Complex Dynamics as Foundation of Relativistic Spacetime

Ervin Goldfain

Abstract

We explore the idea that Minkowski spacetime and the principle of locality reflect the asymptotic properties of *self-organized criticality* (SOC). Both properties arise from demanding that the scaling behavior of space and time coordinates follows the power-law distribution of Gaussian random walks.

Key words: complex dynamics, self-organized criticality, minimal fractal manifold, Minkowski spacetime, the principle of locality, Gaussian random walks.

1. Introduction

SOC can be traced back to a theory developed in the mid-1980's aimed to explain how *complex nonlinear systems* with extended degrees of freedom can reproduce the power-law behavior of certain physical observables [1-2]. Nowadays, SOC is understood as a generic model for self-sustained critical behavior in large-scale systems evolving outside equilibrium. The trademark signature of SOC is two-fold:

a) it occurs in global ensembles of multiple interacting components,

b) it is characterized by a power-law distribution of "avalanche" sizes.

SOC is a powerful analytical tool whose applications extend over several fields dealing with *nonlinear dissipative systems*, ranging from astrophysics, natural hazards, magnetospheric physics, computer science, biophysics, and social sciences.

Elaborating from the SOC framework, the goal of our brief report is to confirm that,

1) space and time coordinates are on *equal footing*, with no physical distinction between x_i, $i = 1, 2, 3$ and $x_0 = t$ in natural units $(c = 1)$.

2) the *principle of locality* emerges as asymptotic case of the *finite-size scaling ansatz* (FSS) introduced in the next paragraph.

2. The FSS ansatz and its asymptotic connection to spacetime

The scaling behavior of avalanches can be derived from the statistical attributes of several observables of interest [1-2] : e.g., the size s of the avalanche , the area a of the avalanche, the avalanche duration t and the linear size of the avalanche r. The probability distribution associated with these observables obeys the FSS ansatz

$$P(\eta, L_\eta) \sim \eta^{-\tau_\eta} \Phi(\eta / \eta_c) \text{ for } \eta >> 1,\ L_\eta >> 1$$

$$\eta_c(L_\eta) \sim L_\eta^{D_\eta} \text{ for } L_\eta >> 1 \tag{1}$$

in which

$$\eta = (s, a, t, r) \tag{2}$$

Here, L_η denotes the uppermost limit of η, whose cutoff value is set by η_c. The parameters τ_η and D_η represent the avalanche-size exponent and avalanche dimension, respectively, and their specific values fix the *universality* class of the SOC process described by (1).

Proceeding along these lines, we find it reasonable to conjecture that the conventional "space" and "time" coordinates may be assigned a statistical meaning outside the effective framework of Quantum Field Theory and General Relativity. In particular, inspired by the concept of minimal fractal manifold (MFM) [6], both space and time coordinates are interpreted as intrinsically *random variables,* echoing the behavior of parameters s and t at energies near or exceeding the Fermi scale. On this basis, we write

$$x = (x_\mu), \;\; \mu = 0,1,2,3 \tag{3}$$

$$\varepsilon = 4 - D = \frac{\Delta x}{x} << 1 \tag{4}$$

in which D represents the dimensionality of ordinary 3+1 spacetime and Δx is the resolution of coordinate measurements. The most natural identification is provided by

$$s \to X = \frac{x}{\Delta x} = \varepsilon^{-1} >> 1 \tag{5}$$

On account of (3) – (5), (1) turns into

$$P(X,L) = X^{-\tau_X} \Phi(X/X_c), \quad X >> 1, \;\; L >> 1 \tag{6a}$$

$$X_c = L^{D_X}, \quad L >> 1 \tag{6c}$$

The contribution of the cutoff function Φ becomes negligible when X falls far below its cutoff value, i.e.

$$\Phi(X/X_c) = const., \quad X << X_c \tag{7}$$

The key assumption of this work is that the limit (7) of the power-law (6) reproduces the probability distribution of *Gaussian random walks* (GRW). The rationale behind this assumption lies on two premises:

a) GRW is statistically self-similar, its distribution is stationary and isotropic [5]. The latter property is consistent with space-reflection and time-inversion symmetries of classical and quantum dynamics.

b) the GRW is a common denominator of basic stochastic processes, including Brownian motion, diffusion and quantum mechanical paths [5].

It is known that the dimension of the GRW is equal to $D_{RW} = 2$ [3, 5]. It follows that, on a minimal fractal manifold defined by a scale-dependent deviation $\varepsilon = 4 - D$, the value of the avalanche size exponent entering (6a) is expected to be

$$\tau_X = D_{RW} \pm \varepsilon \tag{8}$$

In the limit $L \to \infty$, the conditional probability of measuring X' for a given X assumes the form [2]

$$P_{X'X}(X', X) \sim \delta(X' - X^{\gamma_{X'X}}) \tag{9}$$

where

$$\gamma_{X'X} = \frac{\tau_X - 1}{\tau_{X'} - 1} \tag{10}$$

It is apparent from (5), (6a) and (8) that the conventional limit of smooth four-dimensional spacetime $(\varepsilon = 0)$ implies that the scaling behavior of space and time is *identical*. It also implies that (10) is equal to one and that (9) is no longer sensitive to the continuous variation of ε with the energy scale. As a result, the measurement of X' is independent of the measurement of X, a statement reflecting the *principle of locality* of relativistic physics [4].

We close by noting that our findings here are consistent with the body of ideas discussed in [7-11].

References

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