Collatz Conjecture is Proved in this Article

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Abstract

First, it is necessary to expound certain of basic concepts relating to prove this conjecture, after that, the author applies the mathematical induction to complete this proof. Also, before the beginning of this proof, prepares several judging criteria, so as to use them in following classified proofs after operations according to the operational rule. After accomplish proofs of certain of stepwise classification for integers, emphatically proves *15+12c* and *19+12c*, so the entire proof for the conjecture was finished.

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Contents

1. Introduction

The Collatz conjecture also called the *3x+1* mapping, *3n+1* problem, Hasse's algorithm, Kakutani's problem, Syracuse algorithm, Syracuse problem, Thwaites conjecture and Ulam's problem, etc.

Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collatz in 1937; [1].

2. Certain of Basic Concepts

The Collatz conjecture states that take any positive integer *n*, if *n* is even, divide it by *2* to get *n/2*; if *n* is odd, multiply it by *3* and add *1* to get *3n+1*. Repeat the process indefinitely, then, no matter what positive integer you start with, you will always eventually reach a result of *1*; [2] and [3].

Let us regard aforesaid operational stipulations as the operational rule.

Begin with any positive integer⁄integer's expression to operate by the operational rule continuously, then, form successive integers⁄integer's expressions. We regard such consecutive integers⁄integer's expressions and synclastic arrowheads among them as an operational route.

Thereinafter, let us use a capital letter to express a certain positive integer such as "*P*", and use a capital letter plus subscript "*ie*" to express a certain positive integer's expression such as P_{ie}, C_{ie} etc.

If $P_{ie}\,/\,P$ exists at an operational route, then may term the operational route "an operational route via $P_{ie}\,/\,P$".

Generally speaking, integer's expressions at an operational route have a common variable or some variables which can be transformed into a variable.

Two operational routes via P_{ie} branch from P_{ie} or an integer's expression after pass operations of P_{ie}.

3. Mathematical Induction that Proves the Conjecture

Prove the Collatz conjecture by the mathematical induction [4], as follows.

(1) From 1→4→2→1; 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1; 6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 8→4→2→1; 9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 10→5→16→8→4→2→1; 11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 12→6→3→10→5→16→8→4→2→1; 13→40→20→10→5→16→8→4→2→1; 14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 15→46→23→70→35→106→53→160→80→40→20→10→5→16→8→4→2→1; 16→8→4→2→1; 17→52→26→13→40→20→10→5→16→8→4→2→1; 18→9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1 and 19→58→29→88→44→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1, get that every positive integer≤*19* suits the conjecture.

(2) Suppose that n suits the conjecture, where n is an integer≥*19*.

(3) Prove which $n+1$ suits the conjecture likewise.

4. Several Judging Criteria

Before make the proof, it is necessary to prepare judging criteria concerned:

Theorem 1· If an integer's expression at an operational route via P_{ie} is less than P_{ie}, and P_{ie} contains $n+1$, then P_{ie} and $n+1$ suit the conjecture.

For example, if $P_{ie}=31+3^2\eta$, and P_{ie} contains $n+1$, where $\eta\geq 0$, then from $27+2^3\eta\to 82+3\times 2^3\eta\to 41+3\times 2^2\eta\to 124+3^2\times 2^2\eta\to 62+3^2\times 2\eta\to 31+3^2\eta>27+2^3\eta$, conclude that P_{ie} and $n+1$ suit the conjecture.

For another example, if $P_{ie}=5+2^2\mu$, and P_{ie} contains $n+1$, where $\mu\geq 0$, then from $5+2^2\mu\to 16+3\times 2^2\mu\to 8+3\times 2\mu\to 4+3\mu<5+2^2\mu$, conclude that P_{ie} and $n+1$ suit the conjecture.

Proof· Suppose that there is C_{ie} at an operational route via P_{ie}, and $C_{ie}<P_{ie}$. Since P_{ie} and C_{ie} exist at an operational route, then, when their common variable equals some fixed value such that $P_{ie}=n+1$, let $C_{ie}=m$, so it has $m<n+1$. As thus, operations of $n+1$ can pass operations of m at the operational route via $n+1$ to reach 1 since every positive integer$<n+1$ has been supposed to suit the conjecture.

