Units and Reality

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Abstract.

With this paper the author shows that the transformation of the fundamental physical constants c, G, h, e and k_c into systems of units, which differ fundamentally from the International System of Units (SI), is a powerful tool to free the numeric values of the constants from their arbitrariness, caused by the historical choice of units. The arbitrariness of units certainly does not refer to all the careful definition, harmonisation and calibration of units by the international metrological community, but rather to the arbitrary scaling of the size of units in the physical sense.

By transferring the fundamental physical constants into systems of units, with extraordinary scales (e.g. Planck scale) or natural scales like the Proton´s dimensions or the value of the Hubble constant, one can show the true character of the fundamental physical constants. Such transformations uncover correlations - sought for a long time – between the important dimensionless constants, such as $137.036 = \frac{2\epsilon_0 ch}{e^2}$, $1836.15 = \frac{m_p}{m_e}$ or $2.2717 * 10^{39} = \frac{e^2}{4\pi\epsilon_0 G\, m_p\, m_e}$ on the one hand and the constants with dimensions on the other hand.

Amongst other things, the author was able to discover the following examples of fascinating numeric correlations:

$$|c| \approx \frac{(2\pi/\alpha)^4}{m_p/m_e} \ , \ m_p{}^3 \approx \frac{Hh^2}{cG} * (1836.15)^{3/5}, \ |G| \approx \frac{2\pi/\alpha}{(m_p/m_e)^4} \ , \ \left|\frac{e^2}{4\pi\epsilon_0}\right| \approx \frac{(m_p/m_e)^5}{(2\pi/\alpha)^{15}} \ .$$

All in all, the paper describes many (numeric) correlations, which could help to find new physical understanding. Apart from that promising fact, the interconnections between the various systems of units and the underlying principles are very revealing, because one should know the effects of changing the physical scales.

$$m_p/m_e = 1836.15 \approx 2\pi/\alpha$$

Investigation:

The numerical value of the natural constants depends on the units selected for the basic physical quantities. By using the SI units metre, second and kilogram to measure length, time and mass, we obtain the known numerical values for the speed of light c of $2.99792*10^8$ m/s, for the gravitational constant G of $6.6743*10^{-11}$ m^3/kgs^2 and for Planck's constant h of $6.62607*10^{-34}$ kgm^2/s.

Seen in this way, the numerical values of the natural constants are linked to the arbitrarily chosen units and thus have a certain arbitrariness. This does not mean, however, that their values do not contain valuable information about the nature and function of nature. In the past it has been shown again and again, that the numerical values of natural constants are related in interesting and previously unknown ways. For example, the speed of light c results from the electric field constant ϵ_0 and the magnetic field constant μ_0:

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Amongst other things, the discovery of this connection created the foundation for the physics of electromagnetism, which is the basis for broad areas of modern technology.

So the question arises, how far the numerical values of the natural constants can be freed from the arbitrariness caused by the choice of scale units and whether further valuable connections between the natural constants can be revealed by this? To answer this question, it seems reasonable to study the behaviour of the natural constants when the basic units are changed and thereby to decipher possible regularities. A first step in this direction can be seen in the change to the so-called Planckian system of units, which is often carried out by physicists. As is generally known, the so-called Planck units for length, time and mass can be formed with c, G and h:

the square of the Planck length $l_{pl}{}^2 = \frac{Gh}{c^3}$

the square of the Planck time $t_{pl}{}^2 = \frac{Gh}{c^5}$

the square of the Planck mass $m_{pl}{}^2 = \frac{ch}{G}$

For the Planck units the following values are then obtained: $l_{pl} = 4.05*10^{-35}$ m, $t_{pl} = 1.35*10^{-43}$ s, $m_{pl} = 5.46*10^{-8}$ kg. Conversely, for the metre, second and kilogram in Planck units the following values are obtained: $1m = 2.47*10^{34}\ l_{pl}$, $1s = 7.40*10^{42}\ t_{pl}$, $1kg = 1.83*10^{7}\ m_{pl}$.

If the constants c, G and h are transferred to Planck's system of units, the numerical values of these constants are all 1. i.e. $c_{pl} = 1\ l_{pl}/t_{pl}$, $G_{pl} = 1\ l_{pl}{}^3/m_{pl}t_{pl}{}^2$ $h_{pl} = 1\ m_{pl}l_{pl}{}^2/t_{pl}$. This is already a remarkable fact, but is more of the same possible?

To investigate this, it seems appropriate to include the electromagnetic constants in the considerations. The elementary charge e of $1.60218*10^{-19}$ As and the Coulomb constant $k_c = \frac{1}{4\pi\epsilon_0}$ of $8.98755 * 10^9 \frac{kgm^3}{A^2s^4}$. The product of e^2 and k_c, i.e. $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ stands for the electromagnetic force and has the value $2.30708 * 10^{-28} \frac{kgm^3}{s^2}$.

In Planck's system of units

$$1\frac{kgm^3}{s^2} = \frac{1.833*10^7*(2.468*10^{34})^3}{(7.400*10^{42})^2}\frac{m_{pl}*l_{pl}{}^3}{t_{pl}{}^2} = 5.0341 * 10^{24}\frac{m_{pl}*l_{pl}{}^3}{t_{pl}{}^2}$$, and therefore

$$\frac{e^2}{4\pi\epsilon_0} = 2.30708 * 10^{-28} * 5.0341 * 10^{24}\frac{m_{pl}*l_{pl}{}^3}{t_{pl}{}^2} = 1.1614 * 10^{-3}\frac{m_{pl}*l_{pl}{}^3}{t_{pl}{}^2}.$$

Since $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ is the product of e^2 and k_c, you now have the freedom to set either e=1 or k_c =1 by choosing the appropriate units. If e=1 then k_c = $1.1614*10^{-3}$ and vice versa $e=1.1614*10^{-3}$ if k_c=1. So you can see that in Planck's system of units at least one electromagnetic constant must be unequal to 1. It is remarkable that $1.1614*10^{-3}$ is the same as $\alpha/2\pi$, that is $1/(2\pi *137.036)$ or $\frac{e^2}{4\pi\epsilon_0 ch}$. $\alpha/2\pi$ therefore has the same value of $1.1614*10^{-3}$ in the SI system as in the Planckian system. This must be the case, however, because $\alpha/2\pi$ is a dimensionless quantity and should therefore be independent of the choice of unit system.

By changing to the Planckian system, it is therefore possible to "distil" the value of the fine structure constant α from the electromagnetic constants. This does not explain why α has the value of 1/137.036, but it shows how α is related to the natural constants.

However, the Planckian system is not the only system of units that can be defined by means of natural constants. With the elementary charge e, the Coulomb constant k_c and the constants c, G and h, further systems of units can be developed based on integer powers of these natural constants. For this purpose, dimensional analyses are needed to investigate how the SI units metre, second and kilogram can still be replaced by the natural constants. The result, which is shown here, gives a small excerpt of the basically infinitely many possibilities. The first line lists possibilities for the square of the unit of length, the second line lists squares for possible units of time and the third line lists squares for possible units of mass:

$$(1)\ l_x{}^2 = \frac{Ge^2}{c^4 4\pi\epsilon_0},\quad \mathbf{\frac{Gh}{c^3}},\quad \frac{Gh^2 4\pi\epsilon_0}{c^2\, e^2},\quad \frac{Gh^3(4\pi\epsilon_0)^2}{c\, e^4},\quad \frac{Gh^4(4\pi\epsilon_0)^3}{e^6},\quad \frac{Gh^5 c(4\pi\epsilon_0)^4}{e^8}$$

$$(2)\ t_y{}^2 = \frac{Ge^2}{c^6 4\pi\epsilon_0},\quad \mathbf{\frac{Gh}{c^5}},\quad \frac{Gh^2 4\pi\epsilon_0}{c^4\, e^2},\quad \frac{Gh^3(4\pi\epsilon_0)^2}{c^3\, e^4},\quad \frac{Gh^4(4\pi\epsilon_0)^3}{c^2 e^6},\quad \frac{Gh^5(4\pi\epsilon_0)^4}{ce^8}$$

$$(3)\ m_z{}^2 = \frac{e^2}{G\, 4\pi\epsilon_0},\quad \mathbf{\frac{ch}{G}},\quad \frac{c^2h^2 4\pi\epsilon_0}{G\, e^2},\quad \frac{c^3h^3(4\pi\epsilon_0)^2}{G\, e^4},\quad \frac{c^4h^4(4\pi\epsilon_0)^3}{G\, e^6},\quad \frac{c^5h^5(4\pi\epsilon_0)^4}{G\, e^8}$$

The bold options in each row represent the Planck units. The Planck's system is the only one in which no electromagnetic constants occur. It is also noteworthy that two adjacent elements differ from each other by the factor $2\pi/\alpha$. If one considers that the terms above always represent the square of a unit of length, a unit of time or a unit of mass, then between two adjacent l_x, t_y or m_z are factors of $\sqrt{2\pi/\alpha}$ = 29.343.

In the series of unit systems shown above, are there, in addition to the Planckian system, other unit systems in which as many natural constants as possible assume the value 1?

