On some Ramanujan formulas: mathematical connections with ϕ and several parameters of Quantum Geometry, String Theory and Particle Physics. II

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Abstract

In this paper we have described and analyzed some Ramanujan expressions. We have obtained several mathematical connections with ϕ and various parameters of Quantum Geometry, String Theory and Particle Physics.

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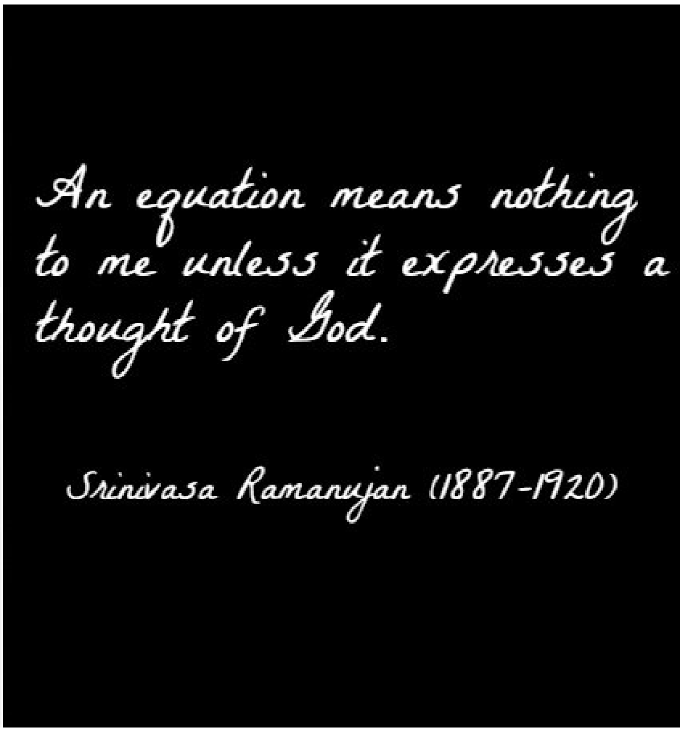


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

On Climbing Scalars in String Theory
E. Dudas, N. Kitazawa and A. Sagnotti - arXiv:1009.0874v1 [hep-th] 4 Sep 2010

We have that:

On the other hand, for $\gamma < 1$ the second solution describes a scalar that emerges from the Big Bang while climbing down the potential, at a speed in "parametric" time that eventually approaches from above the limiting value (2.12), but it disappears altogether as $\gamma \to 1$. However, the suggestive analogy with eqs. (1.1) holds only insofar as one refers to the "parametric" time τ, or equivalently to t, since in all cases the scalar comes eventually to rest in terms of the cosmological time. Keeping this in mind, the complete solutions for $\gamma < 1$ are

$$\begin{aligned} ds^2 &= e^{\frac{2a_0}{D-1}} \left|\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right|^{\frac{2}{(1+\gamma)(D-1)}} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{2}{(1-\gamma)(D-1)}} d\mathbf{x}\cdot d\mathbf{x} \\ &- e^{-2\gamma\varphi_0} \left|\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right|^{-\frac{2\gamma}{1+\gamma}} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{2\gamma}{1-\gamma}} \left(\frac{d\tau}{M\beta}\right)^2 , \qquad (2.13) \\ e^{\varphi} &= e^{\varphi_0} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{1}{1+\gamma}} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{-\frac{1}{1-\gamma}} \end{aligned}$$

for the *climbing* scalar, and

$$\begin{aligned} ds^2 &= e^{\frac{2a_0}{D-1}} \left|\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right|^{\frac{2}{(1+\gamma)(D-1)}} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{2}{(1-\gamma)(D-1)}} d\mathbf{x}\cdot d\mathbf{x} \\ &- e^{-2\gamma\varphi_0} \left|\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right|^{-\frac{2\gamma}{1+\gamma}} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{2\gamma}{1-\gamma}} \left(\frac{d\tau}{M\beta}\right)^2 , \qquad (2.14) \\ e^{\varphi} &= e^{\varphi_0} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{1}{1+\gamma}} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{-\frac{1}{1-\gamma}} \end{aligned}$$

From:

$$e^{\varphi} = e^{\varphi_0} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{1}{1+\gamma}} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{-\frac{1}{1-\gamma}}$$

For $\gamma = 1/2$, $\varphi = \sqrt{3} / 2$ and a positive value of τ ($\tau = 3$), we obtain:

exp((sqrt3)/2) = x* ((([sinh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * ((((([cosh (3/2*sqrt(1-1/4))])))))^1/(1-1/2)

Input:

$$\exp\left(\frac{\sqrt{3}}{2}\right) = x \sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)} \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}$$

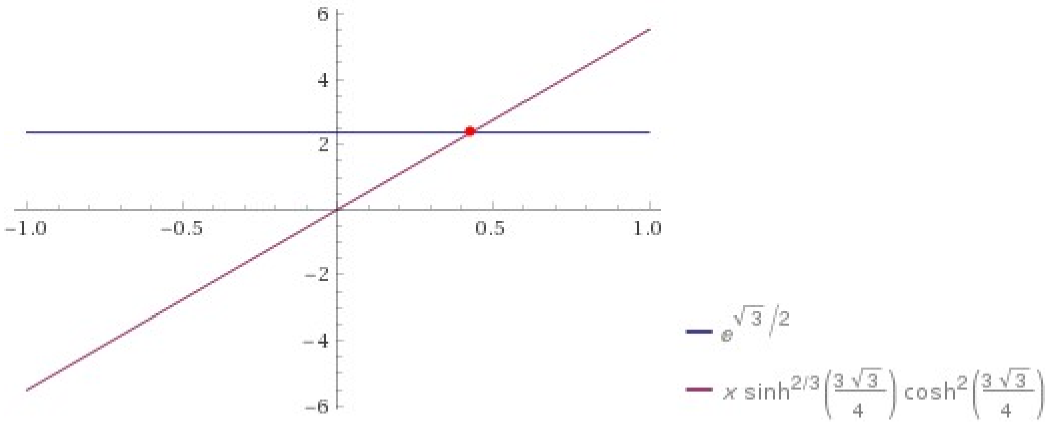
sinh(x) is the hyperbolic sine function

cosh(x) is the hyperbolic cosine function

Exact result:

$$e^{\sqrt{3}/2} = x \sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\cosh^2\left(\frac{3\sqrt{3}}{4}\right)$$

Plot:



Alternate forms:

$$e^{\sqrt{3}/2} = \frac{1}{2}\left(x \sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right) + x \sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\cosh\left(\frac{3\sqrt{3}}{2}\right)\right)$$

$$e^{\sqrt{3}/2} = \frac{1}{4}\left(\frac{1}{2}\left(e^{(3\sqrt{3})/4} - e^{-(3\sqrt{3})/4}\right)\right)^{2/3}\left(e^{-(3\sqrt{3})/4} + e^{(3\sqrt{3})/4}\right)^2 x$$

$$e^{\sqrt{3}/2} = x\left(2\sinh\left(\frac{\sqrt{3}}{4}\right) + -i\right)^2\left(2\sinh\left(\frac{\sqrt{3}}{4}\right) + i\right)^2 \sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\left(\cosh\left(\frac{\sqrt{3}}{8}\right) - i\sinh\left(\frac{\sqrt{3}}{8}\right)\right)^2\left(\cosh\left(\frac{\sqrt{3}}{8}\right) + i\sinh\left(\frac{\sqrt{3}}{8}\right)\right)^2$$

Solution:

$x \approx 0.43098$

0.43098

Solution:

$$x = \frac{e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)}$$

Indeed:

(e^(sqrt(3)/2) sech^2((3 sqrt(3))/4))/(sinh^(2/3)((3 sqrt(3))/4)) * ((([sinh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * (((((([cosh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2)

Input:

$$\frac{e^{\sqrt{3}/2}\,\mathrm{sech}^2\left(\frac{1}{4}\left(3\sqrt{3}\right)\right)}{\sinh^{2/3}\left(\frac{1}{4}\left(3\sqrt{3}\right)\right)}\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}$$

sech(x) is the hyperbolic secant function

sinh(x) is the hyperbolic sine function

cosh(x) is the hyperbolic cosine function

Exact result:

$$e^{\sqrt{3}/2}$$

Decimal approximation:

2.377442675236164788244760758100045419327253742216647458987...