If their common variable equals any value, then any value of P_{ie} can too be operated to 1 via the matching value of C_{ie}, so P_{ie} and $n+1$ suit the conjecture.

Lemma 1· If an integer's expression at an operational route suits the conjecture, then each and every integer's expression at the operational route suits the conjecture too.

Theorem 2: If an operational route via Q_{ie} and an operational route via P_{ie} intersect, P_{ie} contains $n+1$, and that an integer's expression at the operational route via Q_{ie} is less than P_{ie}, then P_{ie} and $n+1$ suit the conjecture, where $P_{ie}\neq Q_{ie}$.

For example, $P_{ie}=63+3\times 2^8\varphi$, and P_{ie} contains $n+1$, where $\varphi\geq 0$, then from

$63+3\times 2^8\varphi\rightarrow 190+3^2\times 2^8\varphi\rightarrow 95+3^2\times 2^7\varphi\rightarrow 286+3^3\times 2^7\varphi\rightarrow 143+3^3\times 2^6\varphi\rightarrow 430+3^4\times 2^6\varphi\rightarrow$

$215+3^4\times 2^5\varphi\rightarrow 646+3^5\times 2^5\varphi\rightarrow 323+3^5\times 2^4\varphi\rightarrow 970+3^6\times 2^4\varphi\rightarrow 485+3^6\times 2^3\varphi\rightarrow 1456+3^7\times 2^3\varphi$

$\rightarrow 728+3^7\times 2^2\varphi\rightarrow 364+3^7\times 2\varphi\rightarrow 182+3^7\varphi\rightarrow \ldots$

$\uparrow 121+3^6\times 2\varphi\leftarrow 242+3^6\times 2^2\varphi\leftarrow 484+3^6\times 2^3\varphi\leftarrow 161+3^5\times 2^3\varphi\leftarrow 322+3^5\times 2^4\varphi$

$\leftarrow 107+3^4\times 2^4\varphi\leftarrow 214+3^4\times 2^5\varphi\leftarrow 71+3^3\times 2^5\varphi\leftarrow 142+3^3\times 2^6\varphi\leftarrow 47+3^2\times 2^6\varphi<63+3\times 2^8\varphi$,

conclude that P_{ie} and $n+1$ suit the conjecture.

Proof: Suppose that D_{ie} at an operational route via Q_{ie} is less than P_{ie}, also the operational route via Q_{ie} and an operational route via P_{ie} intersect at A_{ie}. Then, when their common variable equals some fixed value such that $P_{ie}=n+1$, let $D_{ie}=\mu$ and $A_{ie}=\xi$, so it has $\mu<n+1$. As thus, operations of ξ can pass operations of μ at the operational route via ξ to reach 1 on Lemma 1. In addition, operations of $n+1$ can pass ξ with the aid of operations of μ to reach 1.

If their common variable equals any value, then a value of P_{ie} can too be operated to 1 via matching values of A_{ie} and D_{ie}, so P_{ie} and $n+1$ suit the conjecture.

Lemma 2· If an operational route via Q_{ie} and an operational route via P_{ie} are at indirect connection, P_{ie} contains $n+1$, and that an integer's expression at the operational route via Q_{ie} is less than P_{ie}, then P_{ie} and $n+1$ suit the conjecture.

What is called the indirect connection? Such as an operational route via Q_{ie} intersects an operational route via R_{ie}, and the operational route via R_{ie} intersects an operational route via P_{ie}, yet the operational route via Q_{ie} intersects not the operational route via P_{ie}, then the operational route via Q_{ie} and the operational route via P_{ie} are at the indirect connection exactly.

Lemma 3· If an integer's expression at an operational route suits the conjecture, then each and every integer's expression at each and every operational route which directly intersects the operational route and indirectly connects to the operational route suits the conjecture.

For example, suppose that C_{ie} at an operational route via A_{ie} suits the conjecture, also the operational route via A_{ie} and an operational route via B_{ie} intersect at X_{ie}, so every integer's expression including X_{ie} at the operational route via A_{ie} suits the conjecture on Lemma 1.