A further system of units in which all but one of the 5 natural constants under consideration take the value 1 is the following:

$$l_x^2 = \frac{Gh^4(4\pi\epsilon_0)^3}{e^6},\ t_y^2 = \frac{Gh^6(4\pi\epsilon_0)^5}{e^{10}} \text{ and } m_z^2 = \frac{e^2}{G\,4\pi\epsilon_0} \text{ with}$$

$l_x = 1.0235*10^{-30}$ m, $t_y = 2.9397*10^{-36}$ s, $m_z = 1.8593*10^{-9}$ kg.

What is striking about this system is that c does not occur in the definition of l_x, t_y and m_z. In this system, c takes the value $2\pi/\alpha$ = 861.02. G, h and $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ each take the value 1, therefore e and k_c can also be set to 1 by selecting the appropriate units. Obviously all natural constants used to define this system of units can take the value 1. Planck's system is defined without electromagnetic constants and the electromagnetic constants cannot all take the value 1.

A system of units defined without h, should therefore lead to the result that only h cannot take the value 1 and this is really the case:

$l_x^2 = \frac{Ge^2}{c^4 4\pi\epsilon_0}$, $t_y^2 = \frac{Ge^2}{c^6 4\pi\epsilon_0}$ and $m_z^2 = \frac{e^2}{G\,4\pi\epsilon_0}$

with $l_x = 1.3806*10^{-36}$ m, $t_y = 4.6053*10^{-45}$ s and $m_z = 1.8593*10^{-9}$ kg.

In this system, h takes the value $2\pi/\alpha$ = 861.02. c, h and $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ each take the value 1, therefore e and k_c can also be set to 1 by selecting the appropriate units.

As dimensional analyses show, a system of units cannot be defined without G, because with c, h and $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ only a dimensionless number like $2\pi/\alpha = \frac{e^2}{4\pi ch\epsilon_0}$ = 861.02 but no l_x, t_y or m_z can be formed. Nevertheless there is a system of units in which G takes the value $2\pi/\alpha$ = 861.02 and c and h take the value 1: $l_x^2 = \frac{Ge^2}{c^4 4\pi\epsilon_0}$, $t_y^2 = \frac{Ge^2}{c^6 4\pi\epsilon_0}$ and $m_z^2 = \frac{c^2h^2 4\pi\epsilon_0}{G\,e^2}$ with $l_x = 1.3806*10^{-36}$ m, $t_y = 4.6053*10^{-45}$ s and $m_z = 1.6009*10^{-6}$ kg. However, in this system $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ also cannot take the value 1 and then amounts to $1.1614*10^{-3}$. Since $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$ is the product of e^2 and k_c, you now have the freedom to set either e=1 or k_c=1 by choosing the appropriate units. If e=1, $k_c = 1.1614*10^{-3}$ and vice versa if $e=1.1614*10^{-3}$, k_c=1.

Basics for a variation tool:

In order to be able to test different unit systems based on integer powers of the natural constants used here, I have created an electronic variation tool. This is based on the fact that neighbouring l_x, t_y or m_z each differ by a factor a = $\sqrt{2\pi/\alpha}$ = 29.343 (see above).

l_x, t_y and m_z are therefore defined as follows using the Planck units l_{pl}, t_{pl} and m_{pl}:

$$l_x = l_{pl} * a^x \;,\; t_y = t_{pl} * a^y \text{ and } m_z = m_{pl} * a^z$$

in reverse: $l_{pl} = l_x * a^{-x}$, $t_{pl} = t_y * a^{-y}$ and $m_{pl} = m_z * a^{-z}$

for the speed of light in $\text{system}_{x,y,z}$ the following applies:

$$c_{x,y,z} = c_{pl} * a^{y-x} = 1 * a^{y-x}$$

because $c = c_{pl}\frac{l_{pl}}{t_{pl}} = 1\frac{l_{pl}}{t_{pl}} = \frac{l_x * a^{-x}}{t_y * a^{-y}} = a^{y-x}\frac{l_x}{t_y}$.

As long as x=y the speed of light keeps the value c_{pl} = 1. With x=23.4359 and y=29.2123 the known value of the speed of light is $2.99 * 10^8 = 29.343^{29.2123-23.4359}$ because $1\text{m} = 29.343^{23.4359} l_{pl} = 2.47 * 10^{34} l_{pl}$ and $1\text{s} = 29.343^{29.2123} t_{pl} = 7.40 * 10^{42} t_{pl}$.

If the ratio of proton mass to electron mass m_p/m_e = 1836.15 is represented as the power of $2\pi/\alpha$ = 861.02, the equation $1836.15 \approx (2\pi/\alpha)^{1.112} \approx 861.023^{1.112}$ results. Since 2.888 = 4 - 1.112, the ratio for the value of the speed of light in the SI system to the value in the Planckian system can be represented as follows $c/c_{pl} = 2.99 * 10^8 = 29.343^{29.2123-23.4359} = 29.343^{5.7764} = 29.343^{2*2.888} = 861.023^{2.888} = \frac{861.023^4}{861.023^{1,112}} \approx \frac{861.023^4}{1836.15} = \frac{(2\pi/\alpha)^4}{m_p/m_e}$. So for the value of the speed of light c in the SI system, the interesting relationship $\mathbf{c/c_{pl}} \approx \frac{\mathbf{(2\pi/\alpha)^4}}{\mathbf{m_p/m_e}}$ results, where c_{pl} = 1.

When considering what this could mean, one of the things to look for is what $2\pi/\alpha$ could stand for in this context. A possible interpretation would be the following: The gravitational force between two Planck masses (m_{pl}^2 = ch/G) at a distance r, $F_{pl} = \frac{ch}{r^2}$. The electromagnetic force between two elementary charges at a distance r, $F_e = \frac{e^2}{r^2 4\pi\epsilon_0}$. The ratio of these two forces $F_{pl}/F_e = \frac{4\pi\epsilon_0 ch}{e^2} = \frac{2\pi}{\alpha}$. According to this, the above context could be transformed, for example into $\mathbf{c/c_{pl}} \approx \frac{\mathbf{(F_{pl}/F_e)^4}}{\mathbf{m_p/m_e}}$.

Next we will look at how the gravitational constant behaves in the system of units with $l_x = l_{pl} * a^x$, $t_y = t_{pl} * a^y$ and $m_z = m_{pl} * a^z$ and further $l_{pl} = l_x * a^{-x}$, $t_{pl} = t_y * a^{-y}$ and $m_{pl} = m_z * a^{-z}$. For the gravitational constant in $\text{system}_{x,y,z}$ then $G_{x,y,z} = G_{pl} * a^{2y+z-3x} = 1 * a^{2y+z-3x}$ applies.

because $G = G_{pl}\frac{{l_{pl}}^3}{m_{pl} * {t_{pl}}^2} = 1\frac{{l_{pl}}^3}{m_{pl} * {t_{pl}}^2} = \frac{{l_x}^3 * a^{-3x}}{m_z * a^{-z} * {t_y}^2 * a^{-2y}} = a^{2y+z-3x} * \frac{{l_x}^3}{m_z * {t_y}^2}$.

As long as x=y=z the gravitational constant keeps the value G_{pl} = 1. With x=23.4359, y=29.2123 and z= 4.9493, the known value of the gravitational constant of $6.674 * 10^{-11} = 29.343^{2*29.2123+4.9493-3*23.4359}$ results, because

$1m = 29.343^{23.4359} l_{pl} = 2.47 * 10^{34} l_{pl}$, $1s = 29.343^{29.2123} t_{pl} = 7.40 * 10^{42} t_{pl}$ and $1kg = 29.343^{4.9493} m_{pl} = 1.83 * 10^{7} m_{pl}$.

In the same way we want to look at how Planck's quantum of action behaves in the system of units with $l_x = l_{pl} * a^x$, $t_y = t_{pl} * a^y$ and $m_z = m_{pl} * a^z$ and further $l_{pl} = l_x * a^{-x}$, $t_{pl} = t_y * a^{-y}$ and $m_{pl} = m_z * a^{-z}$. For the Planckian quantum of action in $\text{system}_{x,y,z}$ the following applies:

$$h_{x,y,z} = h_{pl} * a^{y-2x-z} = 1 * a^{y-2x-z}$$

because $$h = h_{pl} \frac{m_{pl} * l_{pl}^2}{t_{pl}} = 1 \frac{m_{pl} * l_{pl}^2}{t_{pl}} = \frac{m_z * a^{-z} * l_x^2 * a^{-2x}}{t_y * a^{-y}} = a^{y-2x-z} * \frac{m_z * l_x^2}{t_y} .$$

As long as x=y=-z the Planckian quantum of action keeps the value h_{pl} = 1. With x=23.4359, y=29.2123 and z= 4.9493, the known value of Planck's constant is $6.626 * 10^{-34} = 29.343^{29.2123-2*23.4359-4.9493}$, because

$1m = 29.343^{23.4359} l_{pl} = 2.47 * 10^{34} l_{pl}$, $1s = 29.343^{29.2123} t_{pl} = 7.40 * 10^{42} t_{pl}$ and $1kg = 29.343^{4.9493} l_{pl} = 1.83 * 10^{7} m_{pl}$.

Systems of units and their effect on natural constants:

As shown above, the natural constants c, G and h can be represented with a= $\sqrt{2\pi/\alpha}$ = 29.343 as well as $l_x = l_{pl} * a^x$, $t_y = t_{pl} * a^y$ and $m_z = m_{pl} * a^z$:

$$c_{x,y,z} = c_{pl} * a^{y-x}$$

$$G_{x,y,z} = G_{pl} * a^{2y+z-3x}$$

$$h_{x,y,z} = h_{pl} * a^{y-2x-z}$$

where c_{pl}, G_{pl} and h_{pl} each have the value 1.