2.3774426752…

Property:

$e^{\sqrt{3}/2}$ is a transcendental number

Alternative representations:

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\,\mathrm{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(e^{\sqrt{3}/2}\sqrt[\frac{3}{2}]{\frac{1}{2}\left(-e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\sqrt[\frac{1}{2}]{\frac{1}{2}\left(e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\left(\frac{1}{\cos\left(-\frac{3i\sqrt{3}}{4}\right)}\right)^2\right)\Big/\left(\frac{1}{2}\left(-e^{-\left(3\sqrt{3}\right)/4}+e^{\left(3\sqrt{3}\right)/4}\right)\right)^{2/3}$$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\frac{e^{\sqrt{3}/2}\ \sqrt[\frac{1}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{3}{2}]{\frac{1}{2}\left(-e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\ \left(\frac{2\,e^{(3\sqrt{3})/4}}{1+e^{(6\sqrt{3})/4}}\right)^2}{\left(\frac{1}{2}\left(-e^{-(3\sqrt{3})/4}+e^{(3\sqrt{3})/4}\right)\right)^{2/3}}$$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(e^{\sqrt{3}/2}\ \sqrt[\frac{1}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{3}{2}]{\frac{1}{2}\left(-e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\ \left(\frac{2}{e^{-(3\sqrt{3})/4}+e^{(3\sqrt{3})/4}}\right)^2\right)\Big/\left(\frac{1}{2}\left(-e^{-(3\sqrt{3})/4}+e^{(3\sqrt{3})/4}\right)\right)^{2/3}$$

Series representations:

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(4\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}\right)^2 \left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^2\right)\Big/\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{3}}{4}\right)\right)^{2/3}$$

for $\left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{(3\sqrt{3})/4}\right)$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(4\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)i^2\right.$$

$$\left.\left(\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}\right)^2\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}\right)^2\right)\Bigg/$$

$$\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{3}}{4}\right)\right)^{2/3} \quad \text{for } \left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{\left(3\sqrt{3}\right)/4}\right)$$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(4\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=1}^{\infty}(-1)^k q^{-1+2k}\right)^2\right.$$

$$\left.\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\right)\Bigg/$$

$$\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{3}}{4}\right)\right)^{2/3} \quad \text{for } \left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{\left(3\sqrt{3}\right)/4}\right)$$

and:

exp((sqrt3)/2)

Input:

$$\exp\left(\frac{\sqrt{3}}{2}\right)$$

Exact result:

$$e^{\sqrt{3}/2}$$

Decimal approximation:

2.3774426752361647882447607581000454193272537422166474589870...

2.3774426752…

Property:

$e^{\sqrt{3}/2}$ is a transcendental number

Series representations:

$$e^{\sqrt{3}/2} = e^{1/2\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}}$$

$$e^{\sqrt{3}/2} = \exp\left(\frac{1}{2}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{\sqrt{3}/2} = \exp\left(\frac{\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\, 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \text{ for } (0<\gamma<-\text{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From

$$e^{\varphi} = e^{\varphi_0} \left[\cosh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{1}{1+\gamma}} \left[\sinh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{-\frac{1}{1-\gamma}}$$

we obtain:

exp((sqrt3)/2) = x* ((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * ((((([sinh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2)

Input:

$$\exp\left(\frac{\sqrt{3}}{2}\right) = x \sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}$$

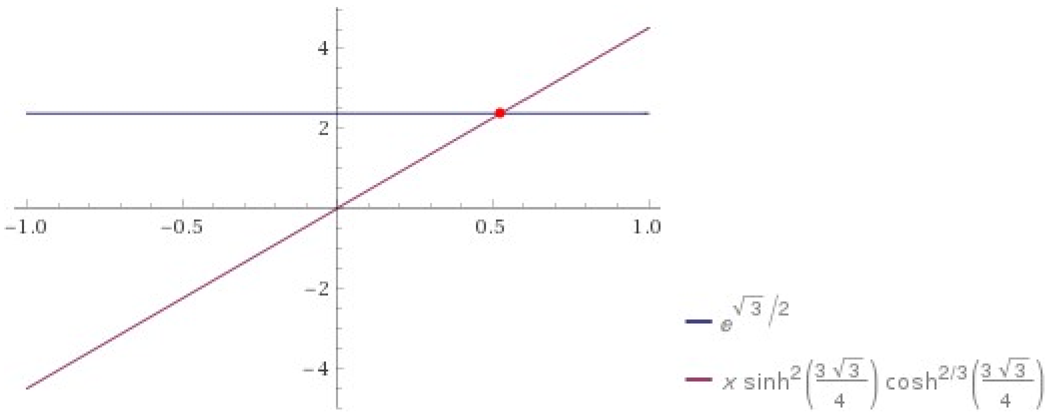
cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Exact result:

$$e^{\sqrt{3}/2} = x \sinh^2\left(\frac{3\sqrt{3}}{4}\right)\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)$$

Plot:



Alternate forms:

$$e^{\sqrt{3}/2} = \frac{1}{2}\left(x\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\cosh\left(\frac{3\sqrt{3}}{2}\right) - x\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\right)$$

$$e^{\sqrt{3}/2} = \frac{1}{4}\left(e^{(3\sqrt{3})/4} - e^{-(3\sqrt{3})/4}\right)^2 \left(\frac{1}{2}\left(e^{-(3\sqrt{3})/4} + e^{(3\sqrt{3})/4}\right)\right)^{2/3} x$$

$$e^{\sqrt{3}/2} = 4x\sinh^2\left(\frac{\sqrt{3}}{8}\right)\cosh^2\left(\frac{\sqrt{3}}{8}\right)\left(2\cosh\left(\frac{\sqrt{3}}{4}\right)-1\right)^2\left(1+2\cosh\left(\frac{\sqrt{3}}{4}\right)\right)^2\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)$$

Solution:

$x \approx 0.52578$

0.52578

and:

(e^(sqrt(3)/2) csch^2((3 sqrt(3))/4))/(cosh^(2/3)((3 sqrt(3))/4)) * ((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * (((((([sinh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2)

Input:

$$\frac{e^{\sqrt{3}/2}\,\operatorname{csch}^2\left(\frac{1}{4}\left(3\sqrt{3}\right)\right)}{\cosh^{2/3}\left(\frac{1}{4}\left(3\sqrt{3}\right)\right)}\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$e^{\sqrt{3}/2}$

Decimal approximation:

2.3774426752361647882447607581000454193272537422166474589 87...

2.3774426752…

Property:

$e^{\sqrt{3}/2}$ is a transcendental number

Alternative representations:

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\frac{e^{\sqrt{3}/2}\ \sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \left(\frac{2\,e^{(3\sqrt{3})/4}}{-1+e^{(6\sqrt{3})/4}}\right)^2}{\cos^{2/3}\left(-\frac{3\,i\sqrt{3}}{4}\right)}$$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\frac{e^{\sqrt{3}/2}\ \sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{\frac{1}{2}\left(-e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\ \left(\frac{2\,e^{(3\sqrt{3})/4}}{-1+e^{(6\sqrt{3})/4}}\right)^2}{\cos^{2/3}\left(-\frac{3\,i\sqrt{3}}{4}\right)}$$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(e^{\sqrt{3}/2}\ \sqrt[\frac{1}{2}]{\frac{1}{2}\left(-e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\ \sqrt[\frac{3}{2}]{\frac{1}{2}\left(e^{-3/2\sqrt{1-1/4}}+e^{3/2\sqrt{1-1/4}}\right)}\ \left(\frac{2\,e^{(3\sqrt{3})/4}}{-1+e^{(6\sqrt{3})/4}}\right)^2\right)\Big/\left(\frac{1}{2}\left(e^{-(3\sqrt{3})/4}+e^{(3\sqrt{3})/4}\right)\right)^{2/3}$$

Series representations:

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(16\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^2 \left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\Bigg)\Bigg/\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{16}\right)^k\sqrt{3}^{2k}}{(2k)!}\right)^{2/3}$$

for $\left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{(3\sqrt{3})/4}\right)$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(16\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^2 \left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\Bigg)\Bigg/ \left(I_0\left(\frac{3\sqrt{3}}{4}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{3}}{4}\right)\right)^{2/3}$$

for $\left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{(3\sqrt{3})/4}\right)$

$$\frac{\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)\left(e^{\sqrt{3}/2}\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)} =$$

$$\left(4\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=1}^{\infty}q^{-1+2k}\right)^2\right.$$
$$\left.\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{2}{3}\right)^{-1-2k}\sqrt{\frac{3}{4}}^{1+2k}}{(1+2k)!}\right)^2\right)\Big/$$
$$\left(I_0\left(\frac{3\sqrt{3}}{4}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{3}}{4}\right)\right)^{2/3} \text{ for } \left(x\in\mathbb{R} \text{ and } x<0 \text{ and } q=e^{(3\sqrt{3})/4}\right)$$

Now, we analyze:

exp((sqrt3)/2) * 1/ (((((((([sinh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * (((((([cosh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2)))))

Input:

$$\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}$$

sinh(x) is the hyperbolic sine function

cosh(x) is the hyperbolic cosine function

Exact result:

$$\frac{e^{\sqrt{3}/2}\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)}$$

sech(x) is the hyperbolic secant function

Decimal approximation:

0.4309835568597609083708882361733597818830846076674428050 50...