Like the reason, since X_{ie} suits the conjecture, so every integer's expression at the operational route via B_{ie} suits the conjecture, and so on and so forth, to the extent that the operational route via A_{ie} and an operational route via P_{ie} are at indirect connection, then every integer's expression at the operational route via P_{ie} suits the conjecture.

5. Initial Proofs Classified

By now, let us start to prove the Collatz conjecture gradually, ut infra.

Proof: According to fore-prepared theorems and lemmas, on balance, must classify positive integers, then find out a relation between each class which possibly contains $n+1$ and another class which is less than the former, to prove that $n+1$ suits the conjecture.

It is well known, positive integers are divided into positive even numbers and positive odd numbers.

For any even number $2k$ with $k\geq 1$, from $2k\to k < 2k$, conclude that if $n+1$ is an even number, then $n+1$ suits the conjecture on Theorem 1.

Secondly, for positive odd numbers out of first step of the mathematical induction, divide them into 2 genera, i.e. $5+4k$ and $7+4k$, where $k\geq 4$.

For $5+4k$, from $5+4k\to 16+12k\to 8+6k\to 4+3k<5+4k$, conclude that if $n+1\in 5+4k$, then $n+1$ suits the conjecture on Theorem 1.

Further divide $7+4k$ into 3 sorts: $15+12c$, $19+12c$ and $23+12c$, where $c\geq 0$.

For $23+12c$, from $15+8c\to 46+24c\to 23+12c>15+8c$, conclude that if $n+1\in 23+12c$, then $n+1$ suits the conjecture on Theorem 1.

For $15+12c$ and $19+12c$ when $c=0$, they were proved to suit the conjecture in advance. So only need us to prove $15+12c$ and $19+12c$ where $c\geq 1$.

6. Proving 15+12c and 19+12c emphatically

For $15+12c$ and $19+12c$ where $c\geq 1$, we will operate them right along, so that expound the relation that they act in accordance with fore-prepared several judging criteria.

Firstly, operate *15+12c* by the operational rule successively, as follows.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥

♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)

c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)

d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   **…**

♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**

f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**

g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)

♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**

f=2g: 304+1458g→152+729g ↓→**…**

...

...

♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**   **…**

e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**

f=2g: 20+243g↓→g=2h: 10+243h (**6**)   **…**

g=2h+1:790+1458h→395+729h↑→ **…**

Annotation:

(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0.

(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

In the course of operation for *15+12c/19+12c* by the operational rule, if an operational result is less than a kind of *15+12c/19+12c*, and that it first appears at an operational route of *15+12c/19+12c*, then let us term the operational result "№1 satisfactory operational result". Hereupon conclude 6 kinds of *15+12c* derived from №1 satisfactory operational results at the bunch of operational routes of *15+12c* monogamously, ut infra.

1. From c=2d+1 and d=2e+1, get c=2d+1=2(2e+1)+1=4e+3, so 15+12c=51+48e=

$51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to116+3^3\times2^2e\to58+3^3\times2e\to$

29+27e where mark (**1**), and 29+27e < 51+48e, thus if $n+1 \in 51+48e$, then 51+48e and $n+1$ suit the conjecture on Theorem *1*.

2. From c=2d+1, d=2e and e=2f+1, get c=2d+1=4e+1=4(2f+1)+1=8f+5, so 15+12c= 75+96f=$75+3\times2^5$f→$226+3^2\times2^5$f→$113+3^2\times2^4$f→$340+3^3\times2^4$f→$170+3^3\times2^3$f→$85+3^3\times2^2$f →$256+3^4\times2^2$f→$128+3^4\times2^1$f→64+81f where mark (**2**), and 64+81f < 75+96f, thus if $n+1 \in 75+96f$, then *75+96f* and $n+1$ suit the conjecture on Theorem *1*.

3. From c=2d, d=2e+1 and e=2f+1, get c=2d=4e+2=4(2f+1)+2=8f+6, so 15+12c= 87+96f=$87+3\times2^5$f→$262+3^2\times2^5$f→$131+3^2\times2^4$f→$394+3^3\times2^4$f→$197+3^3\times2^3$f→$592+3^4\times2^3$f→$296+3^4\times2^2$f→$148+3^4\times2^1$f→74+81f where mark (**3**) , and 74+81f < 87+96f, thus if $n+1 \in 87+96f$, then *87+96f* and $n+1$ suit the conjecture on Theorem *1*.