If one inserts for x=3k, for y=5k and for z=-k into the equations for c, G and h, where k can be any real number, then it can be seen that G and h still retain the value 1. Only the value for c changes depending on k. We have already considered above what the result is when k=1:

$l_x^2 = \frac{Gh^4(4\pi\epsilon_0)^3}{e^6}$, $t_y^2 = \frac{Gh^6(4\pi\epsilon_0)^5}{e^{10}}$ and $m_z^2 = \frac{e^2}{G\, 4\pi\epsilon_0}$ with l_x = 1.0235*10^{-30} m, t_y = 2.9397*10^{-36} s, m_z = 1.8593*10^{-9} kg. In this system, c takes the value 2π/α = 861.02. G, h and $e^2 k_c = \frac{e^2}{4\pi\epsilon_0}$ each take the value 1.

If we insert x=y=z into the equations for c, G and h, we can see that c and G still retain the value 1. Only the value for h changes depending on x, y and z.

The same is the case if we take a mass $m_z = m_{pl} * a^z$, its (half) Schwarzschild radius $R_{sch} = \frac{Gm_z}{c^2}$ and the time $t_y = \frac{R_{sch}}{c}$ as units of measurement. From $\frac{R_{sch}{}^2}{l_{pl}{}^2} = \frac{G^2 m_z{}^2 c^3}{c^4 Gh} = \frac{Gm_{pl}{}^2 * a^{2z}}{ch} = a^{2z}$ it follows that $R_{sch} = l_x = l_{pl} * a^z$. Furthermore $t_y = \frac{R_{sch}}{c} = \frac{l_{pl} * a^z}{c} = t_{pl} * a^z$. Thus l_x, t_y and m_z all have the same exponent for a, which corresponds to the condition x=y=z. Thus, if we take black holes as a system of measurement, h becomes smaller the larger the "black hole scale" is. c and G retain the value 1 regardless of the size of the black hole.

If we insert x=y=-z into the equations for c, G and h, we can see that c and h still retain the value 1. Only the value for G changes depending on x, y and z.

The same is the case if we take a mass $m_z = m_{pl} * a^z$, its Compton wavelength $\lambda = \frac{h}{c\, m_z}$ and the time $t_y = \frac{\lambda}{c}$ as units of measurement. From $\lambda^2 = \frac{h^2}{c^2 m_z{}^2} = \frac{h^2}{c^2 m_{pl}{}^2 * a^{2z}} = \frac{Gh^2}{c^2 ch * a^{2z}} = \frac{Gh}{c^3 * a^{2z}} = l_{pl}{}^2 * a^{-2z}$ it follows that $\lambda = l_x = l_{pl} * a^{-z}$. Furthermore $t_y = \frac{\lambda}{c} = \frac{l_{pl} * a^{-z}}{c} = t_{pl} * a^{-z}$. Thus l_x, t_y have the exponent -z and m_z has the exponent z which corresponds to the condition x=y=-z. Thus, if we take Compton waves as a system of measurement, G becomes smaller the larger the scale "Compton wave" is. c and h keep the value 1 regardless of the size of the Compton wave.

By choosing a suitable system of units it is therefore possible to study how the change of one or more basic unit(s) affects the values of the natural constants and which systems of units are characterised by very special properties.

In the light of what has been studied so far, the commonly used SI units seem relatively arbitrary, but are they really? One metre is of the same size scale as our body dimensions and not a power of ten smaller or larger. One second lasts about as long as a heartbeat or a leisurely movement of our limbs and not a power of ten shorter or longer. We can consume one kilogram in a sumptuous meal through food and drink or sweat it out in a mediocre sports unit, but not a power of ten more. One power of ten less would correspond to a very meagre meal or a short exercise unit without endurance effect.

In this respect, the familiar SI units of metre, second and kilogram correspond to our everyday standards. But they give us the very "inelegant" values for the natural constants from $2.99792*10^8$ m/s for the speed of light to $6.62607*10^{-34}$ kgm^2/s for the Planckian quantum of action.

Therefore, it is helpful to look at how the values of the natural constants change when measured at other scales. If the natural constants are measured on the scale of a proton (proton mass, Compton wavelength of the proton, time scale = Compton wavelength of the proton/velocity of light), i.e.

$l_x = h/cm_z = 1.3228*10^{-15}$ m, $t_y = l_x/c = 4.4123*10^{-24}$ s, $m_z = 1.6709*10^{-27}$ kg and $1m = 1/l_x = 7.5599*10^{14}\ l_x$, $1s = 1/t_y = 2.2664*10^{23}\ t_y$, $1kg = 1/m_z = 5.9848*10^{26}\ m_z$ the gravitational constant is given by the value

$$G_p = G * 1\frac{m^3}{kg*s^2} = 6.6738*10^{-11}*\frac{(7.5599*10^{14})^3}{5.9848*10^{26}*(2.2664*10^{23})^2} = 9.3799*10^{-40}\ \frac{{l_x}^3}{m_z*{t_y}^2}\ .$$

The speed of light attains the value 1: $c_p = c*1\frac{m}{s} = 2.9979*10^8*\frac{7.5599*10^{14}}{2.2664*10^{23}} = 1\ \frac{l_x}{t_y}$.

And the Planck's quantum of action, as expected, also attains the value 1:

$$h_p = h*1\frac{kg*m^2}{s} = 6.6261*10^{-34}*\frac{5.9848*10^{26}*(7.5599*10^{14})^2}{2.2664*10^{23}} = 1\frac{m_z{l_x}^2}{t_y}$$

If the ratio of proton to electron mass $m_p/m_e = 1836.15$ is divided by $2\pi/\alpha = 861.02$, the result is 2.13. Likewise 2.13 is obtained by multiplying the ratio of electromagnetic force to gravitational force by the ratio of the gravitational constant on the proton scale to that in the Planck scale.

$$\frac{F_e}{F_g}*\frac{G_p}{G_{pl}} = \frac{e^2}{4\pi\epsilon_0 Gm_pm_e}*\frac{G_p}{G_{pl}} = 2.2712*10^{39}*9.3799*10^{-40} = 2.13$$

According to this $\frac{m_p}{m_e} = 1836.15 \approx \frac{2\pi}{\alpha}*\frac{e^2}{4\pi\epsilon_0 Gm_pm_e}*\frac{G_p}{G_{pl}} = \frac{4\pi\epsilon_0 ch}{e^2}*\frac{e^2}{4\pi\epsilon_0 Gm_pm_e}*\frac{G_p}{G_{pl}} = \frac{ch}{Gm_pm_e}*\frac{G_p}{G_{pl}}$ where $G_{pl} = 1$.

The result $\mathbf{\frac{m_p}{m_e} = 1836.15 \approx \frac{ch}{Gm_pm_e}*\frac{G_p}{G_{pl}}}$ is a remarkable correlation, in which the term $\mathbf{\frac{ch}{Gm_pm_e}}$ corresponds to the ratio of the gravitational force between two Planck masses to the gravitational force between a proton and an electron. Slightly modified, the formula turns into $\mathbf{\frac{G{m_p}^2}{ch} \approx \frac{G_p}{G_{pl}}}$, where the term $\frac{G{m_p}^2}{ch}$ corresponds to the square of the proton mass divided by the square of the Planck mass.

If one measures the natural constants at a black hole with radius corresponding to the Compton wavelength of the proton (length scale = Compton wavelength of the proton, time scale = Compton wavelength of the proton/velocity of light, mass of a black hole with radius Compton wavelength), thus $l_x = h/cm_p = 1.3228*10^{-15}$ m, $t_y = l_x/c = 4.4123*10^{-24}$ s, $m_z = rc^2/G = ch/Gm_p = 1.7814*10^{12}$ kg and $1m = 1/l_x = 7.5599*10^{14}\ l_x$, $1s = 1/t_y = 2.2664*10^{23}\ t_y$, $1kg = 1/m_z = 5.6137*10^{-13}\ m_z$ then the Planckian quantum of action is given by

$$h_{pb} = h*1\frac{kg*m^2}{s} = 6.6261*10^{-34}*\frac{5.6137*10^{-13}*(7.5599*10^{14})^2}{2.2664*10^{23}} = 9.3799*10^{-40}\frac{m_z{l_x}^2}{t_y}\ .$$

The speed of light attains the value 1: $c_{pb} = c*1\frac{m}{s} = 2.9979*10^8*\frac{7.5599*10^{14}}{2.2664*10^{23}} = 1\ \frac{l_x}{t_y}$.

And the gravitational constant according to the above, also attains the value 1:
$$G_{pb} = G * 1\frac{m^3}{kg*s^2} = 6.6738 * 10^{-11} * \frac{(7.5599*10^{14})^3}{5.6137*10^{-13}*(2.2664*10^{23})^2} = 1\ \frac{{l_x}^3}{m_z*{t_y}^2}\ .$$

The proton mass and the mass of a proton-sized black hole are therefore symmetrical in terms of G and h:

$|G_p| = |h_{pb}| = 9.3799*10^{-40}$.

