\nu} \tag{77}$$

In accordance with the definition 3.12 we obtain the desired geometrical representation of the stress-energy momentum tensor. In other words, the conclusion that

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times g_{\mu\nu} \times T \tag{78}$$

Rearranging equation according to the definition 3.2 it is

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \tag{79}$$

According to the theorem 3.2 the equation can be simplified. According to the theorem 3.3, we obtain the geometrized Einstein's field equation as

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times g_{\mu\nu} \times T \tag{80}$$

[53] WEINBERG, Steven: Anthropic Bound on the Cosmological Constant, in: Physical Review Letters 22 (1987), 2607–2610. (Publisher: American Physical Society)

Rearranging this equation it is

$$\left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times g_{\mu\nu} \times T\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(R_{\mu\nu}\right) \tag{81}$$

Rearranging terms, it is

$$\left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{2}\right)\right) \times g_{\mu\nu} - \left(R_{\mu\nu}\right) \tag{82}$$

Multiplying by the inverse metric $g^{\mu\nu}$ we obtain

$$\left(\Lambda \times g_{\mu\nu}\right) \times g^{\mu\nu} \equiv \left(\left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{2}\right)\right) \times g_{\mu\nu} \times g^{\mu\nu} - \left(R_{\mu\nu}\right) \times g^{\mu\nu} \tag{83}$$

According to the definition 3.10 and the definition 3.11, this equation simplifies as

$$\left(\Lambda \times 4\right) \equiv \left(\left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{2}\right)\right) \times 4 - (R) \tag{84}$$

Dividing the equation by 4 yields

$$\Lambda \equiv \left(\left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{2}\right)\right) - \left(\frac{R}{4}\right) \tag{85}$$

or

$$\Lambda \equiv \left(\left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{2 \times R}{2 \times 2}\right)\right) - \left(\frac{R}{4}\right) \tag{86}$$

or

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{2 \times R}{4}\right) - \left(\frac{R}{4}\right) \tag{87}$$

**The exact value of Einstein's cosmological constant** can be calculated as

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T\right) + \left(\frac{R}{4}\right) \tag{88}$$

with the consequence that the conclusion is true.

**QUOD ERAT DEMONSTRANDUM.**

**Theorem 3.5** (**ANTI COSMOLOGICAL CONSTANT $\underline{\Lambda}$).**

The value of the anti-cosmological constant $\underline{\Lambda}$[54] can be calculated very precisely.
**CLAIM.**
In general, the value of the anti-cosmological constant $\underline{\Lambda}$[55] is given by

$$\underline{\Lambda} \equiv R - \left( \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) + \left( \frac{R}{4} \right) \right) \tag{89}$$

**PROOF BY MODUS PONENS.**
**If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{90}$$

is true, **then** the following conclusion

$$\underline{\Lambda} \equiv R - \left( \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) + \left( \frac{R}{4} \right) \right) \tag{91}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{92}$$

is true. Multiplying this premise by Ricci scalar (see definition 3.10), we obtain

$$(+1) \times (R) \equiv (+1) \times (R) \tag{93}$$

or

$$R \equiv R \tag{94}$$

Adding $\Lambda$ and subtracting $\Lambda$, the cosmological constant, it is

$$R - \Lambda + \Lambda \equiv R - \Lambda + \Lambda \tag{95}$$

or

$$R - \Lambda + \Lambda \equiv R \tag{96}$$

According to our definition 3.10 it is

$$\underline{\Lambda} + \Lambda \equiv R \tag{97}$$

or

$$\underline{\Lambda} \equiv R - \Lambda \tag{98}$$

The exact value of the cosmological constant was calculated by theorem 3.4. The value of the cosmological constant $\Lambda$ can be inserted into the equation before. **The exact value of the anti cosmological constant** can be calculated as

$$\underline{\Lambda} \equiv R - \left( \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) + \left( \frac{R}{4} \right) \right) \equiv \frac{3 \times R}{4} - \left( \left( \frac{2 \times \pi \times \gamma}{c^4} \right) \times T \right) \tag{99}$$

with the consequence that the conclusion is true.

**QUOD ERAT DEMONSTRANDUM.**

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## Discussion

Einstein was one of the first to use explicitly the term "**unified field theory**"in the title[56] of his publication in 1925. In the following, Einstein himself published more than thirty technical papers on this topic. However, Einstein's unified field theory program, besides of his justified insistence on the possibility and desirability of a unified field theory, required a substantial amount of new mathematical preliminaries and methods[57] and was on the level of the mathematical possibilities at his time technically in vain.
At the heart of any enterprise to geometrize all fundamental interactions and to provide a completely geometrized[58] theory of relativity is the assumption too that geometrization could eventually lead to a unification of all known physical interactions. This publication has been able to geometrize Einstein's field equation is completely. A unified field theory program is endangered by the cosmological constant $\Lambda$ too, the energy density of space, or vacuum energy, and the uncertainties associated with the same. To day, there is some experimental evidence (Perlmutter et al.[59] **Supernova Cosmology Project** and Riess et al.[60] **High-Z Supernova Search Team**) that the expansion of the universe is accelerating, implying the possibility of a positive nonzero value for the cosmological constant $\Lambda$. This publication provides the theoretical possibility to calculate even the value of an anti cosmological constant $\underline{\Lambda}$ too.

## Conclusion

Finally, in combination with other already published[61] papers, the field equations of Einstein's general theory of relativity are geometrized completely.

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## Author contributions statement

Ilija Barukčić is the only author of this manuscript.

## Conflict of Interest Statement

Ilija Barukčić declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest. There are no conflict of interest.

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