Modeling the Albedo Advantage in Global Warming And an Albedo-Planck Parameter

Alec Feinberg,
DfRSoft Research,
(Please feel free to provide any helpful preprint comments to dfrsoft@gmail.com)

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Abstract In this paper, we model global warming (GW) using a re-radiation factor and use the Planck's feedback parameter to verify consistency. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to the GHG re-radiation effects. The albedo affect is found to have a 2.6 times larger impact on global warming. In our simple model, we additionally define a handy Planck-albedo feedback parameter having a convenient value of 1W/m^2/^{o}K/Δ%albedo. An alternate way to assess the Planck parameter was also found.

1 Introduction

Although global warming is highly complex, often it is helpful to work with a simplified model. We create a model that uses a re-radiation factor which helps to quantify significant differences between changes in the global albedo versus greenhouse gas forcing. We use the Planck's feedback parameter to verify model consistency. This model illustrates a reasonable way to view the Earth's energy budget; it is likely useful as a teaching aid, it provides a number of useful insights in climatology sensitivity estimates and demonstrates the relative advantage of solar geoengineering solutions over GHG reduction in GW mitigation [1]. In working the model, we also find a handy Planck-albedo parameter that may be useful to climatologists [2].

2. Data and Method

In order to introduce the re-radiation surface model, it is helpful to initially look at the Planck feedback parameter as it plays a key role in verifying modeling.

2.1 Overview of Planck Feedback Parameter

Estimates on Planck's feedback parameter are varied, typically between -3.8W/m^2/^{o}K and -3.21W/m^2/^{o}K with some values as large as -7.1W/m^2/^{o}K [3]. The IPCC AR4 [4] list a value of -3.21W/m^2/^{o}K. Numerous authors have developed different expressions [3]. A typical estimate uses

$$F_{TOA} = (1-\alpha)\, S_o/4 - \sigma(\beta T_S)^4 = (1-\alpha)\, S_o/4 - R_{LWR} \quad (1)$$

where S_o=1361W/m^2, F_{TOA} is the radiation budget at the top of the atmosphere, R_{LWR} is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described below. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA}/\partial T_S = -\partial R_{LWR}/\partial T_S \quad (2)$$

This result is

$$\lambda_o = -4\beta^4\sigma T_S^3 = -4\beta\sigma T_{TOA}^3 \quad (3)$$

where β varies from 0.876 to 0.887 (averaging=0.8815) and Ts=288^{o}K [4]. This yields -3.37W/m^2/^{o}K<λ_o<-3.21W/m^2/^{o}K. However, from Eq. 3, β is often taken as the ratio

$$\beta = T_{TOA}/T_S = 255°K/288°K = 0.8854 \quad (4)$$

A common assessment uses T_{TOA}=255^{o}K, so that λ_o =-3.33W/m^2/^{o}K. Another expression developed by Schlesinger [5] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o\left(1-\alpha\right)/T_S \quad (5)$$

When S_o=1361, 0.294118<α<0.3, and Ts=288 ^{o}K then -3.308W/m^2/^{o}K >λ_o>-3.3358W/m^2/^{o}K , respectively.

2.2 Estimating Planck's Parameter with an Albedo Method

Consider a global albedo change corresponding to 1^oK rise from solar absorption. Since we are only concerned with an albedo change that corresponds to the surface temperature we can write

$$F_{TOA}=0=(1-\alpha)E_o-\sigma(T_S)^4 \tag{6}$$

where $E_o=S_o/4$. Then a 1^oK change is

$$\Delta T_S=T_2-T_1=\left(\frac{E_o}{\sigma}\left(1-\alpha_2\right)\right)^{1/4}-\left(\frac{E_o}{\sigma}\left(1-\alpha_1\right)\right)^{1/4}=1°K \tag{7}$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when $E_o=340W/m^2$. This corresponds to an absorption of

$$\Delta E_o=E_o\left\{\left(1-\alpha_2\right)-\left(1-\alpha_1\right)\right\}=E_o\left(\alpha_1-\alpha_2\right)=3.784W/m^2 \tag{8}$$

Since this is for a 1^oK rise, then it can also be written as

$$\lambda_{1K}=3.784W/m^2/^oK \tag{9}$$

We note this is related to the surface value, then

$$\lambda_{1K}=-4\sigma T_S^3 \tag{10}$$

By comparison to above we have

$$\lambda_o=\lambda_{1K}\,\beta=-3.784W/m^2/^oK=-3.349W/m^2/^oK \tag{11}$$

This is very close to the $-3.33\ W/m^2/^oK$ value obtained in the traditional manner.