If the natural constants are measured with cosmological scales (radius of the visible universe, time scale = radius of the visible universe / speed of light ≈ age of the universe, mass of a particle with Compton wavelength corresponding to the radius of the visible universe), i.e. $l_x = c/H = R = 1.2867*10^{26}$ m (if the Hubble constant $H = 2.33*10^{-18}\ s^{-1}$), $t_y = 1/H = R/c = 4.2918*10^{17}$ s, $m_z = Hh/c^2 = h/cR = 1.7178*10^{-68}$ kg and $1m = 1/l_x = 7.7721*10^{-27}\ l_x$, $1s = 1/t_y = 2.3300*10^{-18}\ t_y$, $1kg = 1/m_z = 5.8214*10^{67}\ m_z$ the gravitational constant is given by the value:

$$G_c = G * 1\frac{m^3}{kg*s^2} = 6.6738 * 10^{-11} * \frac{(7.7721*10^{-27})^3}{5.8214*10^{67}*(2.3300*10^{-18})^2} = 9.9139 * 10^{-122}\ \frac{{l_x}^3}{m_z*{t_y}^2}\ .$$

The speed of light attains the value 1: $c_c = c * 1\frac{m}{s} = 2.9979 * 10^8 * \frac{7.7721*10^{-27}}{2.3300*10^{-18}} = 1\ \frac{l_x}{t_y}$.

And the Planckian quantum of action, as expected, also attains the value 1:

$$h_c = h * 1\frac{kg*m^2}{s} = 6.6261 * 10^{-34} * \frac{5.8214*10^{67}*(7.7721*10^{-27})^2}{2.3300*10^{-18}} = 1\frac{m_z{l_x}^2}{t_y}$$

Another possibility arises when using the following scales of cosmological dimension: radius of the visible universe, time scale = radius of the visible universe / speed of light ≈ age of the universe, mass of a black hole with the radius of the visible universe.

Then $l_x = c/H = R = 1.2867*10^{26}$ m (if the Hubble constant $H = 2.33*10^{-18}\ s^{-1}$), $t_y = 1/H = R/c = 4.2918*10^{17}$ s, $m_z = c^3/HG = Rc^2/G = 1.7327*10^{53}$ kg and $1m = 1/l_x = 7.7721*10^{-27}\ l_x$, $1s = 1/t_y = 2.3300*10^{-18}\ t_y$, $1kg = 1/m_z = 5.7713*10^{-54}\ m_z$. This results in the following value for Planck's quantum of action:

$$h_{cb} = h * 1\frac{kg*m^2}{s} = 6.6261 * 10^{-34} * \frac{5.7713*10^{-54}*(7.7721*10^{-27})^2}{2.3300*10^{-18}} = 9.9139 * 10^{-122}\frac{m_z{l_x}^2}{t_y}\ .$$

The speed of light attains the value 1: $c_{cb} = c * 1\frac{m}{s} = 2.9979 * 10^8 * \frac{7.7721*10^{-27}}{2.3300*10^{-18}} = 1\ \frac{l_x}{t_y}$.

The gravitational constant in this system, as expected, also attains the value 1:

$$G_{cb} = G * 1\frac{m^3}{kg*s^2} = 6.6738 * 10^{-11} * \frac{(7.7721*10^{-27})^3}{5.7713*10^{-54}*(2.3300*10^{-18})^2} = 1\ \frac{{l_x}^3}{m_z*{t_y}^2}\ .$$

Amazing symmetries are revealed here between the unit systems "cosmic Compton wave" and "cosmic black hole", since the magnitude of the gravitational constant in the system "cosmic Compton wave" is equal to the magnitude of

Planck's quantum of action in the system "cosmic black hole": $|G_c| = |h_{cb}| = 9.9139*10^{-122}$. The values of the natural constants are no longer arbitrary here, but seem to follow previously undiscovered laws.

With the constants c, h, G and H = c/R a dimensionless number n can be formed: $n = \frac{c^5}{H^2Gh} = \frac{R^2c^3}{Gh} = 1.0087 * 10^{121} = \frac{{R_c}^2{c_c}^3}{G_ch_c} = \frac{1}{|G_c|} = \frac{1}{9.9139*10^{-122}} = \frac{1}{|h_{cb}|} = \frac{{R_{cb}}^2{c_{cb}}^3}{G_{cb}h_{cb}}$.

R in the unit systems "cosmic Compton wave" with index $_c$ and "cosmic black hole" with index $_{cb}$ each has the value 1: $R_c = R_{cb} = 1$. Since c_c, c_{cb}, h_c and G_{cb} also have the value 1 as calculated above, the net result is the equation $n = \frac{R^2c^3}{Gh} = \frac{1}{G_c} = \frac{1}{h_{cb}}$.

n can be considered as the total information content of the visible universe in bits (see [1]). Since G in the Planck's system has the value G_{pl} =1, the previous equation can also be converted to the form $n = \frac{R^2c^3}{Gh} = \frac{G_{pl}}{G_c}$. With the above derived relationship $\frac{G{m_p}^2}{ch} = \frac{G_p}{G_{pl}}$ or $G_{pl} = \frac{chG_p}{G{m_p}^2}$ the following results $\frac{R^2c^3}{Gh} = \frac{chG_p}{G{m_p}^2G_c}$ =>

$${m_p}^2 = \frac{h^2G_p}{R^2c^2G_c} = \frac{H^2h^2G_p}{c^4G_c} \Rightarrow \mathbf{m_p = \frac{h}{cR} * \sqrt{\frac{G_p}{G_c}} = \frac{Hh}{c^2} * \sqrt{\frac{G_p}{G_c}}} \ .$$

The term $\frac{h}{cR}$ or $\frac{Hh}{c^2}$ represents the mass of a particle with Compton wavelength corresponding to the radius of the visible universe or the unit of mass in the unit system "cosmic Compton wave", which is why we can consequently also call this term m_c. The previous equation can then be transformed into $\mathbf{\frac{G_p}{G_c} = \frac{{m_p}^2}{{m_c}^2}}$ => $\frac{9.3799*10^{-40}}{9.9139*10^{-122}} = \frac{(1.6709*10^{-27})^2}{(1.7178*10^{-68})^2} \Rightarrow 9.4614 * 10^{81} = 9.4614 * 10^{81}$.

The gravitational constant in the unit system proton to the gravitational constant in the unit system "cosmic Compton wave" behaves analogously to the square of the mass unit in the unit system proton to the square of the mass unit in the unit system "cosmic Compton wave".

The above equation can be extended further to

$$\mathbf{\frac{h_{pb}}{h_{cb}} = \frac{G_p}{G_c} = \frac{{m_p}^2}{{m_c}^2} = \frac{{m_{cb}}^2}{{m_{pb}}^2} = \frac{R^2}{{\lambda_p}^2} = \frac{c^4{m_p}^2}{H^2h^2}} \Rightarrow$$

$$\frac{9.3799*10^{-40}}{9.9139*10^{-122}} = \frac{(1.6709*10^{-27})^2}{(1.7178*10^{-68})^2} = \frac{(1.7327*10^{53})^2}{(1.7814*10^{12})^2} = \frac{(1.2867*10^{26})^2}{(1.3228*10^{-15})^2} = \frac{(2.9979*10^8)^4*(1.6709*10^{-27})^2}{(2.3300*10^{-18}*6.6261*10^{-34})^2} =$$

$9.4614 * 10^{81}$,

with m_p = proton mass, $m_c = h/cR = Hh/c^2$, $m_{cb} = Rc^2/G$, $\lambda_p = h/cm_p$ and

$m_{pb} = \lambda_pc^2/G = ch/Gm_p$.

Systems of units that only affect the speed of light:

If we insert x=3k, y=5k and z=-k into the equations for c, G and h as shown above

$$c_{x,y,z} = c_{pl} * a^{y-x} = 1 * a^{2k} = a^{2k}$$

$$G_{x,y,z} = G_{pl} * a^{2y+z-3x} = 1 * a^{0k} = 1$$

$$h_{x,y,z} = h_{pl} * a^{y-2x-z} = 1 * a^{0k} = 1$$

where a= $\sqrt{2\pi/\alpha}$ = 29.343 and k can be any real number, then it can be seen that G and h still retain the value 1. Only the value for c changes depending on k. The corresponding scale units for this are:

$l_x = l_{pl} * a^{3k}$, $t_y = t_{pl} * a^{5k}$ and $m_z = m_{pl} * a^{-k}$.