0.4309835568…

Alternate forms:

$$e^{\sqrt{3}/2}\operatorname{csch}^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\operatorname{sech}^2\left(\frac{3\sqrt{3}}{4}\right)$$

$$\frac{4\,e^{(5\sqrt{3})/2}\left(\frac{2}{e^{(3\sqrt{3})/2}-1}\right)^{2/3}}{\left(1+e^{(3\sqrt{3})/2}\right)^2}$$

$$\frac{4\,e^{\sqrt{3}/2}\cosh^2\left(\frac{3\sqrt{3}}{4}\right)}{\sinh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)\left(1+\cosh\left(\frac{3\sqrt{3}}{2}\right)\right)^2}$$

csch(x) is the hyperbolic cosecant function

Alternative representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{1}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[\frac{3}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{1}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[\frac{3}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{1}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{3}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}}$$

Series representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{4\,e^{\sqrt{3}/2}\left(\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{3^{3/2+3k}\times 4^{-1-2k}}{(1+2k)!}\right)^{2/3}}$$

for $q = e^{(3\sqrt{3})/4}$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = -\frac{e^{\sqrt{3}/2}\sum_{k=-\infty}^{\infty}\frac{1}{\left(\frac{3\sqrt{3}}{4}+i\left(\frac{1}{2}+k\right)\pi\right)^2}}{\left(\sum_{k=0}^{\infty}\frac{3^{3/2+3k}\times 4^{-1-2k}}{(1+2k)!}\right)^{2/3}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{256\,e^{\sqrt{3}/2}\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(1+2k)}{27+4(\pi+2k\pi)^2}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{3^{3/2+3k}\times 4^{-1-2k}}{(1+2k)!}\right)^{2/3}}$$

Integral representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{8\sqrt[3]{2}\,e^{\sqrt{3}/2}\left(\int_0^{\infty}\frac{t^{(3i\sqrt{3})/(2\pi)}}{1+t^2}\,dt\right)^2}{3\pi^2\left(\int_0^1\cosh\left(\frac{3\sqrt{3}\,t}{4}\right)dt\right)^{2/3}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} =$$

$$\frac{16\, i\, 2^{2/3}\, e^{\sqrt{3}/2}\, \sqrt[3]{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{27/(64\,s)+s}}{s^{3/2}}\,ds}\ \left(\int_0^\infty \frac{t^{(3\,i\sqrt{3})/(2\,\pi)}}{1+t^2}\,dt\right)^2}{3\,\pi^{5/3}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{27/(64\,s)+s}}{s^{3/2}}\,ds} \quad \text{for } \gamma > 0$$

and:

exp((sqrt3)/2) * 1/(((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2)))))

Input:

$$\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{e^{\sqrt{3}/2}\,\operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

0.525779409005096453267215227541871842025138799661923321769...

0.525779409…

Alternate forms:

$$e^{\sqrt{3}/2} \operatorname{csch}^2\left(\frac{3\sqrt{3}}{4}\right) \operatorname{sech}^{2/3}\left(\frac{3\sqrt{3}}{4}\right)$$

$$\frac{4\, e^{(5\sqrt{3})/2} \left(\frac{2}{1+e^{(3\sqrt{3})/2}}\right)^{2/3}}{\left(e^{(3\sqrt{3})/2} - 1\right)^2}$$

$$\frac{4\, e^{\sqrt{3}/2} \sinh^2\left(\frac{3\sqrt{3}}{4}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{3}}{4}\right) \left(1 - \cosh\left(\frac{3\sqrt{3}}{2}\right)\right)^2}$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Alternative representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}\, i \sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2} - \frac{3}{2}\, i \sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\, i \sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2} + \frac{3}{2}\, i \sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

Series representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{4e^{\sqrt{3}/2}\left(\sum_{k=1}^{\infty} q^{-1+2k}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{27}{16}\right)^k}{(2k)!}\right)^{2/3}} \quad \text{for } q = e^{(3\sqrt{3})/4}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{432e^{\sqrt{3}/2}\left(\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{27+16k^2\pi^2}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{27}{16}\right)^k}{(2k)!}\right)^{2/3}}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{e^{\sqrt{3}/2}\left(8+27\sum_{k=1}^{\infty}\frac{(-1)^k}{\frac{27}{16}+k^2\pi^2}\right)^2}{108\left(\sum_{k=0}^{\infty}\frac{\left(\frac{27}{16}\right)^k}{(2k)!}\right)^{2/3}}$$

The sum of the two results is:

0.43098355685976090837 + exp((sqrt3)/2) * 1/(((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2)))))

Input interpretation:

$$0.43098355685976090837 + \exp\left(\frac{\sqrt{3}}{2}\right) \times \frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

0.95676296586485736164...

0.956762965…..result that is very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}} \approx 0.9568666373$$

Alternative representations:

$$0.43098355685976090837000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} =$$

$$0.43098355685976090837000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

$$0.43098355685976090837000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} =$$

$$0.43098355685976090837000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

$$0.430983556859760908370000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}=$$

$$0.430983556859760908370000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}$$

Series representations:

$$0.430983556859760908370000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}=$$

$$\left(0.250000000000000000000000\left(1.00000000000000000000000\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)+1.72393422743904363348000\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)\right)\Bigg/\left(\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)\quad\text{for } (x\in\mathbb{R}\text{ and }x<0)$$

$$0.4309835568597609083700000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} =$$

$$\left(0.2500000000000000000000000\left(1.000000000000000000000000\exp\left(\frac{1}{2}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+1.7239342274390436334800000\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)\right)\Big/\left(\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)$$

$$0.43098355685976090837 0000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} =$$

$$\left(0.4309835568597609083700 \left(1.000000000000000000000000\, i \left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{i\pi}{2} + \frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!} + 0.5800685339866502789441 \exp\left(\frac{1}{2}\exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \sqrt[3]{i \sum_{k=0}^{\infty} \frac{\left(-\frac{i\pi}{2} + \frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}}\right)\right)\right) \Big/$$

$$\left(i \left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2 \sum_{k=0}^{\infty} \frac{\left(-\frac{i\pi}{2} + \frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}\right) \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiplying the two results, we obtain:

[((0.43098355685976 * exp((sqrt3)/2) * 1/(((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * ((((([sinh (3/2*sqrt(1-1/4))]))))^1/(1-1/2))))]

Input interpretation:

$$0.43098355685976 \exp\left(\frac{\sqrt{3}}{2}\right) \times \frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)} \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

0.22660227981664...

0.22660227981664…

Alternative representations:

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)} \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)} \sqrt[\frac{1}{2}]{-i \cos\left(\frac{\pi}{2} - \frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)} \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)} \sqrt[\frac{1}{2}]{i \cos\left(\frac{\pi}{2} + \frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}}$$

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}}$$

Series representations:

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{0.107745889214940000 \exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \frac{0.107745889214940000 \exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty} I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}} = \left(0.107745889214940000\right.$$

$$\left.\exp\left(\frac{1}{2}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}\right)\right) /$$

$$\left(\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)$$

From which:

1+[((0.43098355685976 * exp((sqrt3)/2) * 1/(((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) * (((((([sinh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2))))]^1/3

Input interpretation:

$$1+\sqrt[3]{0.43098355685976 \exp\left(\frac{\sqrt{3}}{2}\right) \times \frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

1.60966054733343...

1.60966054733343…

Alternative representations:

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} = 1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} = 1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} = 1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

Series representations:

$$1+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} =$$

$$0.475846526232789172\left(2.10151791569618265+1.00000000000000000\right.$$

$$\left.\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$1+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} =$$

$$0.475846526232789172\left(2.10151791569618265+1.00000000000000000\right.$$

$$\left.\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}} =$$

$$0.475846526232789172\left(2.10151791569618265+1.00000000000000000\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}\right)^{2/3}}}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

and:

1+[((0.43098355685976 exp((sqrt3)/2) 1/(((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2))))]^1/3+3^2/10^3

Input interpretation:

$$1+\sqrt[3]{0.43098355685976\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}+\frac{3^2}{10^3}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

1.61866054733343...

