Reflections on the Foundations of Quantum Mechanics

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Abstract

In this paper the quantised version of Newton Second Law is derived assuming merely the existence of de Broglie matter-waves and their basic properties. At the same time we keep an eye towards interpretations of quantum mechanics and will realise that the two most different interpretations –Copenhagen interpretation and the de Broglie-Bohm theory– owe their difference to two fundamentally different approaches to 'Harmonisation'. In this regard we shall see that the guiding equation of the de Broglie-Bohm theory currently found in literature is not the most complete equation possible; as a result we answer one of the important questions in interpreting quantum mechanics, namely that 'when does the concept of classical path (trajectory) makes sense in quantum mechanics?'

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1 Introduction

One of the most important constituents of our current understanding of the universe is the theory of quantum mechanics, on its exemplary utility in analysing microphysics we have almost no doubt. Apart from the measurement problem, which is almost certain to be resolved only by a complete change of paradigm, the theory faces dead-ends in three main categories:

- Significance of Potentials, in light of the Aharonov-Bohm effect.
- Perturbation theory, from the standpoint of epistemology it is not always applicable and leads to divergences requiring the ad-hoc remedies of renormalisation. From an ontological perspective, a realist physicist tend to believe that nature is not perturbative.
- Application to Dissipative systems. This problem is not a fundamental one as we tend to believe that non-conservative forces exist only to represent a whole lot of complicated stories going on in a 'lower' level when we choose to be in a domain which is 'not fundamental enough'. Nevertheless from a practical point of view it is inevitable to consider such problems in domains like Molecular and Atomic physics.

In this paper we shall only deal with the first of these issues, plus some matters of interpretation of quantum theory; it is possible that the present work can help in solving the second and third problem –as it is applicable to general forces– but unless we can show that our approach is able to solve renormalisation issues in the quantum field theories, we prefer not to claim that.

1.1 $\epsilon = \hbar\omega$

From the considerations of Planck [1] and Einstein [2] we can at most say *Monochromatic* radiation of low density (*within the range of validity of Wien's radiation formula*) behaves thermodynamically as though it consisted of a number of independent energy quanta of magnitude $2R\beta\omega/2\pi N$.[1] There are two critical assumptions

- The wave under consideration is a monochromatic (harmonic) one.
- We are within the range of validity of the Wien's radiation law (Wien's approximation). i.e. the frequencies with which we are working are bounded below by ω_{min}.

Yet a wave in general does not know any of the concepts *frequency* or *wave number*. These are concepts which appear when we choose *a* basis

[1] from the english translation of Einstein's paper. I changed ν to $\omega/2\pi$ for the sake of uniformity of notation.

–the Fourier basis– for our function space; by the principle of relativity, **A physical theory should not depend on our choice of basis**[2], so $\epsilon = \hbar\omega$ must be *a* special case of *the* –still unknown– quantum condition, on which 'hypotheses non fingo'. Apart from incompleteness in its current form, it encounters serious inconsistencies. First, if according to de Broglie

$$mc^2 = \hbar\omega$$

then for a massless particle it means

$$\hbar\omega = 0 \Longrightarrow \omega = 0$$

which undermines the whole idea! one of the most important experimental evidences of $E = \hbar\omega$ is for photons in the celebrated Photoelectric effect. Thus either photons are not massless, which is a serious difficulty leading to many conceptual problems, revival of the Luminiferous aether among them, or $E = \hbar\omega$ is not a complete law of nature.

Second, in this form there is a tension with relativity since a particle's intrinsic energy must transform as –E is its rest-frame energy–

$$E \to \gamma E$$

while for $E = \hbar\omega$ we have

$$E' = \hbar\frac{\omega}{\gamma}$$

This problem was realised by de Broglie himself and he gave a partial solution to it in his thesis[3], called the theorem of phase harmony, *le théorème de l'harmonie des phases*:

A periodic phenomenon is seen by a stationary observer to exhibit the frequency $\omega = mc^2/\hbar\gamma$ that appears constantly in phase with a wave having frequency $\omega = m\gamma c^2/\hbar$ propagating in the same direction with velocity $v = c\gamma$.

But the tension with relativity is still there! since $v = c\gamma \geq c$. essentially, the theorem of phase harmony prefers to make nature non-local than consistent with special relativity. so we have here **chosen** non-locality and superluminal signaling. we shall not be surprised to see this non-locality returning as we further develop quantum mechanics. quantum mechanics is non-local **by assumption**.