2.3 Top of the Atmosphere and Beta

From Eq. 1

$$R_{LWR}=\sigma(\beta T_S)^4=\sigma(T_S)^4 \tag{13}$$

giving

$$\beta^4 R_{TOA,T_S}=R_{TOA,T_{TOA}} \tag{14}$$

We will need this expression later when showing model consistency with the Planck feedback parameter.

2.4 Re-radiation GHG GW Model

Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950 as due to blackbody radiation with the addition of GHG re-radiation so

$$P_{Total\_1950}=P_\alpha+P_{GHG}=P_\alpha+f_1P_\alpha \tag{15}$$

where $P_\alpha=S_o\left\{0.25x(1-Albedo)\right\}$ and $S_o=1361W/m^2$. Here we have a fraction of the blackbody radiation is reradiated by the GHGs so f_1 is a re-radiation parameter. In 2019 due to global warming, modeling is more complex. However in this view it can be modeled in a similar way

$$P_{Total\,2019}=P_{\alpha'}+Pf_{GHG'+Feedback}=P_{\alpha'}+f_2P_{\alpha'} \tag{16}$$

where. Here $P_{GHG'+Feedback}$ includes GHG and its increase comprising also of water-vapor increase, lapse rate effect and other effect such as an increase in snow-ice albedo change that are hard to separate out. That is some of this feedback is related to GHG increases and some is related to albedo change. $P_{\alpha'}$ represents any albedo change due to UHI absorption increases, cloud absorption change, ice and snow melting and so forth that can be discerned.

The re-radiation model connects the absorption to re-radiation. We use a linear f parameter that indicates the fraction of P_α power that must be re-radiated back to obtain the observed temperature. To be clear, f is just a fractional parameter. In 1950 it is some function of the GHGs (with no feedbacks). In 2019 it is more complex and includes feedback effects. However, it primarily related to GHGs re-radiation since $P_{GHG}\approx P_{GHG'+Feedback}$.

We then write

$$P_{Total} = \sigma T^4 \text{ and } P_\alpha = \sigma T_\alpha^4 \tag{17}$$

We will find that $T_\alpha / T \approx \beta$.

2.5 Balancing Pout and Pin

Although Eq. 15 is reasonably simple, it turns out that *f1* is a uniquely defined values obtained by balancing the energy in and the energy out.

2.5.1 Balancing Pout and Pin in 1950

In order to balance the energy in with the energy out, *f* can be shown to have a unique value. In 1950 from Eq. 15 and 17

$$P_{Total\_1950} = P_\alpha + P_{GHG} = P_\alpha + fP_\alpha \tag{18}$$

In equilibrium the radiation that leaves must balance what comes in

$$\begin{aligned} Out &= (1-f)P_\alpha + (1-f)P = (1-f)P_\alpha + (1-f)\{P_\alpha + fP_\alpha\} \\ &= (1-f)\{2P_\alpha + fP_\alpha\} = 2P_\alpha - fP_\alpha - f^2 P_\alpha = In = P_\alpha \end{aligned} \tag{19}$$

The radiation that comes in is just P_α which is the term on the RHS. In 1950 the value of *f* solves the quadratic equation

$$f^2 + f - 1 = 0, \ \ yielding \ \ f = 0.618 \tag{20}$$

2.5.1 Warming Imbalance in 2019

The re-radiation parameters f_1 and f_2 are connected and from Eq. 15 and 16 we have

$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = f_1 + \Delta f \tag{21}$$

In this way f_2 is a function of f_1=0.618 and the differences in the global warming residuals Δf.

3.0 Results and Discussion

Since the re-radiation parameter $f_1 = 0.618$, in order to obtain T_{1950}=13.89°C (287.038°K), the key adjustable parameter in our model turns out to be the Earth's albedo. This value requires an albedo value of 0.3008 (see Table 1). This is a reasonable and similar to values cited in the literature [11].