If we use for l_x = c/H = R = 1.2867*10^{26} m i.e. the radius of the visible universe (if the Hubble constant H = 2.33*10^{-18} s^{-1}), then k = 13.7426, because

$l_x = 4.0512 * 10^{-35} * 29.343^{3*13.7426} = 1.2867 * 10^{26}\text{m}$. For the time scale this results in $t_y = 1.3513 * 10^{-43} * 29.343^{5*13.7426} = 9.2731 * 10^{57}\text{s}$ and for the mass unit

$m_z = 5.4557 * 10^{-8} * 29.343^{-13.7426} = 3.7116 * 10^{-28}\text{kg}$. The speed of light becomes:

$$c_R = c_{k=13.7426} = a^{2k} = 29.343^{2*13.7426} = 2.1607 * 10^{40} \frac{l_x}{t_y} .$$

If we use l_x = λ_p = h/cm_p = 1.3228*10^{-15} m, i.e. the Compton wavelength of the proton, then k = 4.4324, because $l_x = 4.0512 * 10^{-35} * 29.343^{3*4.4324} = 1.3228 * 10^{-15}\text{m}$. For the time scale this results in $t_y = 1.3513 * 10^{-43} * 29.343^{5*4.4324} = 4.5074 * 10^{-11}\text{s}$ and for the mass unit $m_z = 5.4557 * 10^{-8} * 29.343^{-4.4324} = 1.7069 * 10^{-14}\text{kg}$. The speed of light becomes:

$$c_{\lambda p} = c_{k=4.4324} = a^{2k} = 29.343^{2*4.4324} = 1.0216 * 10^{13} \frac{l_x}{t_y}.$$

The third power of the ratio of $c_R/c_{\lambda p}$ gives the number 9.4614*10^{81}, known from above:

$$\frac{c_R{}^3}{c_{\lambda p}{}^3} = \frac{(2.1607*10^{40})^3}{(1.0216*10^{13})^3} = (2.1150 * 10^{27})^3 = 9.4614 * 10^{81} = \frac{h_{pb}}{h_{cb}} = \frac{G_p}{G_c} => \frac{\mathbf{c_R}^3}{\mathbf{c_{\lambda p}}^3} = \frac{\mathbf{h_{pb}}}{\mathbf{h_{cb}}} = \frac{\mathbf{G_p}}{\mathbf{G_c}} .$$

If we use m_z = m_p = 1.6709*10^{-27} kg, i.e. the proton mass, then k = 13.2973, because $m_z = 5.4557 * 10^{-8} * 29.343^{-13.2973} = 1.6709 * 10^{-27}\text{kg}$. For the linear scale this results in $l_x = 4.0512 * 10^{-35} * 29.343^{3*13.2973} = 1.4102 * 10^{24}\text{m}$ and for the time scale in $t_y = 1.3513 * 10^{-43} * 29.343^{5*13.2973} = 5.0149 * 10^{54} s$. The speed of light becomes: $c_p = c_{k=13.2973} = a^{2k} = 29.343^{2*13.2973} = 1.0661 * 10^{39} \frac{l_x}{t_y}$

If we use $m_z = Hh/c^2 = h/cR = 1.7178*10^{-68}$ kg, i.e. the mass of a particle with Compton wavelength corresponding to the radius of the visible universe (if the Hubble constant $H = 2.33*10^{-18} s^{-1}$), then k = 41.2277, because

$m_z = 5.4557 * 10^{-8} * 29.343^{-41.2277} = 1.7178 * 10^{-68} kg$. For the linear scale this results in $l_x = 4.0512 * 10^{-35} * 29.343^{3*41,2277} = 1.2978 * 10^{147} m$ and for the time scale in $t_y = 1.3513 * 10^{-43} * 29.343^{5*41,2277} = 4.3667 * 10^{259} s$. The speed of light becomes: $c_c = c_{k=41.2277} = a^{2k} = 29.343^{2*41.2277} = 1.0087 * 10^{121} \frac{l_x}{t_y}$

The ratio of c_c/c_p gives the number $9.4614*10^{81}$, known from above:

$$\frac{c_c}{c_p} = \frac{1.0087*10^{121}}{1.0661*10^{39}} = 9.4614 * 10^{81} = \frac{h_{pb}}{h_{cb}} = \frac{G_p}{G_c} = \frac{c_R{}^3}{c_{\lambda p}{}^3} = \frac{m_p{}^2}{m_c{}^2}$$

with m_p = proton mass and $m_c = h/cR = Hh/c^2$.

Systems of Units and the Conjecture of Paul Dirac:

According to an assumption made by Paul Dirac, there is a numerical relationship between the sizes of the proton and the universe of approximately the following kind [2]:

$$\frac{\mathbf{m_{cb}}}{\mathbf{m_p}} \approx \frac{\mathbf{R^2}}{\mathbf{\lambda_p}^2} => \frac{1.7327*10^{53}}{1.6709*10^{-27}} \approx\approx \frac{(1.2867*10^{26})^2}{(1.3228*10^{-15})^2} => 1.0370 * 10^{80} \approx\approx 9.4614 * 10^{81} => 1 \approx\approx 91.24,$$

where in the context of the unit systems under consideration, the quantities and abbreviations used here mean the following: m_p = proton mass, $m_{cb} = Rc^2/G$, R = c/H = radius of the visible universe (when the Hubble constant $H = 2.33*10^{-18} s^{-1}$) and $\lambda_p = h/cm_p$.

As you can see, depending on which values are used for the dimension and total mass of the visible universe and for the size of the proton (here the Compton wavelength λ_p instead of the proton radius $r_p = 0.877*10^{-15}$ m measured in the scattering experiment), there is a greater or lesser inaccuracy in the conjecture. With the quantities used here, there is a factor of 91.24 between the left and right side of the assumption. With the measured proton radius it would be even greater.

In addition to the number $9.4614*10^{81}$, which we have already noted several times, the numbers $1.0370*10^{80}$ and 91.24 also appear as ratios between the various systems of units. For example, the ratio of the mass of a black hole with a radius corresponding to the Compton wavelength of the proton to the mass of a particle with the Compton wavelength corresponding to the radius of the visible universe is equal to 1.0370*10^{80} : $\frac{m_{pb}}{m_c} = \frac{1.7814*10^{12}}{1.7178*10^{-68}} = 1.0370*10^{80}$, so for the proton mass it follows that

$$\mathbf{m_p} = \frac{\mathbf{m_c * m_{cb}}}{\mathbf{m_{pb}}} \text{ or } m_p * m_{pb} = m_c * m_{cb} = {m_{pl}}^2.$$

A black hole mass of the size of the visible universe can therefore contain as many proton masses as a proton-sized black hole can contain masses of the Compton wavelength corresponding to the size of the visible universe. This reminds of the analogy between solar systems and atoms, although this analogy, like there, applies only with essential limitations. The above formulas also show that the mass of a Compton wave multiplied by the mass of a black hole of the same size gives a constant value that is equal to the square of the Planck mass. The correlation applies to all possible size scales, not only cosmic or proton-like ones, and is valid because the mass of a Planck-length Compton wave has the same mass as a Planck-length black hole.

An example where the number 91.24 and its powers appear is the ratio of the two systems of units for calculating c_R and c_p. As we have seen above, the basic units of the c_R-system are the radius of the visible universe l_x = l_R = c/H = R = 1.2867*10^{26} m, t_y = t_R = 9.2731*10^{57} s and m_z = m_R = 3.7116*10^{-28} kg. Accordingly 1m_R = 7.7718*10^{-27} l_R, 1s_R = 1.0784*10^{-58} t_R, 1kg_R = 2.6943*10^{27} m_R and c_R = 2.1607*10^{40} l_R/t_R.

As we have also considered above, the basic units of the c_p-system are the proton mass m_z = m_p = 1.6709*10^{-27} kg, l_x = l_p = 1.4102*10^{24} m and t_y = t_p = 5.0149*10^{54} s. Accordingly 1m_p = 7.0912*10^{-25} l_p, 1s_p = 1.9941*10^{-55} t_p, 1kg_p = 5.9848*10^{26} m_p and c_p = 1.0661*10^{39} l_p/t_p.

The ratio of $\frac{l_R}{l_p} = \frac{1.2867*10^{26}}{1.4102*10^{24}} = 91.24$. Since in these systems the scale units must behave as follows: $l_x = l_{pl} * a^{3k}$, $t_y = t_{pl} * a^{5k}$ and $m_z = m_{pl} * a^{-k}$, it follows that

$$\frac{l_R}{l_p} = \frac{a^{3kR}}{a^{3kp}} = 29.343^{3*(kR-kp)} \text{ , } \frac{t_R}{t_p} = \frac{a^{5kR}}{a^{5kp}} = 29.343^{5*(kR-kp)} \text{ , } \frac{m_R}{m_p} = \frac{a^{-kR}}{a^{-kp}} = 29.343^{-(kR-kp)}.$$

If $\frac{l_R}{l_p} = 91.24$, then $\frac{t_R}{t_p} = 91.24^{5/3} = 1849.11$ and $\frac{m_R}{m_p} = 91.24^{-1/3} = 0.2221$, which must be checked:

$$\frac{t_R}{t_p} = \frac{9.2731*10^{57}}{5.0149*10^{54}} = 1849.11 \text{ and } \frac{m_R}{m_p} = \frac{3.7116*10^{-28}}{1.6709*10^{-27}} = 0.2221 = 1849.11^{-1/5} = 91.24^{-1/3} .$$

Taking into account $c_{x,y,z} = c_{pl} * a^{2k}$ it follows that $\frac{c_R}{c_p} = \frac{a^{2kR}}{a^{2kp}} = 29.343^{2*(kR-kp)}$. If $\frac{l_R}{l_p} = 29.343^{3*(kR-kp)} = 91.24$, it follows that $\frac{c_R}{c_p} = 91.24^{2/3} = 20.27 = 1849.11^{2/5}$.

On trial: $\frac{c_R}{c_p} = \frac{2.1607*10^{40}}{1.0661*10^{39}} = 20.27$.