4. From c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, get c=2d+1=4e+1=8f+1=8(2g+1)+1= 16g+9=16(2h+1)+9=32h+25,So15+12c=315+384h=$315+3\times2^7$h→$946+3^2\times2^7$h→$473+3^2\times2^6$h →$1420+3^3\times2^6$h→$710+3^3\times2^5$h→$355+3^3\times2^4$h→$1066+3^4\times2^4$h→$533+3^4\times2^3$h→$1600+3^5\times2^3$h→ $800+3^5\times2^2$h→$400+3^5\times2^1$h→200+243h where mark (**4**), and 200+243h< 315+ 384h, thus if $n+1 \in 315+384h$, then *315+384h* and $n+1$ suit the conjecture on Theorem *1*.

5. From c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, get c=2d=2(2e+1)=4e+2=8f+2=8(2g+1) +2=16g+10=32h+10,So 15+12c=135+384h=$135+3\times2^7$h→$406+3^2\times2^7$h→$203+3^2\times2^6$h →$610+3^3\times2^6$h→$305+3^3\times2^5$h→$916+3^4\times2^5$h→$458+3^4\times2^4$h→$229+3^4\times2^3$h→$688+3^5\times2^3$h →$344+3^5\times2^2$h→86+243h where mark (**5**), and 86+243h<135+384h, thus if $n+1 \in 135+384h$, then *135+384h* and $n+1$ suit the conjecture on Theorem *1*.

6. From c=2d, d=2e, e=2f, f=2g and g=2h, get c=2d=32h, so 15+12c=15+384h=15+ 3×2^7h→$46+3^2\times2^7$h→$23+3^2\times2^6$h→$70+3^3\times2^6$h→$35+3^3\times2^5$h→$106+3^4\times2^5$h→$53+3^4\times2^4$h→60+

$3^5\times2^4$h→80+$3^5\times2^3$h→40+$3^5\times2^2$h→10+243h where mark (**6**), and 10+243h<15+384h, thus if *n+1*∈*15+384h* then *15+384h* and *n+1* suit the conjecture on Theorem *1*.

Secondly, operate *19+12c* by the operational rule successively, as follows.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**) e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (**γ**)
e=2f+1:526+486f ♦

g=2h: 119+243h (**δ**) …
f=2g+1:238+243g↑→g=2h+1:1444+1458h→722+729h↑→…
♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→… …
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→…
e=2f+1:202+243f↑→f=2g+1:1336+1458g→…
♠322+486e→161+243e↑→e=2f:484+1458f→…

♦526+486f→263+243f↓→f=2g: 790+1458g→…
f=2g+1: 253+243g↓→g=2h+1: 248+243h (**ζ**)
g=2h: 760+1458h→…

Like that, conclude 6 kinds of *19+12c* derived monogamously from №1 satisfactory operational results at the bunch of operational routes of *19+12c*:

1. From c=2d and d=2e, get c=2d=4e, so 19+12c=19+48e=19+3×2^4e→58+$3^2\times2^4$e→29+$3^2\times2^3$e →88+$3^3\times2^3$e→44+$3^3\times2^2$e→22+$3^3\times2$e→11+27e where mark (**α**), and 11+27e<19+48e, thus if *n+1*∈*19+48e*, then *19+48e* and *n+1* suits the conjecture on Theorem *1*.

2. From c=2d, d=2e+1 and e=2f, get c=2d=2(2e+1)=4e+2=8f+2, so 19+12c= 43+96f=43+3×2^5f→130+$3^2\times2^5$f→65+$3^2\times2^4$f→196+$3^3\times2^4$f→98+$3^3\times2^3$f→49+$3^3\times2^2$f →148+$3^4\times2^2$f→74+$3^4\times2^1$f→37+81f where mark (**β**), and 37+81f < 43+96f, thus if *n+1*∈*43+96f*, then *43+96f* and *n+1* suit the conjecture on Theorem *1*.