For the moment let us neglect these inconsistencies and focus on developing quantum mechanics further, being aware that we should not expect to arrive at a fully consistent theory!

1.2 Role of Potentials in quantum mechanics

Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V\psi(\mathbf{x},t)$$

contains only V, not its gradient. Taking Schrödinger equation as a fundamental law of nature, not derivable from the most fundamental law of

[2] This is not exactly the principle of relativity but essentially it is of the same spirit.

classical mechanics, $\mathbf{F} = d\mathbf{p}/dt$, this might lead one to think that potentials, and not local forces, are the primary ontological entities in nature. But potentials are fundamentally non-local and Aharonov and Bohm, in [4] devised of an experiment which showed that such non-localities, can have observable effects[3], if schrödinger equation is a fundamental law of nature. But is it? We shall argue that Schrödinger equation is **not** a fundamental law of nature, at least not more fundamental than Newton Second Law. All that we can say is that the fundamental laws of nature are the Newton Second Law and the de Broglie relation for matter-waves. These two laws would suffice to build the whole edifice of quantum mechanics. We will not embroil ourselves in subtleties of trying to analyse the Aharonov-Bohm setup, neither do we embark on trying to see what mechanism is at work to account for the apparent peculiarities. Instead we take the most simple-minded approach: Forget everything you know or like about the Aharonov-Bohm effect. In the Aharonov-Bohm setup all that there is, is a free particle. What does a quantised Newton second law say about a free particle?
In what follows we shall pave the way to prove that Schrödinger equation is **derivable** from Newton Second Law, assuming only the existence of de Broglie matter-waves and their basic properties.

1.3 Harmonisation

What should we do then if we are desparate enough to assume every wave as harmonic? In order to develop quantum mechanics we accept this as a bare *fact* and *brute-force* everything to be harmonic, by giving harmonicity to all waves; a process which we refer to as *Harmonisation*. Let us see how we can find an expression for frequency and wavevector in terms of the wavefunction itself[4], assuming all waves are harmonic,

$$\psi(\mathbf{x}, t) = e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

if we apply the gradient operator to both sides we have,

$$\nabla\psi = i\mathbf{k}\psi$$

and similarly for the (partial) time derivative operator,

$$\frac{\partial}{\partial t}\psi = -i\omega\psi$$

so we have two possibilites for harmonisation. One possibility leads to ordinary quantum mechanics –Copenhagen interpretation– and the other, to a more complete formulation of the de Broglie-Bohm theory.

[3] Many physicists share the position that the AB effect is a real quantum-topological effect, unexplainable by classical physics, supported by observations. On this issue much has been written, both physical and philosophical. Among most interesting ones is Timothy H. Boyer's[5] seminal paper. We prefer to remain neutral about this controversy and only trust what equations say, for it is quite natural for experiments –even more today, because of extreme technical difficulties and limitations– to be misinterpreted or even, to be wrong.

[4] Note that this is not the most general harmonic wave we can write. Moreover notice that in these definitions only forward-in-time waves are considered. it is not clear whether this preference of time direction affects the theory; this being said it should not be a big surprise if this time-asymmetry shows up somewhere later.

1.3.1 First approach: de Broglie-Bohm theory

In this approach which we call *Number-Division* approach, we use the following definition

$$\mathbf{k} := -i\frac{\nabla\psi}{\psi}. \tag{1}$$

But this definition has singularities, when $\psi = 0$. Nevertheless, this is the approach which eventually leads to the de Broglie-Bohm theory.[5] To see this we use the polar representation of a complex-valued function $\psi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{C}$

$$\psi(\mathbf{x}, t) = R(\mathbf{x}, t)e^{iS(\mathbf{x},t)/\hbar}$$

where $R : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ and $S : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$. Now we apply the quantum condition (de Broglie relation)[6]

$$\mathbf{p} = \hbar\mathbf{k} \Longrightarrow \mathbf{p} = -i\hbar\frac{\nabla\psi}{\psi} = -i\frac{1}{\psi}\left(\nabla Re^{iS(\mathbf{x},t)/\hbar} + iR\nabla Se^{iS(\mathbf{x},t)/\hbar}\right)$$

$$\mathbf{p} = -i\frac{\nabla R}{R} + \nabla S \tag{2}$$

While in the context of de Broglie-Bohm theory (see, for example[6])

$$\mathbf{p} = \nabla S$$

so there is a missing term:

$$-i\frac{\nabla R}{R}.$$

In the literature one usually uses the non-relativistic momentum $\mathbf{p} = m\mathbf{v}$ to find the velocity field of the particle as

$$\mathbf{v} = \frac{\nabla S}{m}$$

the complete relation, however, is

$$\mathbf{v} = -i\frac{\nabla R}{mR} + \frac{\nabla S}{m} \tag{3}$$

But classical velocity cannot be complex. The only remedy would be to let $\nabla R = 0$, $(R \neq 0)$ which gives

$$\psi(\mathbf{x}, t) = f(t)e^{iS(\mathbf{x},t)}$$

where $f(t)$ is the constant of (partial) integration. Using the Copenhagenian terminology, we can conclude that the concept of a classical path makes sense only for a probability density function uniform in $\mathbf{x}$.

[5] We can now see why in the de Broglie-Bohm theory one can have a **number** for velocity/momentum of the particle; because the way the theory proceeds is aimed at avoiding using linear operators and eigenvalues.

[6] Assuming here $\psi \in \mathcal{C}^1$, at least.

1.3.2 Second approach: orthodox quantum mechanics

The second –and more aesthetically appealing– approach would be to 'promote' wavevector to a linear operator $\hat{\mathbf{k}} \in \mathcal{L}(L^2(\mathbb{R}))$ defined as

$$\hat{\mathbf{k}} = -i\nabla \tag{4}$$

and say instead that we are dealing with an eigenvalue problem, which is exactly what people do in orthodox quantum mechanics. Similarly for frequency we have

$$\hat{\omega} = i\frac{\partial}{\partial t} \tag{5}$$

in $(+,-,-,-)$ metric signature we know the four-gradient covariant vector is given as,

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right)$$

and the four-wavevector K_μ by,

$$K_\mu = \left(\frac{\omega}{c}, -\mathbf{k}\right).$$

therefore if we promote the four-wavevector K_μ to an operator we have

$$\hat{K}_\mu = \left(\frac{\hat{\omega}}{c}, -\hat{\mathbf{k}}\right) = \left(\frac{i}{c}\frac{\partial}{\partial t}, i\nabla\right)$$

so the complete covariant quantum condition would be

$$\hat{p}_\mu = \hbar\hat{K}_\mu = i\hbar\partial_\mu. \tag{6}$$

It is important to notice two important radical changes that are *forced* upon us once we assume all waves as harmonic:

1. i, bringing about complex numbers, which are hard to interpret physically and were sofar assumed to be only *tools*. Now they are more than tools. We cannot assume all waves as harmonic unless we pay the price: complex numbers.
2. **Eigenvalue problem and Linear Operators**, bringing about Hilbert spaces.

Now that we have these, unlike Schrödinger[7], without appealing to any opto-mechanical analogies we can get the Schrödinger and Klein-Gordon equations[7].
From the special-relativistic energy relation

$$E \cdot E = c^2\mathbf{p}\cdot\mathbf{p} + m^2c^4$$

Applying the de Broglie operator equation we have

$$(\hbar\omega)^2 = \hbar^2c^2\mathbf{k}\cdot\mathbf{k} + m^2c^4$$

$$-\hbar^2\partial_t^2 = \hbar^2c^2\nabla^2 + m^2c^4 \tag{7}$$

[7] It is of course nothing new, and familiar to every physicist, we are only trying to change the perspective.

which is the Klein-Gordon operator equation.
For a massive non-relativistic particle in a potential V, approximately we have

$$E = \frac{\mathbf{p}\cdot\mathbf{p}}{2m} + V$$

which –as we know– yields the schrödinger equation, after using the quantum condition $\mathbf{p} = -i\hbar\nabla$:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V\psi(\mathbf{x},t).$$

If we insist on this solution we must as well have a definition for the propagation velocity of a general wave. It must be

$$\hat{c} := \partial_t\nabla^{-1}$$

which is not generally defined.
Another possibility is

$$c := \pm i\frac{\partial_t\psi}{|\nabla\psi|} \tag{8}$$

which has a sign ambiguity. It might have two possibile remedies: Either we should have only one kind of waves (backward-in-time *or* forward-in-time) but not both; which treats time asymmetrically.
In the second remedy we use second derivatives to define

$$c := \sqrt{\frac{\partial_t^2\psi}{\nabla^2\psi}} \tag{9}$$

which is again problematic due to its singularities and multivaluedness of complex functions.