In 2019, the average temperature of the Earth is T_{2019}=14.84°C (287.99°K). Here we are not sure of the albedo since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [6] is 0.294118. However, this would represent a 3% change since 1950 which may be an overestimation. In our assessment, we will assume a 1% change. Then the f_2 parameter is adjusted to *0.6324* in order to obtain T_{2019}. Results are provided in the Table 1. The results yields $P_{Total1950}=384.9177\ W/m^2$ and $P_{Total1950}=390.024$ W/m². We find that

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.097 W/m^2 \tag{22}$$

and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95°C \tag{23}$$

which is the observed surface temperature increase since 1950.

The table below summarizes model results for the specified albedos and setting the model to the observed Earth's surface temperatures.

Table 1 Model results

Year	T(°K)	T_α(°K)	f_1, f_2	α, α'	P_α, $P_{\alpha'}$ (W/m²)	P_{GHG} (W/m²) $P_{GHG'+feedback}$	P_{Total} (W/m²)
2020	287.989	254.78	0.6324	29.779	238.927	149.870	390.024
1950	287.0395	254.51	0.618	30.08	237.903	147.024	384.918
Δ2020-1950	0.95	0.27	**1.44%**	-0.3 **(1%)**	**1.024**	2.846	**5.097**

To show model consistency, the forcing change 5.097 W/m^2 resulting in a 0.95°K rise, should agree with what is expected from Planck's feedback parameter. From Eq. 14 it is evident that

$$\beta^4 \Delta R_{TOA} = 5.097 \text{ x } \beta^4 = 3.132 \text{W/m}^2 \tag{24}$$

This illustrates the consistency of the simple re-radiation model. Then Planck's feedback parameter temperature rise is in agreement with what is observed

$$3.139 \text{W/m}^2 \text{ x } (1/3.3)^{\circ}\text{K/W/m}^2 = 0.949^{\circ}\text{K at } T_s \tag{25}$$

3.1 Why the Re-radiation Parameter is Significant

In Table 1, the measure of $\Delta f = 1.44\%$ fractional increase is due to re-radiation change. This is significance. It is not out of line with GHGs change. Although, awkward to compare, one can note the ratio of CO_2 from 2019 to 1950 is 412/312=1.32. Given other increase in GHGs and feedbacks, the parameter f, a linear aspect of the model, illustrates reasonable characteristic of the climate.

Therefore f is an estimate of climate re-radiation and Δf an estimate of climate change from a different perspective. It is a measure of GHG increase, and is generally helpful in looking at how our climate is working. Specifically, it is given by Eq. 21. Furthermore, we can deduce an albedo strength.

3.2 The Albedo Advantage

We can look at an important ratio, the re-radiation created by the albedo effect compared to GHGs in 1950 from

$$\frac{fP_\alpha + fP_T}{P_{GHG}} = \frac{fP_\alpha + fP_\alpha + fP_{GHG}}{P_{GHG}} = \frac{fP_\alpha + fP_\alpha + f^2 P_\alpha}{fP_\alpha} = \frac{fP_\alpha + fP_\alpha + f^2 P_\alpha}{fP_\alpha} = \frac{1.62}{0.62} = 2.62 \tag{26}$$

In general, albedo forcing has a higher impact factor in climate forcing, 2.6 times larger than P_{GHG} and is a key reason that UHIs, cloud coverage, snow and ice melting, can create significant climate effects. Appendix A puts this important impact factor in layman's terms.

In this view, an albedo solution is advantageous having significant potential for reversing global warming or ignoring it, as in UHIs likely can create serious issues. Therefore, trying to control global warming by reducing GHGs is important. However, certainly an albedo approach is more advantageous. It reduces both initial absorption and its potential re-radiation. Its impact rating can be taken as 162% compared to re-radiation f with a 62% impact by comparison according to Eq. 26, yielding a 2.6 times higher advantage. It is important to realize that because the albedo solution can highly impact GW and reverse trends, it is also vital in preventing a tipping point from occurring.