1.61866054733343…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}+\frac{3^2}{10^3}}=$$

$$1+\frac{9}{10^3}+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

$$1+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}+\frac{3^2}{10^3}=$$

$$1+\frac{9}{10^3}+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

$$1+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}+\frac{3^2}{10^3}=$$

$$1+\frac{9}{10^3}+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}}$$

Series representations:

$$1+\sqrt[3]{\frac{0.430983556859760000 \exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}+\frac{3^2}{10^3}=$$

$$0.47584652623278917\left(2.1204315769374483+1.0000000000000000\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}+\frac{3^2}{10^3}=$$

$$0.47584652623278917\left(2.1204315769374483+1.0000000000000000\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$1+\sqrt[3]{\frac{0.430983556859760000\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}+\frac{3^2}{10^3} =$$

$$0.47584652623278917\left(2.1204315769374483+1.0000000000000000\sqrt[3]{\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}\right)^{2/3}}}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

From the following division of the two results, we obtain:

1/2* [((exp((sqrt3)/2) /(((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) (((((([sinh (3/2*sqrt(1-1/4))]))))))^1/(1-1/2))))] / (0.43098355685976)

Input interpretation:

$$\frac{1}{2} \times \frac{\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}{0.43098355685976}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

0.60997618196392...

0.60997618196392...

Alternative representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2 \times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} = \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2 \times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} =$$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2\times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

Series representations:

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} =$$

$$\frac{0.290034266993333125\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} =$$

$$\frac{0.290034266993333125\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2} =$$

$$\left(0.290034266993333125\right.$$

$$\left.\exp\left(\frac{1}{2}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}\right)\right)/$$

$$\left(\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)$$

From which:

3^2/10^3+1+1/2* [((exp((sqrt3)/2) /(((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))]))))^1/(1-1/2))))] / (0.43098355685976)

Input interpretation:

$$\frac{3^2}{10^3}+1+\frac{1}{2}\times\frac{\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}{0.43098355685976}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

1.6189761819639...

1.6189761819639…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=1+\frac{9}{10^3}+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2\times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=1+\frac{9}{10^3}+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2\times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=1+\frac{9}{10^3}+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{2\times 0.430983556859760000\left(\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\right)}$$

Series representations:

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=$$

$$\left(0.2900342669933331\left(1.0000000000000000\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)+3.4788992709719827\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)\right)\Big/$$

$$\left(\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\,\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=$$

$$\left(0.2900342669933331\left(1.0000000000000000\exp\left(\frac{1}{2}\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2\,(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)+\right.\right.$$

$$\left.\left.3.4788992709719827\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{\,2k}}{(2k)!}\right)^{2/3}\right)\right)\Big/$$

$$\left(\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{\,2k}}{(2k)!}\right)^{2/3}\right)$$

$$\frac{3^2}{10^3}+1+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\left(\left(\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\;\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\right)0.430983556859760000\right)2}=$$

$$\left(1.0090000000000000\right.$$

$$\left(1.0000000000000000\, i\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2 \sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}+\right.$$

$$0.28744724181698030\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt[3]{i\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}}\right)\right)\Bigg/$$

$$\left(i\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+\frac{3\sqrt{\frac{3}{4}}}{2}\right)^{1+2k}}{(1+2k)!}\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

From the previous expression

$$0.43098355685976090837+\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}$$

we obtain also:

((((0.430983556859 + exp((sqrt3)/2) 1/(((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2))))))))^1/64

Input interpretation:

$$\sqrt[64]{0.430983556859+\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

0.99930961962882...

0.99930961962882….. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and from which, we obtain:

log base 0.999309619628(((((((0.430983556859 + exp((sqrt3)/2) 1/(((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2))))))))))

Input interpretation:

$$\log_{0.999309619628}\left(0.430983556859 + \exp\left(\frac{\sqrt{3}}{2}\right) \times \frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log_b(x)$ is the base- b logarithm

Result:

63.9999999...

63.9999… ≈ 64

Alternative representations:

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) = \frac{\log\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)}{\log(0.9993096196280000)}$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) = \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) = \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

Series representations:

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\,\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) = -\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-0.5690164431410000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\sinh^2\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)}\right)^k}{k}}{\log(0.9993096196280000)}$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) =$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) =$$

$$\log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

27*log base 0.999309619628((((((0.430983556859 + exp((sqrt3)/2) 1/((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2)))))))))))+1

Input interpretation:

$$27\log_{0.999309619628}\left(0.430983556859 + \exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+1$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log_b(x)$ is the base- b logarithm

Result:

1729.00000...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternative representations:

$$27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 1 =$$

$$1 + \frac{27 \log\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)}{\log(0.9993096196280000)}$$

$$27\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+1=$$

$$1+27\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

$$27\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+1=$$

$$1+27\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

Series representations:

$$27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 1 =$$

$$1 - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-0.5690164431410000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\sinh^2\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)}\right)^k}{k}}{\log(0.9993096196280000)}$$

$$27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 1 =$$

$$1 + 27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2 \left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k \sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 1 =$$

$$1 + 27 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right) + 2\sum_{k=1}^{\infty} I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty} I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

2*log base 0.999309619628((((((0.430983556859 + exp((sqrt3)/2) 1/((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))])))))^1/(1-1/2)))))))))))-3

Input interpretation:

$$2 \log_{0.999309619628}\left(0.430983556859 + \exp\left(\frac{\sqrt{3}}{2}\right) \times \frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) - 3$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log_b(x)$ is the base- b logarithm

Result:

125.000000...

125 result very near to the Higgs boson mass 125.18 GeV

Alternative representations:

$$2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) - 3 =$$

$$-3 + \frac{2 \log\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)}{\log(0.9993096196280000)}$$

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)-3=$$

$$-3+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i\cos\left(\frac{\pi}{2}+\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)-3=$$

$$-3+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i\cos\left(\frac{\pi}{2}-\frac{3}{2}\,i\sqrt{1-\frac{1}{4}}\right)}}\right)$$

Series representations:

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)-3=$$

$$-3-\frac{2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-0.5690164431410000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\sinh^2\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)}\right)^k}{k}}{\log(0.9993096196280000)}$$

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)-3=$$

$$-3+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)-3=$$

$$-3+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

2*log base 0.999309619628((((((0.430983556859 + exp((sqrt3)/2) 1/((((((([cosh (3/2*sqrt(1-1/4))])))^(1/(1+1/2)) ((((([sinh (3/2*sqrt(1-1/4))]))))^1/(1-1/2)))))))))))+11

Input interpretation:

$$2\log_{0.999309619628}\left(0.430983556859+\exp\left(\frac{\sqrt{3}}{2}\right)\times\frac{1}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+11$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log_b(x)$ is the base- b logarithm

Result:

139.000000...

139 result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 11 =$$

$$11 + \frac{2 \log\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)}{\log(0.9993096196280000)}$$

$$2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 11 = 11 + 2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{i \cos\left(\frac{\pi}{2} + \frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}}\right)$$

$$2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right) + 11 = 11 + 2 \log_{0.9993096196280000}\left(0.4309835568590000 + \frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[\frac{3}{2}]{\cos\left(-\frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}\sqrt[\frac{1}{2}]{-i \cos\left(\frac{\pi}{2} - \frac{3}{2} i \sqrt{1-\frac{1}{4}}\right)}}\right)$$

Series representations:

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+11=$$

$$11-\frac{2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-0.5690164431410000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\cosh^{2/3}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\sinh^2\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)}\right)^k}{k}}{\log(0.9993096196280000)}$$

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+$$

$$11=11+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}\right)^k\sqrt{\frac{3}{4}}^{2k}}{(2k)!}\right)^{2/3}}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{\sqrt{3}}{2}\right)}{\sqrt[1+\frac{1}{2}]{\cosh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}\ \sqrt[1-\frac{1}{2}]{\sinh\left(\frac{3}{2}\sqrt{1-\frac{1}{4}}\right)}}\right)+$$

$$11 = 11+2\log_{0.9993096196280000}\left(0.4309835568590000+\frac{\exp\left(\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4\left(I_0\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^{2/3}\left(\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{3\sqrt{\frac{3}{4}}}{2}\right)\right)^2}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

Now, we have that:

Let us consider a four–dimensional effective action described via a superpotential W and a Kähler potential K of the type

$$W = W_0 + a\,e^{-bT}\,,\qquad K = -3\,\ln(T+\bar{T})\,,\tag{3.1}$$

In adapting eqs. (3.2) and (3.3) to the four–dimensional KKLT system [18], the complex field T is to be expanded according to

$$T = e^{\frac{\Phi_t}{\sqrt{3}}} + i\,\frac{\theta}{\sqrt{3}}\,, \tag{3.4}$$

in terms of the canonically normalized scalar Φ_t and the axion θ. As we have anticipated, the last term in eq. (3.3) corresponds precisely to the "critical" value $\gamma = 1$, in the notation of Section 2, so that the relevant portion of the low–energy effective field theory reads

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\,(\partial\Phi_t)^2 - \frac{1}{2}\,e^{-\frac{2}{\sqrt{3}}\Phi_t}\,(\partial\theta)^2 - V(\Phi_T,\theta)\right]\,. \tag{3.5}$$