The "electrifying" thing about this is that $1849.11 \approx \frac{m_p}{m_e} = 1836.15$. This applies with an error of only 0.7%. This could be more than coincidental and provides reason for the assumption that $\frac{c_R{}^5}{c_p{}^5} \approx \frac{m_p{}^2}{m_e{}^2}$ or $\frac{m_p{}^5}{m_R{}^5} \approx \frac{m_p}{m_e}$ applies. If one considers that $m_R{}^3 = \frac{h^2}{GR} = \frac{Hh^2}{Gc} = \frac{(6.62607*10^{-34})^2}{6.6743*10^{-11}*1.2867*10^{26}} = (3.7116 * 10^{-28})^3$, then the above assumption becomes $m_p{}^{12} \approx \frac{h^{10}}{G^5R^5m_e{}^3}$ or $m_p{}^{15} \approx \frac{h^{10}}{G^5R^5} * \frac{m_p{}^3}{m_e{}^3} = \frac{h^{10}}{G^5R^5} * 1836.15^3$ or

$$\mathbf{m_p{}^3 \approx \frac{h^2}{GR} * 1836.15^{3/5}} \ .$$

The value for m_p from this relationship is $1.6686*10^{-27}$ kg. This is a deviation of only 0.14 % from the measured value (if the Hubble constant H = c/R = $2.33*10^{18}$ s^{-1}). Thus we have a numerically good relationship between the number 1836.15 and the proton mass. The assumption $m_p{}^3 \approx \frac{h^2}{GR}$ (without the factor $1836.15^{3/5}$) was already made by Carl Friedrich von Weizsäcker as part of his so-called "Urhypothese" [theory of ur-alternatives] (see [3]).

What is still missing would be a good relationship between the number α = 1/137.036 and the proton mass.

This is obtained by using the formula $m_x{}^3 = \frac{e^2Hh}{4\pi\epsilon_0 c^2 G} = \frac{e^2h}{4\pi\epsilon_0 cGR}$, which I discovered in 2014 (see [4]). In this formula, m_x is an abbreviation for m_p*m_e in the following way: $m_x{}^2 = m_p * m_e$. In full length the formula is therefore:

$$m_p{}^3 * m_e{}^3 = \left(\frac{e^2h}{4\pi\epsilon_0 cGR}\right)^2 = \left(\frac{\alpha}{2\pi}\right)^2 * \left(\frac{h^2}{GR}\right)^2.$$

Combining the last found relationship for m_p with this formula, one obtains a relationship between α = 1/137.036 and m_p in the form $\left(\frac{\alpha}{2\pi}\right)^{2/3} \approx \frac{h^2}{GRm_p{}^3}$ or

$$\mathbf{m_p{}^3 \approx \left(\frac{2\pi}{\alpha}\right)^{2/3} * \frac{h^2}{GR}} \ .$$

The value for m_p from this relationship is $1.6664*10^{-27}$ kg. This is a deviation of only 0.27% from the measured value (when the Hubble constant H = c/R = $2.33*10^{-18}$ s^{-1}). Thus we also have a numerically good relationship between the number α = 1/137.036 and the proton mass. The formula I found in 2014 is more accurate if one assumes H = c/R = $2.33*10^{-18}$ s^{-1} for the Hubble constant. However, it includes both the proton and electron masses, whilst the two new relationships include only the proton mass and could therefore possibly provide more information about the proton alone. Comparing the two assumptions, the following must apply: $\left(\frac{m_p}{m_e}\right)^{3/5} \approx \left(\frac{2\pi}{\alpha}\right)^{2/3}$ or $1836.15^{9/10} \approx 861.02 \ => \ 866.00 \approx 861.02$. The inaccuracy here is 0.58%.

If one combines the relationship found above $c/c_{pl} \approx \frac{(2\pi/\alpha)^4}{m_p/m_e}$ with the approximation between 1836.15 and $2\pi/\alpha$ mentioned here, the result is: $c/c_{pl} \approx \left(\frac{2\pi}{\alpha}\right)^{26/9}$ or $c/c_{pl} \approx \left(\frac{m_p}{m_e}\right)^{13/5}$, whereby the former deviates from the measured value of the speed of light by 0.49% and the latter by 2.17%. On the other hand, the original relationship $c/c_{pl} \approx \frac{(2\pi/\alpha)^4}{m_p/m_e}$ deviates from the measured value by only 0.15%, which is, in addition to its integer exponents, a comparatively important argument for it.

Let's return to Dirac's conjecture in the form used here:

$\frac{m_{cb}}{m_p} \approx \frac{R^2}{{\lambda_p}^2}$ $=>1 \approx\approx 91.24$ and insert the knowledge gained above i.e. the assumption of Weizsäcker

$\frac{{m_p}^3}{{m_R}^3} = \frac{(1.6709*10^{-27})^3}{(3.7116*10^{-28})^3} = \frac{{m_p}^3 GR}{h^2} = \frac{(1.6709*10^{-27})^3*6.6743*10^{-11}*1.2867*10^{26}}{(6.62607*10^{-34})^2} = \frac{4.0061*10^{-65}}{4.3905*10^{-67}} = 91.24$ into Dirac's conjecture, then it becomes an equation, which happily resolves as:

$$\frac{m_{cb}}{m_p} * \frac{{m_p}^3 GR}{h^2} = \frac{R^2}{{\lambda_p}^2} \quad => \frac{Rc^2}{Gm_p} * \frac{{m_p}^3 GR}{h^2} = \frac{R^2 c^2 {m_p}^2}{h^2} \quad => \frac{R^2 c^2 {m_p}^2}{h^2} = \frac{R^2 c^2 {m_p}^2}{h^2} \quad => 1 = 1$$

Discussion of the results:

As we have seen, the transfer of the natural constants c, G, h, e and k_c into unit systems with basic units for length, time and mass, which are fundamentally different from the SI system, is a powerful tool to free the numerical values of the natural constants from the physical arbitrariness inherent in the SI system. Arbitrariness in this context certainly does not refer to the careful definition, coordination and calibration by the international metrological community, but rather to the arbitrary scaling of the units of measurement in the physical sense in comparison to the scales occurring in nature and having the same nature throughout the cosmos, such as the proton.

If one refers the natural constants to systems of units, which in the theory-building of physics are characterised particularly by their basic units compared to others (Planck units etc.) or to natural scales, such as the proton or the Hubble constant, and compares their effects with each other, the real nature of the natural constants becomes apparent and their relationship to each other distils itself from the sometimes confusing nebula of numbers.

Suddenly, long sought-after correlations appear between the dimensionless constants that are important in physics, such as $137.036 = \frac{2\epsilon_0 ch}{e^2}$, $1836.15 = \frac{m_p}{m_e}$ or

$2.2717 * 10^{39} = \frac{e^2}{4\pi\epsilon_0 G\, m_p\, m_e}$ on the one hand, and the dimensional natural constants on the other. In addition, other important insights into the inner connections of the systems of units are revealed. A first important finding in this respect is that in Planck's system c, G and h assume the value 1.

That this would even apply to arbitrary, fictitious values of c, G and h, shall be proved here: the base a is a positive real number, the exponents b, c, d are positive or negative real numbers, the speed of light $c = a^b \frac{m}{s}$, the gravitational constant $G = a^c \frac{m^3}{kgs^2}$ and Planck's constant $h = a^d \frac{kgm^2}{s}$, then:

the Planck length $l_{pl} = \left(\frac{Gh}{c^3}\right)^{1/2} m = \left(\frac{a^c * a^d}{a^{3b}}\right)^{1/2} m = a^{-\frac{3b}{2}+\frac{c}{2}+\frac{d}{2}} m \Rightarrow 1\,m = a^{\frac{3b}{2}-\frac{c}{2}-\frac{d}{2}} l_{pl}$,

the Planck time $t_{pl} = \left(\frac{Gh}{c^5}\right)^{1/2} s = \left(\frac{a^c * a^d}{a^{5b}}\right)^{1/2} s = a^{-\frac{5b}{2}+\frac{c}{2}+\frac{d}{2}} s \Rightarrow 1\,s = a^{\frac{5b}{2}-\frac{c}{2}-\frac{d}{2}} t_{pl}$,

the Planck mass $m_{pl} = \left(\frac{ch}{G}\right)^{1/2} kg = \left(\frac{a^b * a^d}{a^c}\right)^{1/2} kg = a^{\frac{b}{2}-\frac{c}{2}+\frac{d}{2}} kg \Rightarrow 1\,kg = a^{-\frac{b}{2}+\frac{c}{2}-\frac{d}{2}} m_{pl}$,

$$c = a^b \frac{m}{s} = \frac{a^b * a^{\frac{3b}{2}-\frac{c}{2}-\frac{d}{2}}}{a^{\frac{5b}{2}-\frac{c}{2}-\frac{d}{2}}} \frac{l_{pl}}{t_{pl}} = a^0 \frac{l_{pl}}{t_{pl}} = 1 \frac{l_{pl}}{t_{pl}} ,$$

$$G = a^c \frac{m^3}{kgs^2} = \frac{a^c * a^{\frac{9b}{2}-\frac{3c}{2}-\frac{3d}{2}}}{a^{-\frac{b}{2}+\frac{c}{2}-\frac{d}{2}} * a^{\frac{10b}{2}-\frac{2c}{2}-\frac{2d}{2}}} \frac{{l_{pl}}^3}{m_{pl}{t_{pl}}^2} = a^0 \frac{{l_{pl}}^3}{m_{pl}{t_{pl}}^2} = 1 \frac{{l_{pl}}^3}{m_{pl}{t_{pl}}^2} ,$$

$$h = a^d \frac{kgm^2}{s} = \frac{a^d * a^{-\frac{b}{2}+\frac{c}{2}-\frac{d}{2}} * a^{\frac{6b}{2}-\frac{2c}{2}-\frac{2d}{2}}}{a^{\frac{5b}{2}-\frac{c}{2}-\frac{d}{2}}} \frac{m_{pl}{l_{pl}}^2}{t_{pl}} = a^0 \frac{m_{pl}{l_{pl}}^2}{t_{pl}} = 1 \frac{m_{pl}{l_{pl}}^2}{t_{pl}} .$$

As can be seen here, the speed of light, the gravitational constant and the Planck constant measured in Planck units, regardless of what value they would assume in a fictional universe, are always 1. This is due to the dimensionality ($\frac{l_{pl}}{t_{pl}}$, $\frac{{l_{pl}}^3}{m_{pl}{t_{pl}}^2}$ and $\frac{m_{pl}{l_{pl}}^2}{t_{pl}}$) of these constants, i.e. the basic construction of the universe, and not to their concrete values.