3.2 Planck-Albedo Feedback Parameter

The albedo changes in Table 1, is: $\%\Delta\alpha = 1\%$. The albedo ΔP_α change in Table 1 is 1.024W/m^2. We note that we can define a unique Planck-albedo parameter $\lambda_{\%\Delta\alpha} = \Delta P_\alpha / \%\Delta albedo$. To illustrate from Table 1

$$\lambda_{\%\Delta\alpha} = 1.024 \text{ W/m}^2/\Delta\%\text{albedo} = 1.024/1\% \tag{27}$$

This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

$$\lambda_{\%\Delta\alpha\Delta T} \approx 1 W/m^2/\Delta\% albedo/^{\circ}K \tag{28}$$

The parameter was first noted in Feinberg 2020 [2] but is featured here as a modeling tool. We term it the Planck-albedo parameter, since it relates to blackbody (P_α) absorption. This interesting parameter arises from the basic assessment

$$\lambda_{\%\Delta\alpha} = \frac{(\Delta E_o)_\alpha}{\frac{\alpha_1 - \alpha_2}{\alpha_1}100} = \frac{E_o(\alpha_1 - \alpha_2)}{\frac{\alpha_1 - \alpha_2}{\alpha_1}100} = E_o\alpha_1/100 \approx 1W/m^2/\%\Delta albedo \tag{29}$$

where E_o=340 W/m^2 and when α_1 is 29.4118%, the value 1.000W/m^2/Δ%albedo is obtained. We note the value 29.4118% (100/340) is given in AR5 [6]. The parameter's relationship to λ_α is

$$\lambda_\alpha = \lambda_{\%\Delta\alpha\Delta T} \; x \; \%\Delta\alpha \tag{30}$$

and the feedback parameter including f re-radiation is in 2019

$$\lambda_{\alpha}^{\dagger} = \lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha\ x 1.618 \tag{31}$$

4.0 Conclusion

In this paper we provided a simple re-radiation global warming model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.44% due to global warming from 1950 to 2019, illustrating the warming from a different perspective. From the model, the albedo effect was quantified having an impact rating of 162% compared to GHGs with 62%. The albedo effect then yields a 2.6 times higher advantage upon comparison. These results strongly support moving forward with solar geoengineering solutions [2, 7-9].

We also found a handy parameter that we termed the Planck-albedo parameter which is about $\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/{}^{\circ}K$. This can be helpful in quickly estimating the effect of an albedo change on global warming and in assessing λ_{α}. For example, Feinberg 2020 [1] suggested a goal of 1.5% geoengineering albedo change. Using this parameter, an impact of 1.5 Watts/m^2 warming reduction should result. Given a 1.6 reemission factor, this is 2.4W/m^2 improvement. With a reduction in water-vapor feedback, often estimated by a factor of 2 [10], provides an overall resulting effect that could be as high as 4.8W/m^2. Feasibility is discussed in more detail in Feinberg's 2020 paper [1] and other solutions have been proposed [6-9].

Appendix A: Quantifying the Albedo Advantage in Layman's Terms

It may be helpful for the reader to have a layman's view of how the 2.62 factor comes about. Consider the Earth with a roof. The roof represents the GHGs over the Earth and only allows 40% of any energy to leave. Sunlight comes in and some is absorbed and heats the floor to 255^{o}K (-2.3^{o}F very cold). Let's say it takes 100 units of energy. The heat rises but only 40 units of energy can leaves so 60 units comes back and warms the floor some more to 288^{o}K (57^{o}F average temp of Earth). On average the floor is warmed a total of 160 units. The sun keeps warming the floor at 100 units on average and the roof keeps sending back 60. So the roof is responsible for 60 units on average of energy and the floor is warmed up to 160 units on average. We can write this as

Energy units: 160=100+60=100+100x0.6

We see the 100 units is in two places in the equation, while the 60 is only in one place. That is without the floor absorption first the roof cannot keep the Earth warm. Therefore, the floor is responsible for more energy, resulting in160 units and the roof is only 60 units by comparison. The impact factor is

160/60=2.66 floor has this much larger impact

Alternately, for every unit of energy given off, by the floor after absorption it is equivalent to causing 1.6 units of heating while the roof (GHG) is only responsible for 0.6.

How much heat leaves in equilibrium? There was the initial 40 leaving of the 100 initially. As well the floor received a total of 160 units but the roof only let 40% leave that is another 64 (0.4 x 160) leaving. The total leaving is 104 in equilibrium or

104/160=65%, or in terms of the total 160x0.65=105 so roughly 100 comes in and almost same goes out.

This can be refined to 61.8%. Then 100 comes in and 38.2 initially leave, and 61.8 stay so the floor is heated to 161.8. From this 0.382 x 161.8 leaves=61.8 units or energy. The total is 61.8+38.2=100 leaves and another 100 come in for equilibrium.

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