3. From c=2d+1, d=2e+1 and e=2f, get c=2d+1=4e+3=8f+3, so 19+12c=55+96f= $55+3\times2^5f\rightarrow166+3^2\times2^5f\rightarrow83+3^2\times2^4f\rightarrow250+3^3\times2^4f\rightarrow125+3^3\times2^3f\rightarrow376+3^4\times2^3f\rightarrow$ $188+3^4\times2^2f\rightarrow94+3^4\times2^1f\rightarrow47+81f$ where mark (γ), and 47+81f < 55+96f, thus if *n+1*∈*55+96f*, then *55+96f* and *n+1* suit the conjecture on Theorem *1*.

4. From c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h, get c=2d=2(2e+1)=4e+2=4(2f+1)+2= 8f+6=8(2g+1)+6=16g+14=32h+14, so $19+12c=187+384h=187+3\times2^7h\rightarrow562+3^2\times2^7h\rightarrow281+3^2\times2^6h\rightarrow844+3^3\times2^6h\rightarrow422+3^3\times2^5h\rightarrow211+3^3\times2^4h\rightarrow634+3^4\times2^4h\rightarrow317+3^4\times2^3h\rightarrow952+3^5\times2^3h\rightarrow476+3^5\times2^2h\rightarrow238+3^5\times2^1h\rightarrow119+243h$ where mark (δ), and 119+243h<187+384h, thus if *n+1*∈*187+384h*, then *187+384h* and *n+1* suit the conjecture on Theorem *1*.

5. From c=2d+1, d=2e, e=2f+1, f=2g and g=2h+1, get c=2d+1=4e+1=4(2f+1)+1=8f+5= 16g+5=16(2h+1)+5=32h+21, so $19+12c=271+384h=271+3\times2^7h\rightarrow814+3^2\times2^7h\rightarrow407+3^2\times2^6h\rightarrow1222+3^3\times2^6h\rightarrow611+3^3\times2^5h\rightarrow1834+3^4\times2^5h\rightarrow917+3^4\times2^4h\rightarrow2752+3^5\times2^4h\rightarrow1376+3^5\times2^3h\rightarrow688+3^5\times2^2h\rightarrow344+3^5\times2^1h\rightarrow172+243h$ where mark (ε), and 172+243h< 271+ 384h, thus if *n+1*∈*271+384h*, then *271+384h* and *n+1* suit the conjecture on Theorem *1*.

6. From c=2d+1, d=2e+1, e=2f+1, f=2g+1 and g=2h+1, get c=2d+1= 2(2e+1)+1= 4e+3=4(2f+1)+3=8f+7=8(2g+1)+7=16(2h+1)+15=32h+31, so 19+12c=391+384h= $391+3\times2^7h\rightarrow1174+3^2\times2^7h\rightarrow587+3^2\times2^6h\rightarrow1762+3^3\times2^6h\rightarrow881+3^3\times2^5h\rightarrow2644+3^4\times2^5h\rightarrow1322+3^4\times2^4h\rightarrow661+3^4\times2^3h\rightarrow1984+3^5\times2^3h\rightarrow992+3^5\times2^2h\rightarrow496+3^5\times2^1h\rightarrow248+243h$ where mark (ζ), and 248+243h < 391+384h, thus if *n+1*∈*391+384h*, then *391+384h* and *n+1* suit the conjecture on Theorem *1*.

It is obvious that if *n+1*∈a kind of *15+12c /19+12c* derived from a №1 satisfactory operational result, then the kind of *15+12c/19+12c* and *n+1*suit the conjecture on Theorem *1*, of course, not excepting each of proved *12* kinds of *15+12c* plus *19+12c* in advance of this.

It is observed that variables *d*, *e*, *f*, *g*, *h* etc. substituted for *c* to appear within integer's expressions at the bunch's operational routes of *15+12c/ 19+12c*, actually the purpose is in order to avoid the confusion and for convenience. On the contrary, thereinafter, let χ represent intensively variables *d*, *e*, *f*, *g*, *h*, etc., but χ can not represent the variable *c* directly.