In the convenient gauge (2.3) and with the redefinitions

$$\Phi_t = \frac{2}{\sqrt{3}}\,x\,,\quad \theta = \frac{2}{\sqrt{3}}\,y\,,\quad \tau = M\sqrt{\frac{3}{2}}\,t\,, \tag{3.6}$$

where M is a dimensionful quantity related to the energy scale of the potential V, and neglecting

while the scalar potential takes the form

$$\begin{aligned} V \;&=\; \frac{c}{8}\,e^{-2x} + \frac{b}{2}\,e^{-\frac{4x}{3}-b\,e^{\frac{2x}{3}}}\left[(Re\;a\overline{W_0})\,\cos\frac{2by}{3} + (Im\;a\overline{W_0})\,\sin\frac{2by}{3}\right. \\ &+\; \left.\frac{|a|^2}{3}\left(3 + b\,e^{\frac{2x}{3}}\right)\,e^{-b\,e^{\frac{2x}{3}}}\right]\,. \end{aligned} \tag{3.8}$$

$$c = \frac{\gamma}{\sqrt{1-\gamma^2}}\quad,\quad k = \frac{1}{\sqrt{1-\gamma^2}}\left[1-\gamma\left(\gamma+\frac{2}{3}\right)\right]$$

0.72 / sqrt(1-0.72^2)

Input:

$$\frac{0.72}{\sqrt{1-0.72^2}}$$

Result:

1.03750...

1.03750…. = c or c = 1

From

$$a = a_0 + \frac{1}{2}\,\log|\tau-\tau_0| + \frac{(\tau-\tau_0)^2}{4}\,.$$

For $a_0 = 2$, $\tau = 5$, $\tau_0 = 3$, we obtain:

2+1/2 ln(5-3) + ((5-3)^2)/4

Input:

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2$$

$\log(x)$ is the natural logarithm

Exact result:

$$3+\frac{\log(2)}{2}$$

Decimal approximation:

3.3465735902799726547086160607290882840377500671801276270 60...

3.34657359…..= a

Property:

$3+\frac{\log(2)}{2}$ is a transcendental number

Alternate form:

$$\frac{1}{2}(6+\log(2))$$

Alternative representations:

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2=2+\frac{\log_e(2)}{2}+\frac{4}{4}$$

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2=2+\coth^{-1}(3)+\frac{4}{4}$$

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2=2+\frac{1}{2}\log(a)\log_a(2)+\frac{4}{4}$$

Series representations:

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2=$$
$$3+i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\frac{\log(x)}{2}-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k} \quad \text{for } x<0$$

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2 = 3+\frac{1}{2}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{2}+\frac{1}{2}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}$$

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2 = 3+i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\frac{\log(z_0)}{2}-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2 = 3+\frac{1}{2}\int_1^2\frac{1}{t}\,dt$$

$$2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2 = 3-\frac{i}{4\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

Note that:

1/2(((2+1/2 ln(5-3) + ((5-3)^2)/4)))

Input:

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{2}\left(3+\frac{\log(2)}{2}\right)$$

Decimal approximation:

1.673286795139986327354308030364544142018875033590063813530...

1.6732867951…

Property:

$\frac{1}{2}\left(3+\frac{\log(2)}{2}\right)$ is a transcendental number

Alternate forms:

$$\frac{1}{4}(6+\log(2))$$

$$\frac{3}{2}+\frac{\log(2)}{4}$$

Alternative representations:

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{1}{2}\left(2+\frac{\log_e(2)}{2}+\frac{4}{4}\right)$$

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{1}{2}\left(2+\coth^{-1}(3)+\frac{4}{4}\right)$$

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{1}{2}\left(2+\frac{1}{2}\log(a)\log_a(2)+\frac{4}{4}\right)$$

Series representations:

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{3}{2}+\frac{1}{2}i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\frac{\log(x)}{4}-\frac{1}{4}\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\quad\text{for } x<0$$

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{3}{2}+\frac{1}{4}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{4}+\frac{1}{4}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{1}{4}\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}$$

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{3}{2}+\frac{1}{2}i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\frac{\log(z_0)}{4}-\frac{1}{4}\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{3}{2}+\frac{1}{4}\int_1^2\frac{1}{t}\,dt$$

$$\frac{1}{2}\left(2+\frac{1}{2}\log(5-3)+\frac{1}{4}(5-3)^2\right)=\frac{3}{2}-\frac{i}{8\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

From

$$\Phi_t = \frac{2}{\sqrt{3}} x \, , \quad \theta = \frac{2}{\sqrt{3}} y$$

$$T = e^{\frac{\Phi_t}{\sqrt{3}}} + i \frac{\theta}{\sqrt{3}}$$

For x = 1, y = 2 and t = 3, we obtain:

(((exp(2/(sqrt3))))) + ((i*4/(sqrt3)*1/(sqrt3)))

Input:

$$\exp\left(\frac{2}{\sqrt{3}}\right) + i \times \frac{4}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

i is the imaginary unit

Exact result:

$$e^{2/\sqrt{3}} + \frac{4 i}{3}$$

Decimal approximation:

3.1730730616353537098110151283225633260831599935774263618 6... +
1.3333333333333333333333333333333333333333333333333333333... i

Property:

$\frac{4 i}{3} + e^{2/\sqrt{3}}$ is a transcendental number

Polar coordinates:

$r \approx 3.44183$ (radius), $\theta \approx 22.7923°$ (angle)

3.44183 = T

Alternate form:

$$\frac{1}{3}\left(3 e^{2/\sqrt{3}} + 4 i\right)$$

Series representations:

$$\exp\left(\frac{2}{\sqrt{3}}\right)+\frac{i\,4}{\sqrt{3}\,\sqrt{3}}=\frac{4\,i+\exp\left(\frac{2}{\sqrt{2}\,\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt{2}^{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^{2}}{\sqrt{2}^{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^{2}}$$

$$\exp\left(\frac{2}{\sqrt{3}}\right)+\frac{i\,4}{\sqrt{3}\,\sqrt{3}}=\frac{4\,i+\exp\left(\frac{2}{\sqrt{2}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}\right)\sqrt{2}^{2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}{\sqrt{2}^{2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}$$

$$\exp\left(\frac{2}{\sqrt{3}}\right)+\frac{i\,4}{\sqrt{3}\,\sqrt{3}}=$$
$$\frac{16\,i\,\sqrt{\pi}^{2}+\exp\left(\frac{4\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^{2}}{\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^{2}}$$

From

$$V = \frac{c}{8}\,e^{-2x} + \frac{b}{2}\,e^{-\frac{4x}{3}-b\,e^{\frac{2x}{3}}}\left[(Re\;a\overline{W_0})\,\cos\frac{2by}{3} + (Im\;a\overline{W_0})\,\sin\frac{2by}{3} + \frac{|a|^2}{3}\left(3+b\,e^{\frac{2x}{3}}\right)e^{-b\,e^{\frac{2x}{3}}}\right]. \qquad (3.8)$$

we obtain:

1/8 exp(-2) + x/2*exp(-4/3-x*e^(2/3)) [((((3.34657359 cos(4/3)*x + 3.34657359*i*sin(4/3) +(((3.34657359)^2)/3)*(3+exp(2/3)*x)*exp(-e^(2/3)*x)))))]

Input interpretation:

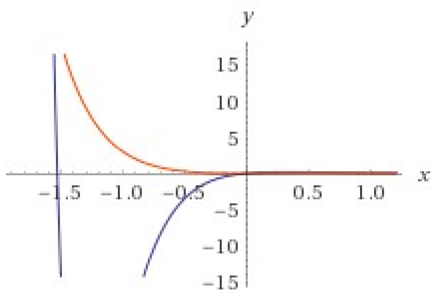
$$\frac{1}{8}\exp(-2)+\frac{x}{2}\exp\left(-\frac{4}{3}-x\,e^{2/3}\right)\left(3.34657359\cos\left(\frac{4}{3}\right)x+3.34657359\,i\sin\left(\frac{4}{3}\right)x+\frac{3.34657359^{2}}{3}\left(3+\exp\left(\frac{2}{3}\right)x\right)\exp\left(-e^{2/3}\,x\right)\right)$$

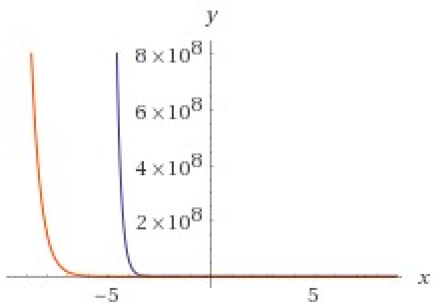
i is the imaginary unit

Result:

$$\frac{1}{8 e^2} + \frac{1}{2} e^{-e^{2/3} x - 4/3} x \left(3.73318 e^{-e^{2/3} x} \left(e^{2/3} x + 3\right) + (0.78724 + 3.25266 i) x\right)$$