A second important finding is that there are systems of units - not only the system of Planck units - in which the natural constants c, G, h, e and k_c all but one take the value 1. The reason why not all constants mentioned can take the value 1 is that then, in addition to c_{pl}, G_{pl} and h_{pl}, the product of the square of the elementary charge e and the Coulomb constant $\frac{e^2}{4\pi\epsilon_0} = \frac{\alpha c_{pl} h_{pl}}{2\pi} = \frac{1}{2\pi * 137.036} = 1.1614 * 10^{-3}$ would have to take the value 1.

This is not possible by a simple transfer to another system of units. This is shown by the continuation of the above proof. In addition to the above assumptions, f is also a positive or negative real number, the product of the square of the elementary charge e and the Coulomb constant is

$$e^2 k_c = \frac{e^2}{4\pi\epsilon_0} \frac{kgm^3}{s^2} = a^f \frac{kgm^3}{s^2} = \frac{a^f * a^{-\frac{b}{2}+\frac{c}{2}-\frac{d}{2}} * a^{\frac{9b}{2}-\frac{3c}{2}-\frac{3d}{2}}}{a^{\frac{10b}{2}-\frac{2c}{2}-\frac{2d}{2}}} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2} = \frac{a^f}{a^b * a^d} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2} = \frac{e^2}{4\pi\epsilon_0 ch} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2} = \frac{\alpha}{2\pi} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2} = 1.1614 * 10^{-3} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2}.$$

So that in the Planckian system $e^2 k_c = \frac{e^2}{4\pi\epsilon_0} = 1$, $\frac{\alpha}{2\pi} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2} = \frac{e^2}{4\pi\epsilon_0 ch} \frac{m_{pl}{l_{pl}}^3}{{t_{pl}}^2}$ would have to be equal to 1.

It is also important in this context that in general - not only in the Planckian system – the natural constant which cannot assume the value 1 at the same time as all the others, then takes the value 2π/α = 861.02 or α/2π = $1.1614*10^{-3}$. Seen in this light, the importance of the fine structure constant α is further enhanced. It is becoming the guiding constant in our universe, because it seems to be unaffected by transformations of units.

A third insight concerns the amazing symmetries between the systems of units. The magnitude of the gravitational constant, measured on the scale "Compton wave" and the magnitude of Planck's quantum of action, measured on the scale "black hole", are equal (for example $|G_c| = |h_{cb}| = 9.9139*10^{-122}$). Antisymmetrically to this, the amount of the speed of light, measured at the scale "Compton wave" is equal to the reciprocal of the amount of the gravitational constant, measured at the scale "Compton wave" or is equal to the reciprocal of the Planck's action quantum, measured on the scale "black hole" (for example $|1/c_c| = |G_c| = |h_{cb}| = 9.\ 9139*10^{-122}$). The corresponding amount ratios are: $\frac{c_c}{c_p} = \frac{G_p}{G_c} = \frac{h_{pb}}{h_{cb}} = 9.4614 * 10^{81}$.

Constant	Amount	Amount ratio	Unit of mass	Mass ratio
c_c	$1/9.9*10^{-122}=1*10^{121}$	$9.4614*10^{81}$	$1.7178*10^{-68}$	1
c_p	$1/9.4*10^{-40}=1.1*10^{39}$	1	$1.6709*10^{-27}$	$9.7266*10^{40}$
G_p	$9.377*10^{-40}$	$9.4614*10^{81}$	$1.6709*10^{-27}$	$9.7266*10^{40}$
G_c	$9.914*10^{-122}$	1	$1.7178*10^{-68}$	1
h_{pb}	$9.377*10^{-40}$	$9.4614*10^{81}$	$1.7814*10^{12}$	1
h_{cb}	$9.914*10^{-122}$	1	$1.7327*10^{53}$	$9.7266*10^{40}$

The situation is different with the units of mass, because the smaller the mass scales by which the speed of light and Planck's quantum of action are measured, the greater their value becomes. Antisymmetrically to it the gravitational constant becomes bigger, the bigger the mass scale is, at which it is measured:

🡡 unit of mass => G🡡, c🡣, h🡣.

The ratio of the natural constants corresponds to the square of the ratio of the corresponding units of mass. The corresponding amount ratios are:

$$\frac{c_c}{c_p} = \frac{G_p}{G_c} = \frac{h_{pb}}{h_{cb}} = 9.4614 * 10^{81} = (9.7766 * 10^{40})^2 = \frac{{m_p}^2}{{m_c}^2} = \frac{{m_{cb}}^2}{{m_{pb}}^2}$$

where $m_c = h/cR$, $m_{cb} = Rc^2/G$ and $m_{pb} = ch/Gm_p$.

A fourth finding is that with the speed of light c, Planck's constant h, the gravitational constant G and the Hubble constant H, a mass (unit) ${m_R}^3 = \frac{Hh^2}{Gc} = \frac{h^2}{GR} = \frac{(6.62607*10^{-34})^2}{6.6743*10^{-11}*1.2867*10^{26}} = (3.7116 * 10^{-28})^3$ can be formed, with a proportionality to the proton mass of $\frac{{m_p}^3 Gc}{Hh^2} = \frac{(1.6709*10^{-27})^3}{(3.7116*10^{-28})^3} = 91.24 \approx 1836.15^{3/5}$ (if the Hubble constant

$H = 2.33*10^{-18}\ s^{-1}$). With the number 91.24 the gap can be filled in Dirac's conjecture:

$\frac{M_u}{m_p} * \frac{{m_p}^3 GR}{h^2} = \frac{R^2}{{\lambda_p}^2}$ (if for $M_u = Rc^2/G$ and for $\lambda_p = h/cm_p$ is set). Furthermore, $\frac{Hh^2}{Gc}$ is also contained in the formula ${m_x}^3 = \frac{e^2 Hh}{4\pi\epsilon_0 c^2 G}$ I found in [4] in 2014:

$${m_x}^3 = (m_p * m_e)^{\frac{3}{2}} = \frac{e^2 Hh}{4\pi\epsilon_0 c^2 G} = \frac{e^2}{4\pi\epsilon_0 ch} * \frac{Hh^2}{Gc} = \frac{\alpha}{2\pi} * \frac{h^2}{GR} .$$

A fifth finding $m_p = \frac{h}{cR} * \sqrt{\frac{G_p}{G_c}} = \frac{Hh}{c^2} * \sqrt{\frac{G_p}{G_c}}$ relates the proton mass to the mass $\frac{h}{cR}$ or $\frac{Hh}{c^2}$. This mass represents a particle with a Compton wavelength corresponding to the radius of the visible universe (when the Hubble constant $H = 2.33*10^{-18}\ s^{-1}$). The previous equation can also be transformed into

$$\frac{G_p}{G_c} = \frac{{m_p}^2 c^2 R^2}{h^2} .$$

The gravitational constant measured on the proton scale (proton mass, Compton wavelength of the proton, time scale = Compton wavelength of the proton/velocity of light) to the gravitational constant measured on the cosmic scale (radius of the visible universe, time scale = radius of the visible universe/velocity of light, mass of a particle with Compton wavelength corresponding to the radius of the visible universe) behaves like the square of the proton mass to the square of a particle mass with Compton wavelength corresponding to the radius of the visible universe.

The mass $\frac{h}{cR}$ or $\frac{Hh}{c^2}$ is also in the formula ${m_x}^3 = \frac{e^2 Hh}{4\pi\epsilon_0 c^2 G}$ I found:

${m_x}^3 = m_x * m_p * m_e = \frac{e^2 Hh}{4\pi\epsilon_0 Gc^2} => (m_p * m_e)^{\frac{1}{2}} = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} * \frac{Hh}{c^2} = \frac{F_e}{F_g} * \frac{h}{cR}$, where $\frac{F_e}{F_g}$ is the ratio of the strength of the electromagnetic force to the gravitational force.

A sixth finding is the result $\frac{m_p}{m_e} = 1836.15 \approx \frac{ch}{G m_p m_e} * \frac{G_p}{G_{pl}}$ (with $G_{pl} = 1$). It is a remarkable relationship in which the term $\frac{ch}{G m_p m_e}$ corresponds to the ratio of the gravitational force between two Planck masses to the gravitational force between a proton and an electron. Slightly modified the formula becomes $\frac{\mathbf{G{m_p}^2}}{\mathbf{ch}} \approx \frac{\mathbf{G_p}}{\mathbf{G_{pl}}}$, whereby the term $\frac{G{m_p}^2}{ch}$ corresponds to the square of the proton mass divided by the square of the Planck mass. G_p is the gravitational constant measured at the proton scale (proton mass, Compton wavelength of the proton, time scale = Compton wavelength of the proton/velocity of light).