After substitute variables *d*, *e*, *f*, *g*, *h* and otherwise by the variable χ, the odevity of part integer's expressions that contain χ at operational routes of *15+12c/19+12c* is still indeterminate. Or rather, for every such integer's expression, both regard it as an odd number to operate, and regard it as an even number to operate. We thus label such integer's expressions "odd-even expressions".

For any odd-even expression at a bunch of operational routes of *15+12c/19+12c*, two kinds of operations synchronize at itself.

After regard an odd-even expression as an odd number to operate, get a greater operational result > itself. Yet after regard it as an even number to operate, get a smaller operational result < itself.

Moreover, pass operations for each odd-even expression, the bunch of operational routes of *15+12c/19+12c* will add an operational route.

Begin with an odd-even expression to operate by the operational rule continuously, every operational route via consecutive greater operational results will be getting longer and longer, and that the sum of constant term plus coefficient of χ of each integer's expression appeared thereon is getting greater and greater on the whole, along continuation of operations. On the other, for a smaller operational result in synchronism with a greater operational result, if it can be divided by 2^{μ} with $\mu \geq 2$ to get an even smaller integer's expression, and the even smaller integer's expression is first less than a kind of $15+12c/19+12c$, then the even smaller integer's expression is the very a №1 satisfactory operational result. Naturally the kind of $15+12c/19+12c$ is derived from the №1 satisfactory operational result, so operations at the operational route may stop at here. If the even smaller integer's expression is not less than any kind of $15+12c/19+12c$, or the smaller operational result is still an odd-even expression, then this needs us continue to operate by the operational rule. By this token, on the one hand, the bunch's operational routes of $15+12c/19+12c$ increase continually always; on the other hand, the bunch's operational routes of $15+12c/19+12c$ reduce continually always.

Begin with $15+12c/19+12c$ to operate by the operational rule on and on, it will continuously educe and discover №1 satisfactory operational results, thus derive more and more proven kinds of $15+12c/19+12c$ from them. But also, can derive at least a kind of $15+12c/19+12c$ from a №1 satisfactory operational result, such as derive only $15+12(1+2^{57}y)$ from $23+3^{38}y$, yet derive $15+12(4+2^{55}\times 3^{2}y)$ and $15+12(8+2^{32}\times 3^{17}y)$ from $61+2^{3}\times 3^{37}y$.

Please, see also their operational routes as listed below.