Plots:





Alternate forms:

$$0.0169169 + e^{-2 e^{2/3} x} x \left(1.47609 + x \left((0.103757 + 0.428696 i) e^{e^{2/3} x} + \left(0.958341 + 8.32667 \times 10^{-17} i\right)\right)\right)$$

$$0.252616 e^{-2 e^{2/3} x} \left((0.410731 + 1.69703 i) e^{e^{2/3} x} x^2 + (5.8432 + 0 i) x + \left(0.0669669 + 1.85871 \times 10^{-18} i\right) e^{2 e^{2/3} x} + 3.79367 x^2\right)$$

$$e^{-e^{2/3} x - 4/3} x^2 \left(1.86659 e^{2/3 - e^{2/3} x} + (0.39362 + 1.62633 i)\right) + 5.59978 e^{-2 e^{2/3} x - 4/3} x + \frac{1}{8 e^2}$$

Expanded form:

$$(0.103757 + 0.428696 i) e^{-e^{2/3} x} x^2 + 0.958341 e^{-2 e^{2/3} x} x^2 + 1.47609 e^{-2 e^{2/3} x} x + \frac{1}{8 e^2}$$

Alternate form assuming x is real:

$$0.958341\, e^{-2 e^{2/3} x} x^2 + 0.103757\, e^{-e^{2/3} x} x^2 + i\left(1.62633\, e^{-e^{2/3} x - 4/3} x^2 + 0\right) + 1.47609\, e^{-2 e^{2/3} x} x + \frac{1}{8 e^2}$$

Numerical roots:

$x \approx -0.0110649 - 0.0000338878\, i\ldots$

$x \approx 0.7903 + 0.767721\, i\ldots$ 0.7903 + 0.767721i

$x \approx 2.86253 - 1.385\, i\ldots$

Series expansion at x = 0:

$$0.0169169 + 1.47609\, x - (4.68795 - 0.428696\, i)\, x^2 + (7.26428 - 0.834986\, i)\, x^3 - (7.07444 - 0.813165\, i)\, x^4 + O(x^5)$$

(Taylor series)

Derivative:

$$\frac{d}{dx}\left(\frac{1}{8 e^2} + \frac{1}{2} e^{-e^{2/3} x - 4/3} x \left(3.73318\, e^{-e^{2/3} x}\left(e^{2/3} x + 3\right) + (0.78724 + 3.25266\, i)\, x\right)\right) = e^{-2 e^{2/3} x}\left(x^2\left(\left(-3.73318 + 1.11022 \times 10^{-16}\, i\right) - (0.202091 + 0.834986\, i)\, e^{e^{2/3} x}\right) + x\left(-3.83336 + (0.207514 + 0.857392\, i)\, e^{e^{2/3} x}\right) + \left(1.47609 + 5.55112 \times 10^{-17}\, i\right)\right)$$

Indefinite integral:

$$\int\left(\frac{\exp(-2)}{8} + \frac{1}{2} x \exp\left(-\frac{4}{3} - x e^{2/3}\right)\left(3.34657359 \cos\left(\frac{4}{3}\right) x + 3.34657359\, i \sin\left(\frac{4}{3}\right) x + \frac{1}{3} \times 3.34657359^2 \left(3 + \exp\left(\frac{2}{3}\right) x\right) \exp\left(-e^{2/3} x\right)\right)\right) dx = (0.103757 + 0.428696\, i)\, e^{-1.94773 x}\left(-0.513417 x^2 - 0.527194 x - 0.270671\right) + e^{-3.89547 x}\left(-0.246014 x^2 - 0.505232 x - 0.129697\right) + 0.0169169 x + \text{constant}$$

Definite integral:

$$\int_{-\frac{4}{3 e^{2/3}}}^{0}\left(\frac{1}{8 e^2} + \frac{1}{2} e^{-4/3 - e^{2/3} x} x \left((0.78724 + 3.25266\, i)\, x + 3.73318\, e^{-e^{2/3} x}\left(3 + e^{2/3} x\right)\right)\right) dx \approx -1.66733\ldots$$

-1.66733…

From

1/(8 e^2) + 1/2 e^(-e^(2/3) x - 4/3) x (3.73318 e^(-e^(2/3) x) (e^(2/3) x + 3) + (0.78724 + 3.25266 i) x)

for x = 0.7903 + 0.767721i , we obtain:

1/(8 e^2) + 1/2 e^(-e^(2/3) (0.7903 + 0.767721i) - 4/3) (0.7903 + 0.767721i) (3.73318 e^(-e^(2/3) (0.7903 + 0.767721i)) (e^(2/3) (0.7903 + 0.767721i) + 3) + (0.78724 + 3.25266 i) (0.7903 + 0.767721i))

Input interpretation:

$$\frac{1}{8e^2} + \frac{1}{2} e^{-e^{2/3}\times(0.7903+0.767721 i)-4/3} \times (0.7903 + 0.767721 i) \left(3.73318 e^{-e^{2/3}\times(0.7903+0.767721 i)} \left(e^{2/3} \times (0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i) \times (0.7903 + 0.767721 i)\right)$$

i is the imaginary unit

Result:

$$-1.27885\ldots \times 10^{-8} + 1.22702\ldots \times 10^{-7} i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.23367\times 10^{-7}$ (radius), $\theta = 95.9501°$ (angle)

$1.23367*10^{-7}$

Alternative representation:

$$\frac{1}{8e^2} + \frac{1}{2} e^{-e^{2/3} (0.7903+0.767721 i)-4/3} (0.7903 + 0.767721 i) \left(3.73318 e^{-e^{2/3} (0.7903+0.767721 i)} \left(e^{2/3} (0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i) (0.7903 + 0.767721 i)\right) =$$

$$\frac{1}{8\exp^2(z)} + \frac{1}{2} \exp^{-\exp^{\frac{2}{3}}(z)(0.7903+0.767721 i)-\frac{4}{3}}(z) (0.7903 + 0.767721 i) \left(3.73318 \exp^{-\exp^{\frac{2}{3}}(z)(0.7903+0.767721 i)}(z) \left(\exp^{\frac{2}{3}}(z) (0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i) (0.7903 + 0.767721 i)\right) \text{ for } z = 1$$

Series representations:

$$\frac{1}{8 e^2}+\frac{1}{2} e^{-e^{2/3} (0.7903+0.767721 i)-4/3} (0.7903+0.767721 i) \left(3.73318 e^{-e^{2/3} (0.7903+0.767721 i)} \left(e^{2/3} (0.7903+0.767721 i)+3\right)+(0.78724+3.25266 i) (0.7903+0.767721 i)\right) =$$

$$0.958552 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-2-2 (0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}} \left(4.61686 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3} + 4.48496 i \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3} + 1.21623 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4/3} + 2.36297 i \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4/3} + 1.14773 i^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4/3} + 0.256475 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}} + 1.55798 i \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}} + 2.30085 i^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}} + i^3 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}} + 0.130405 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2 (0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} 1/k!\right)^{2/3}}\right)$$

$$\frac{1}{8 e^2} + \frac{1}{2} e^{-e^{2/3} (0.7903 + 0.767721 i) - 4/3} (0.7903 + 0.767721 i) \left(3.73318 e^{-e^{2/3} (0.7903+0.767721 i)} \left(e^{2/3} (0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i) (0.7903 + 0.767721 i)\right) =$$

$$0.958552 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{-2-2 (0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}} \left(4.61686 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3} + 4.48496 i \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3} + 1.21623 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4/3} + 2.36297 i \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4/3} + 1.14773 i^2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4/3} + 0.256475 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}} + 1.55798 i \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}} + 2.30085 i^2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}} + i^3 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2/3+(0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}} + 0.130405 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2 (0.7903+0.767721 i) \left(\sum_{k=0}^{\infty} (-1+k)^2/k!\right)^{2/3}}\right)$$

$$\frac{1}{8e^2} + \frac{1}{2} e^{-e^{2/3}(0.7903+0.767721 i)-4/3} (0.7903 + 0.767721 i) \left(3.73318 e^{-e^{2/3}(0.7903+0.767721 i)} \left(e^{2/3}(0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i)(0.7903 + 0.767721 i)\right) =$$

$$0.958552 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{-2-2(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}} \left(4.61686 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3} + 4.48496 i \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3} + 1.21623 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{4/3} + 2.36297 i \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{4/3} + 1.14773 i^2 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{4/3} + 0.256475 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3+(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}} + 1.55798 i \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3+(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}} + 2.30085 i^2 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3+(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}} + i^3 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2/3+(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}} + 0.130405 \left(\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}\right)^{2(0.7903+0.767721 i)\left(\sum_{k=0}^{\infty}(1+2k)/(2k)!\right)^{2/3}}\right)$$