2 Quantisation of Newtonian mechanics

Given 'sufficient' initial conditions, in classical mechanics, Newton second law solves a mechanical problem completely; therefore we expect this to be also the case in our new quantum mechanics.
However, there is a crucial difference in quantum mechanics: in view of de Broglie's wave-particle duality, all classical variables become fields (vector or scalar) in quantum mechanics. It turns out that the classical law of motion (Newton second law and related definitions) is easily extended to the quantum case if we treat our classical dynamical variables as functions of trajectory and time, viz. if $\mathcal{V}$ is a classical dynamical variable, then the correct treatment in quantum mechanics should see it like

$$\mathcal{V} = \mathcal{V}(\mathbf{x}(t), t).$$

2.1 Operator-Eigen approach to Harmonisation

2.1.1 Non-relativistic case

Using chain rule[8]

$$\mathbf{F} = \frac{d\mathbf{p}(\mathbf{x}(t),t)}{dt} = \frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{p} = \frac{\partial \mathbf{p}}{\partial t} + \frac{1}{m}\mathbf{p}\cdot\nabla\mathbf{p} \tag{10}$$

in Euclidean space we have the following identity

$$\mathbf{p}\cdot\nabla\mathbf{p} = \frac{1}{2}\nabla(\mathbf{p}\cdot\mathbf{p})$$

proof:

$$\nabla(\mathbf{p}\cdot\mathbf{p}) = \partial_\mu(p^\rho p_\rho)$$

where $\mu, \rho = 1, 2, 3$.

$$\partial_\mu(p^\rho p_\rho) = (\partial_\mu p_\rho)p^\rho + p_\rho \partial_\mu p^\rho$$

in either term, lower one ρ and raise the other. since $p^\alpha = -p_\alpha$, we can say

$$\partial_\mu(p^\rho p_\rho) = 2p_\rho\partial_\mu p^\rho \Longrightarrow \nabla(\mathbf{p}\cdot\mathbf{p}) = 2\mathbf{p}\cdot\nabla\mathbf{p}$$

thus

$$\mathbf{F} = \frac{\partial \mathbf{p}}{\partial t} + \frac{1}{2m}\nabla(\mathbf{p}\cdot\mathbf{p})$$

If we now quantise this equation using the quantum mechanical operator $\mathbf{p} = -i\hbar\nabla$, and act on ψ, we get the *quantised Newton second law*,

$$\mathbf{F}\psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla\nabla^2\psi - i\hbar\frac{\partial}{\partial t}\nabla\psi \tag{11}$$

If we have a conservative force field $\mathbf{F}(\mathbf{x}) = -\nabla V(\mathbf{x})$, after substitution we have,

$$-\nabla V\psi = -i\hbar\nabla\frac{\partial}{\partial t}\psi - \frac{\hbar^2}{2m}\nabla(\nabla\cdot\nabla)\psi$$

which is

$$i\hbar\nabla\frac{\partial}{\partial t}\psi = \nabla\left(V - \frac{\hbar^2}{2m}\nabla^2\right)\psi \tag{12}$$

i.e. 'the gradient' of the Schrödinger equation.

We now observe that even for a conservative force field, schrödinger equation 'kills' some solutions, because

$$\nabla\left(-\frac{\hbar^2}{2m}\nabla^2 + V - i\hbar\frac{\partial}{\partial t}\right)\psi = 0$$

$$\Longrightarrow \exists g(t) : \mathbb{R}\to\mathbb{R},\ \text{s.t.}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi - i\hbar\frac{\partial}{\partial t}\psi = g(t) \tag{13}$$

This non-homogeneity can be interpreted in two different ways:

[8]We need not worry here about possible non-commutativities familiar from ordianry quantum mechanics. such non-commutativities happen for canonically conjugate (Fourier dual) variables only. no use of Fourier transform/duals is made in our discussion.

- Incompleteness of the theory. Although we expected that Newton second law can solve our new quantum-mechanical problems completely, it is possible that the new quantum theory lacks a law –for whatever reason– to fix $g(t)$.
- The theory demands more information of boundary conditions. We need not to try to 'drop' the gradient; we can solve a *third*-order PDE instead, which of course needs one more boundary condition.

We have seen, therefore, that the Schrödinger equation is essentially the conservation of energy and in comparison to the quantised Newton second law it contains less information. More on this issue will be said in the next section.