Combining the fifth and the sixth findings into a formula, results in

$\frac{G_{pl}}{G_c} = \frac{Rc^2}{G} / \frac{h}{cR} = \frac{R^2c^3}{Gh} = n$, where G_{pl} has the value 1 and n is the total information content of the visible universe in bits (see [1]). The number $n = 1.0087 * 10^{121}$ also results as the quotient of Rc^2/G by h/cR, i.e. the total mass of the visible universe by the mass of a particle with a Compton wavelength corresponding to the radius of the visible universe.

The number n is also in the formula ${m_x}^3 = \frac{e^2Hh}{4\pi\epsilon_0 c^2 G}$ I found in 2014:

$${m_x}^3 = (m_p * m_e)^{\frac{3}{2}} = \frac{e^2Hh}{4\pi\epsilon_0 c^2 G} = \frac{e^2h}{4\pi\epsilon_0 cRG} = \frac{e^2}{4\pi\epsilon_0 ch} * \frac{R^2c^3}{Gh} * \left(\frac{h}{cR}\right)^3 = \frac{\alpha}{2\pi} * n * \left(\frac{h}{cR}\right)^3.$$

As has been shown step by step, it is therefore possible to express the important dimensionless constants $137.036 = \frac{2\epsilon_0 ch}{e^2} = \frac{1}{\alpha}$, $1836.15 = \frac{m_p}{m_e}$ and $2.2717 * 10^{39} = \frac{e^2}{4\pi\epsilon_0 G\, m_p\, m_e}$ as well as the number $1.0087 * 10^{121} = \frac{R^2c^3}{Gh}$ by my formula ${m_x}^3 = \frac{e^2Hh}{4\pi\epsilon_0 c^2 G}$.

Last but not least, a possible finding for the value of the speed of light c in the SI system has been found: $c/c_{pl} \approx \frac{(2\pi/\alpha)^4}{m_p/m_e}$, where c_{pl} = 1. This relationship applies with an accuracy of 0.15% of the measured value of the speed of light, so it is difficult to ignore it as a possible coincidence. It is distinguished from previous relationships by the fact that here one of the most important, if not the most important natural constant, is directly related to the two most important dimensionless numbers in physics. All findings presented so far put two or more different natural constants in relation to each other. It is precisely this fact that gives this relation the potential possibility of a fundamental meaning, but makes it difficult to interpret.

Accepting this relation gives rise to the question: which of the three combined quantities, i.e. the ratio of proton to electron mass, the fine structure constant or the speed of light, is the most fundamental? If one considers that the dimensionless quantities m_p/m_e and $2\pi/\alpha$ are not changed by simple transformations of the system of units, the speed of light should be the least fundamental quantity of the three. The fact that - as we have seen - $2\pi/\alpha$ or the reciprocal value of it always appears when all natural constants except one assume the value 1 speaks for the fact that $2\pi/\alpha$ is more fundamental than the speed of light. Seen in this way, the value of the speed of light would depend on the value of the fine structure constant α and on m_p/m_e.

The remaining question is whether the fine structure constant is really completely independent of the scale units with which it is measured. If this were the case, then the relation in question would also contain a certain statement about our own nature. We, who are about the size of a metre and whose movements take place in about one second, as well as the scales we use and the value of the speed of light (approximately) measured with these scales are determined by the fine structure constant and the ratio of proton to electron mass. A thought that has a certain plausibility inherent in it, because if $2\pi/\alpha$ and

m_p/m_e had different values, then this would of course also have an effect on the possible proportions of intelligent living beings and thus also on the scales with which such beings measure nature.

If, contrary to expectations, the fine structure constant is not (completely) independent of the scale units with which it is measured, then this would be a revolutionary insight that would shake the current foundations of physics.

So if the value of the speed of light is to depend on $2\pi/\alpha$ and m_p/m_e, what is the situation with the other natural constants? By some effort the following best possible approximation with small integer powers can be found for the gravitational constant: $G/G_{pl} \approx \frac{\frac{2\pi}{\alpha}}{\left(\frac{m_p}{m_e}\right)^4} = \frac{861.02}{(1836.15)^4} = 7.5749 * 10^{-11}$, where G_{pl} = 1.

This approximation is valid with an accuracy of 13.5%, so it is much less accurate than the approximation for c. But and this is the remarkable thing, it is antisymmetrical to the approximation for c in terms of the exponents. Whereas in the approximation for c, $2\pi/\alpha$ occurs in the fourth power, in the approximation for G, m_p/m_e occurs in the fourth power. Incidentally, in order to get this relation exactly right, one would only have to use a mass unit of 1.135 kg instead of the mass unit of 1 kilogram, if the units for length and time (metres and seconds) were the same.

Furthermore if 1.0015 m is used instead of 1 m as unit of length and 1.1403 kg instead of 1 kg as unit of mass with the same unit of time of 1 s, both c (= then $2.9933*10^8$) and G (= then $7.5749*10^{-11}$) assume the exact value of the respective approximation. Seen in this light, our SI units are almost perfectly matched to the approximations found. Combining the relations found for c and G in the adapted SI system results in the following new relations:

$\left|\frac{\mathbf{c^4}}{\mathbf{G}}\right| = \left(\frac{\mathbf{2\pi}}{\boldsymbol{\alpha}}\right)^{\mathbf{15}}$, $\left|\frac{\mathbf{c}}{\mathbf{G^4}}\right| = \left(\frac{\mathbf{m_p}}{\mathbf{m_e}}\right)^{\mathbf{15}}$ or $\left|\mathbf{c * G}\right| = \left(\frac{\mathbf{2\pi/\alpha}}{\mathbf{m_p/m_e}}\right)^{\mathbf{5}}$

with the associated checks:

$$\left|\frac{c^4}{G}\right| = \frac{\left(2.9933*10^8\right)^4}{7.5749*10^{-11}} = 1.059 * 10^{44} = (861.02)^{15} = \left(\frac{2\pi}{\alpha}\right)^{15} ,$$

$$\left|\frac{c}{G^4}\right| = \frac{2.9933*10^8}{(7.5749*10^{-11})^4} = 9.091 * 10^{48} = (1836.15)^{15} = \left(\frac{m_p}{m_e}\right)^{15} \text{ and}$$

$$\left|c * G\right| = 2.9933 * 10^8 * 7.5749 * 10^{-11} = 0.02267 = \left(\frac{861.02}{1836.15}\right)^5 = \left(\frac{2\pi/\alpha}{m_p/m_e}\right)^5 .$$

The term $c^4/4G$ (or $c^4/16G$ if the Schwarzschild radius $R = \frac{2GM}{c^2}$ instead of $R = \frac{GM}{c^2}$) corresponds to the gravitational force between two black holes of equal but arbitrary size that touch each other at their Schwarzschild radii. This also applies to two Planck masses, i.e. the smallest black holes, if they touch each other at a distance of 2 Planck lengths:

$$m_{pl}^2 = \frac{ch}{G}\ ,\ l_{pl}^2 = \frac{Gh}{c^3} \Rightarrow F = \frac{Gm_{pl}^2}{4l_{pl}^2} = \frac{Gchc^3}{4GGh} = \frac{c^4}{4G}\ .$$

Now the question arises, of course, what is there to report in this context with regard to h and $e^2k_c = \frac{e^2}{4\pi\epsilon_0}$?

For h I have found the following approximation with the smallest possible integer powers: $h/h_{pl} \approx \frac{\left(\frac{m_p}{m_e}\right)^6}{\left(\frac{2\pi}{\alpha}\right)^{18}} = \frac{(1836.15)^6}{(861.02)^{18}} = 5.6649 * 10^{-34}$. Using 1.0015 m, 1s and 1.1403 kg for the units of measurement results in a value of h = $5.7928*10^{-34}$, from which the value $5.6649*10^{-34}$ deviates by 2.21%. For $\frac{e^2}{4\pi\epsilon_0}$ again I have found the following approximation with the smallest possible integer powers:

$\left|\frac{e^2}{4\pi\epsilon_0}\right| \approx \frac{\left(\frac{m_p}{m_e}\right)^5}{\left(\frac{2\pi}{\alpha}\right)^{15}} = \frac{(1836.15)^5}{(861.02)^{15}} = 1.9694 * 10^{-28}$. Using 1.0015 m, 1s and 1.1403 kg for the units of measurement results in a value of $\frac{e^2}{4\pi\epsilon_0} = 2.0138 * 10^{-28}$, from which the value $1.9694*10^{-28}$ also deviates by 2.21%.

In order that, in addition to c ($2.9933*10^{8}$) and G ($7.5749*10^{-11}$), h ($5.6649*10^{-34}$) and $\frac{e^2}{4\pi\epsilon_0}$ ($1.9694*10^{-28}$) also assume exactly the value of the respective approximation, the units of measurement need only be changed to around 1.0128 m, 1.0112 s and 1.1531 kg. All in all, this is not a very large deviation from the usual SI system and an amazing fact that makes one think.

Altogether, by transforming the system of units, we have thus found a wealth of (numerical) correlations that could help to detect new physical relationships. To do so, it will be necessary to look at these correlations in detail and to test their potential. Even if no new physical findings could be obtained through these correlations, at least the correlations between different systems of units and their structural laws are interesting, because at least one should know what happens when one changes the physical scales ☺.

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