From which:

4ln[1/(8 e^2)+1/2 e^(-e^(2/3) (0.7903+0.767721i) - 4/3) (0.7903+0.767721i) (3.73318 e^(-e^(2/3) (0.7903+0.767721i)) (e^(2/3) (0.7903+0.767721i)+3)+(0.78724+3.25266 i) (0.7903+0.767721i))]

Input interpretation:

$$4 \log\left(\frac{1}{8e^2} + \frac{1}{2} e^{-e^{2/3}\times(0.7903+0.767721 i)-4/3} \times (0.7903 + 0.767721 i) \left(3.73318 e^{-e^{2/3}\times(0.7903+0.767721 i)} \left(e^{2/3} \times (0.7903 + 0.767721 i) + 3\right) + (0.78724 + 3.25266 i) \times (0.7903 + 0.767721 i)\right)\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$-63.632404862739095019251109280002884163821215089721890911\ldots +$
$6.698579761164522145336078814251599235772616863563099306\ldots\, i$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 63.984$ (radius), $\theta = 173.991°$ (angle)

63.984 ≈ 64

Alternative representations:

$$4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(e^{2/3}(0.7903+0.767721i)+3\right)+(0.78724+3.25266i)(0.7903+0.767721i)\right)\right)=$$
$$\left(4\log(a)\log_a\left(\frac{1}{2}(0.7903+0.767721i)\left((0.7903+0.767721i)(0.78724+3.25266i)+3.73318\left(3+(0.7903+0.767721i)e^{2/3}\right)e^{-(0.7903+0.767721i)e^{2/3}}\right)e^{-4/3-(0.7903+0.767721i)e^{2/3}}+\frac{1}{8e^2}\right)=$$
$$4\log(a)\log_a\left(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)\right)$$

$$4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(e^{2/3}(0.7903+0.767721i)+3\right)+(0.78724+3.25266i)(0.7903+0.767721i)\right)\right)=$$
$$\left(4\log_e\left(\frac{1}{2}(0.7903+0.767721i)\left((0.7903+0.767721i)(0.78724+3.25266i)+3.73318\left(3+(0.7903+0.767721i)e^{2/3}\right)e^{-(0.7903+0.767721i)e^{2/3}}\right)e^{-4/3-(0.7903+0.767721i)e^{2/3}}+\frac{1}{8e^2}\right)=$$
$$4\log_e\left(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)\right)$$

$$4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(e^{2/3}(0.7903+0.767721i)+3\right)+(0.78724+3.25266i)(0.7903+0.767721i)\right)\right)=$$

$$\left(-4\,\mathrm{Li}_1\left(1-\frac{1}{2}(0.7903+0.767721i)\left((0.7903+0.767721i)(0.78724+3.25266i)+3.73318\left(3+(0.7903+0.767721i)e^{2/3}\right)e^{-(0.7903+0.767721i)e^{2/3}}\right)e^{-4/3-(0.7903+0.767721i)e^{2/3}}-\frac{1}{8e^2}\right)=\right.$$

$$-4\,\mathrm{Li}_1\left(1-\frac{1}{8e^2}-\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)\Bigg)$$

Series representations:

$$4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(e^{2/3}(0.7903+0.767721i)+3\right)+(0.78724+3.25266i)(0.7903+0.767721i)\right)\right)=$$

$$4\log\left(-1+\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)-$$

$$4\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318\,e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)^{-k}$$

$$4\log\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(e^{2/3}(0.7903+0.767721i)+3\big)+(0.78724+3.25266i)(0.7903+0.767721i)\Big)\Big)=$$
$$8\pi\mathcal{A}\Big\lfloor\frac{1}{2\pi}\arg\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(3+e^{2/3}(0.7903+0.767721i)\big)+(0.7903+0.767721i)(0.78724+3.25266i)\Big)-x\Big)\Big\rfloor+$$
$$4\log(x)-4\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(3+e^{2/3}(0.7903+0.767721i)\big)+(0.7903+0.767721i)(0.78724+3.25266i)\Big)-x\Big)^k x^{-k}\quad\text{for } x<0$$

$$4\log\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(e^{2/3}(0.7903+0.767721i)+3\big)+(0.78724+3.25266i)(0.7903+0.767721i)\Big)\Big)=$$
$$8\pi\mathcal{A}\Big\lfloor\frac{1}{2\pi}\Big(\pi-\arg\Big(\frac{1}{z_0}\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(3+e^{2/3}(0.7903+0.767721i)\big)+(0.7903+0.767721i)(0.78724+3.25266i)\Big)\Big)\Big)-\arg(z_0)\Big)\Big\rfloor+4\log(z_0)-$$
$$4\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(3+e^{2/3}(0.7903+0.767721i)\big)+(0.7903+0.767721i)(0.78724+3.25266i)\Big)-z_0\Big)^k z_0^{-k}$$

Integral representations:

$$4\log\Big(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\Big(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\big(e^{2/3}(0.7903+0.767721i)+3\big)+(0.78724+3.25266i)(0.7903+0.767721i)\Big)\Big)=4\int_1^{\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)}\frac{1}{t}\,dt$$

$$4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}(0.7903+0.767721i)-4/3}(0.7903+0.767721i)\left(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\left(e^{2/3}(0.7903+0.767721i)+3\right)+(0.78724+3.25266i)(0.7903+0.767721i)\right)\right)=\frac{2}{\pi\mathcal{A}}$$

$$\int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1+\frac{1}{8e^2}+\frac{1}{2}e^{-4/3-e^{2/3}(0.7903+0.767721i)}(0.7903+0.767721i)\left(3.73318e^{-e^{2/3}(0.7903+0.767721i)}\left(3+e^{2/3}(0.7903+0.767721i)\right)+(0.7903+0.767721i)(0.78724+3.25266i)\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds \quad \text{for } -1<\gamma<0$$

From:

(**Modular equations and approximations to π** – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

Now, from the following Ramanujan equation

$$64(g_{58}^{24}+g_{58}^{-24})=e^{\pi\sqrt{58}}-24+4372e^{-\pi\sqrt{58}}+\cdots=64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12}+\left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

we have that:

e^(Pi*sqrt58)-24+4372e^(-Pi*sqrt58)

Input:

$$e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}$$

Exact result:

$$-24+4372\,e^{-\sqrt{58}\,\pi}+e^{\sqrt{58}\,\pi}$$

Decimal approximation:

$$2.4591257727999999999999998408281269931200964876685\ldots\times10^{10}$$

24591257727.999 ≈ 24591257728

Property:

$-24+4372\,e^{-\sqrt{58}\,\pi}+e^{\sqrt{58}\,\pi}$ is a transcendental number

Alternate form:

$$e^{-\sqrt{58}\,\pi}\left(4372-24\,e^{\sqrt{58}\,\pi}+e^{2\sqrt{58}\,\pi}\right)$$

Series representations:

$$e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}=e^{-\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}\left(4372-24\,e^{\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}+e^{2\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}\right)$$

$$e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}=\exp\left(-\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(4372-24\,e^{\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\exp\left(2\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}=\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(58-z_0)^k z_0^{-k}}{k!}\right)\left(4372-24\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(58-z_0)^k z_0^{-k}}{k!}\right)+\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(58-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

That is equal to:

64[((5+sqrt29)/2)^12+((5-sqrt29)/2)^12]

Input:

$$64\left(\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}\right)$$

Result:

24591257728

Decimal approximation:

$2.45912577280000000000000000000000000000000000000000\ldots\times 10^{10}$

24591257728

and from which, we obtain:

(((e^(Pi*sqrt58)-24+4372e^(-Pi*sqrt58))))*1/[((5+sqrt29)/2)^12+((5-sqrt29)/2)^12]

Input:

$$\left(e^{\pi\sqrt{58}} - 24 + 4372\, e^{-\pi\sqrt{58}}\right) \times \frac{1}{\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12} + \left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}}$$

Exact result:

$$\frac{-24 + 4372\, e^{-\sqrt{58}\,\pi} + e^{\sqrt{58}\,\pi}}{\frac{\left(5-\sqrt{29}\right)^{12}}{4096} + \frac{\left(5+\sqrt{29}\right)^{12}}{4096}}$$

Decimal approximation:

63.99999999999999999999999958574709821201082469541529302902...