2.1.2 Relativistic case

In a similar manner we can have an educated guess about how to quantise the Relativistic Newton Second Law. Note that in light of special relativity, in the non-relativistic case t was only a parameter; so here that instead of t we have τ we should treat it similarly.

$$F^\mu = \frac{d}{d\tau} P^\mu(x^\nu(\tau), \tau) = (\partial_\nu P^\mu)\frac{dx^\nu}{d\tau} + \frac{\partial P^\mu}{\partial \tau} \tag{14}$$

since

$$\frac{dx^\nu}{d\tau} = U^\nu = \frac{1}{m} P^\nu$$

$$F^\mu = \frac{1}{m} P^\nu(\partial_\nu P^\mu) + \frac{\partial P^\mu}{\partial \tau}$$

as

$$P^\mu = i\hbar\partial^\mu$$

therefore[9] after substitution and acting on $\psi(x^\lambda)$

$$\Longrightarrow \boxed{F^\mu \psi(x^\lambda) = -\frac{\hbar^2}{m}\Box\partial^\mu\psi(x^\lambda) + i\hbar\frac{\partial}{\partial\tau}\partial^\mu\psi(x^\lambda)} \tag{15}$$

2.2 Number approach to Harmonisation

2.2.1 Non-relativistic case

In this case the quantum condition takes the following form:

$$\mathbf{p} = \frac{-i\hbar\nabla\psi}{\psi}$$

as

$$\mathbf{F} = \frac{d\mathbf{p}(\mathbf{x}(t), t)}{dt} = \frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{p} = \frac{\partial \mathbf{p}}{\partial t} + \frac{1}{2m}\nabla(\mathbf{p}\cdot\mathbf{p})$$

$$\mathbf{F} = -i\hbar\frac{\partial}{\partial t}\frac{\nabla\psi}{\psi} - \frac{\hbar^2}{2m}\nabla(\frac{\nabla\psi\cdot\nabla\psi}{\psi^2})$$

[9] Notice an important use we have made here of the commutativity of the covariant derivative of the Minkowski space-time, i.e. ∂_μ to make the wave operator appear. This is obviously not possible in presence of gravity and it can be a reason for difficulties involved with quantising gravity.

If we denote by log the principal values of the complex logarithm function,

$$\mathbf{F} = -i\hbar\frac{\partial}{\partial t}\nabla\log\psi - \frac{\hbar^2}{2m}\nabla(\nabla\log\psi)^2 \tag{16}$$

For a conservative force field $\mathbf{F} = -\nabla V$,

$$\boxed{i\hbar\frac{\partial}{\partial t}\log\psi = -\frac{\hbar^2}{2m}(\nabla\log\psi)^2 + V} \tag{17}$$

2.2.2 Relativistic de Broglie-Bohm theory

$$P_\mu = \frac{i\hbar\partial_\mu\psi}{\psi}$$

as

$$F^\mu = \frac{1}{m}P^\nu\partial_\nu P^\mu + \frac{\partial P^\mu}{\partial\tau} = \frac{-\hbar^2}{m}\frac{\partial^\nu\psi}{\psi}\partial_\nu(\frac{\partial^\mu\psi}{\psi}) + i\hbar\frac{\partial}{\partial\tau}\frac{\partial^\mu\psi}{\psi}$$

If we denote by log the principal values of the complex logarithm function,

$$F^\mu = \frac{-\hbar^2}{m}(\partial^\nu\log\psi)(\partial_\nu\partial^\mu\log\psi) + i\hbar\frac{\partial}{\partial\tau}\partial^\mu\log\psi \tag{18}$$

Analogue to (17), the 'potential form' of this equation for a free field is

$$\boxed{(\frac{\partial\psi}{\partial(ct)})^2 - (\nabla\psi)^2 + (\frac{mc}{\hbar})^2\psi^2 = 0} \tag{19}$$

Here comes the critical observation: in the de Broglie-Bohm approach, operators are already 'fed'! Unlike the Operator-Eigen approach, we cannot here make the wave operator appear. Evidently this is a serious obstacle to quantum electrodynamics, for Dirac's ideas of Clifford algebras –roughly said, taking the 'square root' of the wave operator– are not applicable here.