(**1**) From $15+12(1+2^{57}y)=27+2^{59}\times3y\rightarrow82+2^{59}\times3^2y\rightarrow41+2^{58}\times3^2y\rightarrow124+2^{58}\times3^3y\rightarrow$ $62+2^{57}\times3^3y\rightarrow31+2^{56}\times3^3y\rightarrow94+2^{56}\times3^4y\rightarrow47+2^{55}\times3^4y\rightarrow142+2^{55}\times3^5y\rightarrow71+2^{54}\times3^5y\rightarrow$ $214+2^{54}\times3^6y\rightarrow107+2^{53}\times3^6y\rightarrow322+2^{53}\times3^7y\rightarrow161+2^{52}\times3^7y\rightarrow484+2^{52}\times3^8y\rightarrow242+2^{51}\times3^8y$ $\rightarrow121+2^{50}\times3^8y\rightarrow364+2^{50}\times3^9y^{*}\rightarrow182+2^{49}\times3^9y\rightarrow91+2^{48}\times3^9y\rightarrow274+2^{48}\times3^{10}y\rightarrow137+2^{47}\times$ $3^{10}y\rightarrow412+2^{47}\times3^{11}y\rightarrow206+2^{46}\times3^{11}y\rightarrow103+2^{45}\times3^{11}y\rightarrow310+2^{45}\times3^{12}y\rightarrow155+2^{44}\times3^{12}y$ $\rightarrow466+2^{44}\times3^{13}y\rightarrow233+2^{43}\times3^{13}y\rightarrow700+2^{43}\times3^{14}y\rightarrow350+2^{42}\times3^{14}y\rightarrow175+2^{41}\times3^{14}y\rightarrow$ $526+2^{41}\times3^{15}y\rightarrow263+2^{40}\times3^{15}y\rightarrow790+2^{40}\times3^{16}y\rightarrow395+2^{39}\times3^{16}y\rightarrow1186+2^{39}\times3^{17}y\rightarrow$ $593+2^{38}\times3^{17}y\rightarrow1780+2^{38}\times3^{18}y\rightarrow890+2^{37}\times3^{18}y\rightarrow445+2^{36}\times3^{18}y\rightarrow1336+2^{36}\times3^{19}y\rightarrow$ $668+2^{35}\times3^{19}y\rightarrow334+2^{34}\times3^{19}y^{**}\rightarrow167+2^{33}\times3^{19}y\rightarrow502+2^{33}\times3^{20}y\rightarrow251+2^{32}\times3^{20}y\rightarrow$ $754+2^{32}\times3^{21}y\rightarrow377+2^{31}\times3^{21}y\rightarrow1132+2^{31}\times3^{22}y\rightarrow566+2^{30}\times3^{22}y\rightarrow283+2^{29}\times3^{22}y\rightarrow$ $850+2^{29}\times3^{23}y\rightarrow425+2^{28}\times3^{23}y\rightarrow1276+2^{28}\times3^{24}y\rightarrow638+2^{27}\times3^{24}y\rightarrow319+2^{26}\times3^{24}y\rightarrow$ $958+2^{26}\times3^{25}y\rightarrow479+2^{25}\times3^{25}y\rightarrow1438+2^{25}\times3^{26}y\rightarrow719+2^{24}\times3^{26}y\rightarrow2158+2^{24}\times3^{27}y\rightarrow$ $1079+2^{23}\times3^{27}y\rightarrow3238+2^{23}\times3^{28}y\rightarrow1619+2^{22}\times3^{28}y\rightarrow4858+2^{22}\times3^{29}y\rightarrow2429+2^{21}\times3^{29}y$ $\rightarrow7288+2^{21}\times3^{30}y\rightarrow3644+2^{20}\times3^{30}y\rightarrow1822+2^{19}\times3^{30}y\rightarrow911+2^{18}\times3^{30}y\rightarrow2734+2^{18}\times3^{31}y$ $\rightarrow1367+2^{17}\times3^{31}y\rightarrow4102+2^{17}\times3^{32}y\rightarrow2051+2^{16}\times3^{32}y\rightarrow6154+2^{16}\times3^{33}y\rightarrow3077+2^{15}\times3^{33}y$ $\rightarrow9232+2^{15}\times3^{34}y\rightarrow4616+2^{14}\times3^{34}y\rightarrow2308+2^{13}\times3^{34}y\rightarrow1154+2^{12}\times3^{34}y\rightarrow577+2^{11}\times3^{34}y\rightarrow$ $1732+2^{11}\times3^{35}y\rightarrow866+2^{10}\times3^{35}y\rightarrow433+2^{9}\times3^{35}y\rightarrow1300+2^{9}\times3^{36}y\rightarrow650+2^{8}\times3^{36}y\rightarrow$ $325+2^{7}\times3^{36}y\rightarrow976+2^{7}\times3^{37}y\rightarrow488+2^{6}\times3^{37}y\rightarrow244+2^{5}\times3^{37}y\rightarrow122+2^{4}\times3^{37}y\rightarrow61+2^{3}\times$ $3^{37}y\rightarrow184+2^{3}\times3^{38}y\rightarrow92+2^{2}\times3^{38}y\rightarrow46+2^{1}\times3^{38}y\rightarrow23+3^{38}y$, get №1 satisfactory operational result $23+3^{38}y$ about the kind of $15+12(1+2^{57}y)$.

(**2**) From $15+12(4+2^{55}\times3^2y)=63+2^{57}\times3^3y\rightarrow190+2^{57}\times3^4y\rightarrow95+2^{56}\times3^4y\rightarrow286+2^{56}\times3^5y\rightarrow$

$143+2^{55}\times 3^5y\to 430+2^{55}\times 3^6y\to 215+2^{54}\times 3^6y\to 646+2^{54}\times 3^7y\to 323+2^{53}\times 3^7y\to 970+2^{53}\times 3^8y$ $\to 485+2^{52}\times 3^8y\to 1456+2^{52}\times 3^9y\to 728+2^{51}\times 3^9y\to 364+2^{50}\times 3^9y$* at operational route $27+2^{59}\times 3y\ldots\to 61+2^3\times 3^{37}y < 63+2^{57}\times 3^3y$, get №1 satisfactory operational result $61+2^3\times 3^{37}y$ about the kind of $15+12(4+2^{55}\times 3^2y)$.