63.99999…≈ 64

Property:

$$\frac{-24 + 4372\, e^{-\sqrt{58}\,\pi} + e^{\sqrt{58}\,\pi}}{\frac{\left(5-\sqrt{29}\right)^{12}}{4096} + \frac{\left(5+\sqrt{29}\right)^{12}}{4096}}$$ is a transcendental number

Alternate forms:

$$\frac{-24 + 4372\, e^{-\sqrt{58}\,\pi} + e^{\sqrt{58}\,\pi}}{384\,238\,402}$$

$$-\frac{12}{192\,119\,201} + \frac{2186\, e^{-\sqrt{58}\,\pi}}{192\,119\,201} + \frac{e^{\sqrt{58}\,\pi}}{384\,238\,402}$$

$$\frac{e^{-\sqrt{58}\,\pi}\left(4372 - 24\, e^{\sqrt{58}\,\pi} + e^{2\sqrt{58}\,\pi}\right)}{384\,238\,402}$$

Series representations:

$$\frac{e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}}{\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}}=$$

$$\left(2048\,e^{-\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}\left(4372-24\,e^{\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}+e^{2\pi\sqrt{57}\sum_{k=0}^{\infty}57^{-k}\binom{1/2}{k}}\right)\right)\Big/$$
$$\left(\left(625+150\sqrt{28}^{\,2}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^2+\sqrt{28}^{\,4}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^4\right)\right.$$
$$\left(390\,625+937\,500\sqrt{28}^{\,2}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^2+83\,750\sqrt{28}^{\,4}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^4+\right.$$
$$\left.\left.1500\sqrt{28}^{\,6}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^6+\sqrt{28}^{\,8}\left(\sum_{k=0}^{\infty}28^{-k}\binom{\frac{1}{2}}{k}\right)^8\right)\right)$$

$$\frac{e^{\pi\sqrt{58}}-24+4372\,e^{-\pi\sqrt{58}}}{\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12}+\left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}}=\left(2048\exp\left(-\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$
$$\left.\left(4372-24\,e^{\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\exp\left(2\pi\sqrt{57}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{57}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\right)\Big/$$
$$\left(\left(625+150\sqrt{28}^{\,2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2+\sqrt{28}^{\,4}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)\right.\left(390\,625+\right.$$
$$937\,500\sqrt{28}^{\,2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2+83\,750\sqrt{28}^{\,4}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4+$$
$$\left.\left.1500\sqrt{28}^{\,6}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6+\sqrt{28}^{\,8}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{28}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8\right)\right)$$

$$\frac{e^{\pi\sqrt{58}} - 24 + 4372\, e^{-\pi\sqrt{58}}}{\left(\frac{1}{2}\left(5+\sqrt{29}\right)\right)^{12} + \left(\frac{1}{2}\left(5-\sqrt{29}\right)\right)^{12}} =$$

$$-\left(24 \middle/ \left(\frac{\left(5 - \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096} + \frac{\left(5 + \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096}\right)\right) +$$

$$\left(4372\exp\left(-\pi\exp\left(i\pi\left\lfloor\frac{\arg(58-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (58-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right) \Bigg/$$

$$\left(\frac{\left(5 - \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096} + \frac{\left(5 + \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096}\right) +$$

$$\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(58-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (58-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Bigg/$$

$$\left(\frac{\left(5 - \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096} + \frac{\left(5 + \exp\left(i\pi\left\lfloor\frac{\arg(29-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (29-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}}{4096}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

Thence, we have the following mathematical connection between the two previous formulas:

$$\left(\begin{array}{l}4\log\left(\frac{1}{8e^2}+\frac{1}{2}e^{-e^{2/3}\times(0.7903+0.767721i)-4/3}\times(0.7903+0.767721i)\right.\\ \left(3.73318\,e^{-e^{2/3}\times(0.7903+0.767721i)}\left(e^{2/3}\times(0.7903+0.767721i)+3\right)+\right.\\ \left.\left.(0.78724+3.25266i)\times(0.7903+0.767721i)\right)\right)\end{array}\right) = 63.984 \approx 64$$

$$\left(\frac{-24+4372\,e^{-\sqrt{58}\,\pi}+e^{\sqrt{58}\,\pi}}{\frac{\left(5-\sqrt{29}\right)^{12}}{4096}+\frac{\left(5+\sqrt{29}\right)^{12}}{4096}}\right) = 63.99999\ldots \approx 64$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] *The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

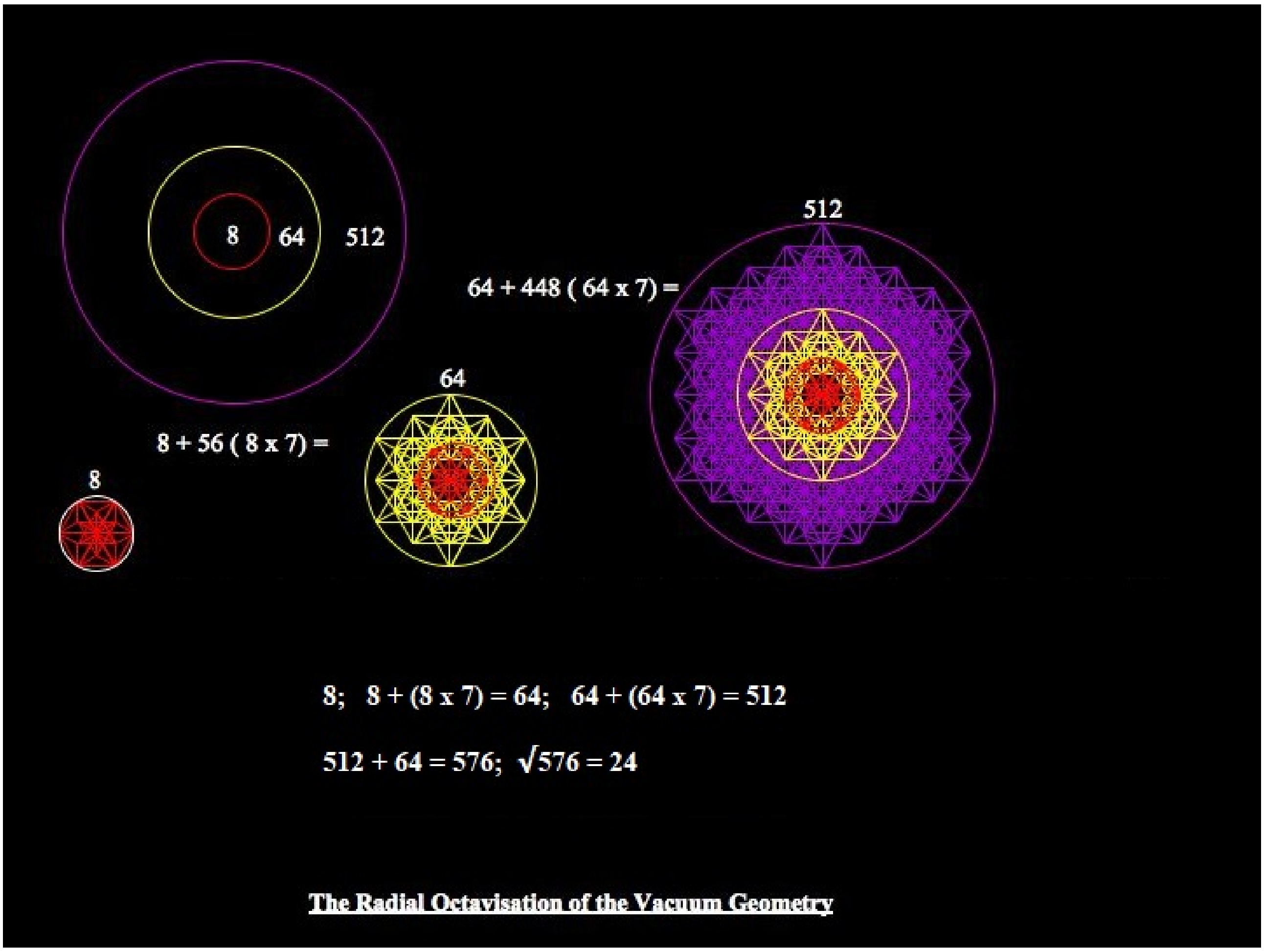
We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

Fig. 1

https://www.pinterest.it/pin/570338740293422619/



Conclusion

From what we have described above, it is possible and plausible that the vacuum geometry is strongly connected to the value of the golden ratio and that Ramanujan's mathematics, especially that described in paragraph 5 of the wonderful paper "Modular equations and approximations to π " (precisely the equation where there is 64, a fundamental number in the vacuum geometry), is strictly connected to the quantum gravity, precisely to the mathematical development of this theory.

References

On Climbing Scalars in String Theory
E. Dudas, N. Kitazawa and A. Sagnotti - arXiv:1009.0874v1 [hep-th] 4 Sep 2010

Modular equations and approximations to π – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372