3 Does the Energy Conservation theorem solve a dynamical problem completely?

It is familiar from classical mechanics that except for a scleronomic system with only one degree of freedom, conservation of energy does not yield to complete integration and solution of the problem.(see [8])
For the case with rheonomic systems (e.g. time-dependent potentials), if

$$E = \frac{1}{2}m\dot{\mathbf{r}}^2 + V(\mathbf{r}, t)$$

$$\Longrightarrow 0 = \frac{dE}{dt} = m\mathbf{a}\cdot\dot{\mathbf{r}} \ + \ \frac{\partial V}{\partial t} + \dot{\mathbf{r}}\cdot\nabla V$$

which is

$$m\mathbf{a}\cdot\dot{\mathbf{r}} \ + \dot{\mathbf{r}}\cdot\nabla V = -\frac{\partial V}{\partial t}.$$

this is all we can say. As an example, take the famous case of a –non-relativistic– charged particle in an electrodynamical field. The equation of motion is given by the Lorentz force law, while it is impossible to derive this force from the Lagrangian

$$\mathcal{L}(\mathbf{r}, \dot{\mathbf{r}}; t) = \frac{1}{2}m\dot{\mathbf{r}}^2 - e(\phi(\mathbf{r}, t) - \dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}, t))$$

if we are going to use **solely** the conservation of energy; more information is needed to find the equation of motion, which is given to us by the Euler-Lagrange equations.
Since we know that the schrödinger equation is essentially the conservation of energy, without any further application of the rest of the Hamiltonian mechanics, i.e. Hamilton's equations, we should be sceptic and careful about its application to time-dependent potentials; no ontological conclusion shall be infered from a possibly-incomplete theory, i.e. what such theory says about physical reality cannot be easily trusted.

4 Free particle and the Aharonov-Bohm problem

In this problem, essentially we have a free particle, $\mathbf{F} = 0$. As we saw earlier, for a free particle, the quantised NSL is

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi - i\hbar\frac{\partial}{\partial t}\psi = f(t) \tag{20}$$

for some differentiable real-valued function $f(t)$.
We do not need any solution of this equation at hand to observe that the solutions will have no dependence on any undefined quantity like potentials whatsoever, let alone electromagnetic ones.
One might say that the function $f(t)$ will 'transmit' the non-local effects of potentials and the problem still remains. However, note that

- As we pointed earlier, the appearance of $f(t)$ can only show our lack of information; either of some yet-unknown law of quantum mechanics or of boundary conditions. even if we neglect this crucial point:
- The function f is only a function of time, and as we are in the domain of non-relativistic classical mechanics, time is an absolute entity *a priori* and cannot be affected by fields evolving in it. even if the electromagnetic potentials altered the geometry of space, they could not affect time![10]
- From the viewpoint of special relativity, potentials defined on spacetime cannot affect the invariant $d\tau$ (proper time parameter).

[10] We are considering here the 'hard' scenario where the time-independent vector potential is responsible for the effect. If we focus on the Electric Aharonov-Bohm effect, where the potential is time-dependent we can immediately resolve the issue by pointing that the schrödinger equation is not even applicable in such case! even if applicable, it will not solve the problem completely, and if we do not know the complete answer of a theory in a particular situation, how can we deduce such important ontological conclusion from it?!

- From the standpoint of general relativity it is true that $T^{\mu\nu}_{EM}$ can alter the structure of spacetime, as a source for Einstein Field Equations. but $T^{\mu\nu}_{EM}$ is expressed entirely in terms of local fields $\mathbf{E}$ and $\mathbf{B}$. This case has no tension with locality: $\mathbf{E}$ and $\mathbf{B}$ alter the geometry of spacetime and the change in geometry transmits to the time parameter of the above non-homogeneous equation (in infinitesimal regions where we can approximate the spacetime as flat and solve the schrödinger equation). This is a perfectly local explanation.

5 Conclusions

In this paper we showed that

- The famous claim of the advocates of the de Broglie-Bohm theory, that quantum mechanical particles do have path is wrong. such particles can have classical path defined for them only in special circumstances.
- Schrödinger equation is derivable from Newton Second Law. It is therefore not more fundamental than Newton Second Law. It is highly unlikely that 'the classical limit problem' makes a valid assumption. classical mechanics is not a special case of quantum mechanics.
- Schrödinger equation is not the most complete quantised equation of motion for de Broglie matter-waves. In particular, it is not applicable to rheonomic systems (e.g. time-dependent potentials) nor applicable in presence of non-holonomic kinematical constraints. If applied in such situations, we will not get a complete solution of the mechanical problem under consideration, because conservation of energy only for scleronomic system leads to a complete solution.
- The so-called Aharonov-Bohm effect cannot exist in the first place. It would be better to refer to it as the Aharonov-Bohm *problem*. Most probably it does not exist as a physical phenomenon.

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