(3) From $15+12(8+2^{32}\times 3^{17}y)=111+2^{34}\times 3^{18}y\to 334+2^{34}\times 3^{19}y$** at operational route $27+2^{59}\times 3y\ldots\to 61+2^3\times 3^{37}y<111+2^{34}\times 3^{18}y$, get №1 satisfactory operational result $61+2^3\times 3^{37}y$ about the kind of $15+12(8+2^{32}\times 3^{17}y)$ as well.

In some cases, an operational route of $15+12c$ and an operational route of $19+12c$ intersect from each other, such as operate $15+12(1+2^{57}y)$ via fifth step to $19+12(1+2^{54}\times 3^2y)$, see also the example (1).

Due to $c\geq 1$, there are infinitely many odd numbers of $15+12c/19+12c$, as thus, probably they belong to infinite many kinds.

Since there is an operational route between each kind of $15+12c/19+12c$ and a №1 satisfactory operational result about the kind of $15+12c/19+12c$, so $15+12c/19+12c$ has how many kinds, then there are how many operational routes.

Yet, for each line segment coincided from one another, either it is regarded as component part of some operational routes, or it is irrespective of the others.

Thus, the bunch's operational routes of $15+12c/19+12c$ are formally similar to networking status. Namely, any two operational routes existed therein, either they intersect directly, or they are at indirect connection.

Since setting up the variable of *c*, such that all kinds of *15+12c/19+12c* collect at the bunch of operational routes of *15+12c/19+12c*; but then, due to the odevity of χ, such that there is likely infinitely many branches at the bunch of operational routes of *15+12c/19+12c*.

Now that №1 satisfactory operational results determine all kinds of *15+12c/19+12c*, therefore, if begin with any kind of *15+12c/19+12c* to operate by the operational rule, then №1 satisfactory operational result about the kind of *15+12c/19+12c* is only in one of following *3* cases.

(**1**) №1 satisfactory operational result about a kind of *15+12c/19+12c* first appears at an operational route which starts with the kind of *15+12c/19+12c*;

(**2**) An operational route which starts with a kind of *15+12c/19+12c* directly intersects another operational route which has №1 satisfactory operational result about the kind of *15+12c/19+12c*;

(**3**) An operational route which starts with a kind of *15+12c/19+12c* indirectly connects to another operational route which has №1 satisfactory operational result about the kind of *15+12c/19+12c*.

In a word, all kinds of *15+12c/19+12c* and every №1 satisfactory operational result must coexist at the bunch of operational routes of *15+12c/19+12c*.

As has been mentioned, there are 6 kinds of *15+12c/19+12c* derived from №1 satisfactory operational results monogamously, and that each of the 6 kinds has been proved to suit the conjecture, undoubtedly they coexist at the bunch of operational routes of *15+12c/19+12c* too.

Unquestionably, for an operational route which starts with any unproved kind of *15+12c/19+12c*, either it intersects directly an operational route which starts with one of proven 6 kinds of *15+12c/19+12c*, or indirectly connects to an operational route which starts with one of proven 6 kinds of *15+12c/19+12c*.

Therefore, each and every unproved kind of *15+12c/19+12c* is proved to suit the conjecture, according to Lemma *3*.

Consequently, if *n+1* belongs within any kind of *15+12c/ 19+12c*, then *n+1* suits the conjecture, according to Theorem *2* or Lemma *2*.

7. Make a summary and reach the conclusion

To sum up, *n+1* has been proved to suit the conjecture, whether *n+1* belongs within which genus, which sort or which kind of odd numbers, or it is exactly an even number.

We likewise can prove positive integers *n+2*, *n+3*, *n+4* etc. up to every positive integer to suit the conjecture in the light of the preceding way of doing the thing.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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