Inverse-transformation of 4-dimensional Rindler spacetime

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ABSTRACT

In special relativity theory, we discovered 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.We try to discover 4-dimensional inverse-transformation of general Rindler space-time.

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I.Introduction

In special relativity theory, we discovered 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames [9].We try to discover 4-dimensional inverse-transformation of general Rindler space-time

At first, 2-dimensional transformation is in Rindler spacetime,

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c}) \quad , x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \tag{1}$$

We know 4-dimensional transformation in Rindler space-time[9],

$$ct = \sinh(\frac{a_0\xi^0}{c})\frac{c^2}{a_0}(1 + \frac{\vec{a}_0}{c^2}\cdot\vec{\xi}) \tag{2}$$

$$\vec{x} = \vec{\xi} + \frac{c^2}{a_0{}^2}\cosh(\frac{a_0\xi^0}{c})\vec{a}_0 - (1 - \cosh(\frac{a_0\xi^0}{c}))\frac{\vec{a}_0\cdot\vec{\xi}}{a_0{}^2}\vec{a}_0 - \frac{c^2}{a_0{}^2}\vec{a}_0 \tag{3}$$

Hence, the proper time is [8]

$$d\tau^2 = dt^2 - \frac{1}{c^2}d\vec{x}\cdot d\vec{x} = dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2)$$
$$= (1 + \frac{\vec{a}_0\cdot\vec{\xi}}{c^2})^2(d\xi^0)^2 - \frac{1}{c^2}((d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2) \tag{4}$$

2. 4-dimensional inverse-transformation in Rindler spacetime

To discover 4-dimensional inverse transformation, if we treat $\frac{\vec{a}_0}{c^2}\cdot Eq(3)$,

$$\frac{\vec{a}_0}{c^2}\cdot\vec{x} = \frac{\vec{a}_0}{c^2}\cdot\vec{\xi} + \cosh(\frac{a_0\xi^0}{c}) - (1 - \cosh(\frac{a_0\xi^0}{c}))\frac{\vec{a}_0\cdot\vec{\xi}}{c^2} - 1 \tag{5}$$

Therefore, Eq(5) is

$$\frac{c^2}{a_0}(1 + \frac{\vec{a}_0}{c^2}\cdot\vec{x}) = \frac{c^2}{a_0}\cosh(\frac{a_0\xi^0}{c})(1 + \frac{\vec{a}_0\cdot\vec{\xi}}{c^2}) \tag{6}$$

Eq(2) is

$$ct = \sinh(\frac{a_0\xi^0}{c})\frac{c^2}{a_0}(1 + \frac{\vec{a}_0}{c^2}\cdot\vec{\xi}) \tag{7}$$

Hence, we compare Eq(6) and Eq(7), we discover 4-dimensional inverse-transformation in Rindler spacetime.

$$\frac{ct}{\frac{c^2}{a_0}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})}=\tanh(\frac{a_0\xi^0}{c}) \quad \rightarrow \xi^0=\frac{c}{a_0}\tanh^{-1}\{\frac{ct}{\frac{c^2}{a_0}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})}\} \tag{8}$$

And,

$$\frac{c^2}{a_0}(1+\frac{\vec{a}_0\cdot\vec{\xi}}{c^2})=\sqrt{\frac{c^4}{{a_0}^2}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})^2-c^2t^2} \tag{9}$$

Hence,

$$\frac{\vec{a}_0\cdot\vec{\xi}}{a_0}=\sqrt{\frac{c^4}{{a_0}^2}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})^2-c^2t^2}-\frac{c^2}{a_0} \tag{10}$$

In this time, if we suppose the condition,

$$\vec{a}_0\cdot\vec{\xi}=a_0\xi\cos\theta, \qquad 0\le\theta\le\pi,\theta\ne\frac{\pi}{2} \ ,$$

$$\left|\vec{\xi}\right|=\xi,\left|\vec{a}_0\right|=a_0 \tag{11}$$

Therefore, we discover 4-dimensional inverse-transformation in Rindler spacetime..

$$\xi^0=\frac{c}{a_0}\tanh^{-1}\{\frac{ct}{\frac{c^2}{a_0}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})}\} \tag{12}$$

$$\xi=\frac{1}{\cos\theta}[\sqrt{\frac{c^4}{{a_0}^2}(1+\frac{\vec{a}_0\cdot\vec{x}}{c^2})^2-c^2t^2} \quad \frac{c^2}{a_0}] \tag{13}$$

3. Conclusion

We know general Rindler coordinate inverse-transformation from 4-dimensional Rindler transformation..

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