Noname manuscript No.
(will be inserted by the editor)

The Photon Existence Paradox

Armin Nikkhah Shirazi

Received: date / Accepted: date

Abstract Special relativity enjoins lightspeed objects from possessing essential characteristics one might naively associate with physically existing objects. Thus, it can be argued, had we not already known of the physical existence of lightspeed objects, such as photons, then we would have interpreted this enjoinment to mean that it is *impossible* for lightspeed objects to exist. From this perspective, the empirical existence of photons poses a paradox. A recently proved equivalence relation shows how this paradox is properly resolved: it is possible for lightspeed objects to exist, but not in spacetime. The interpretation of the paradox depends on which aspect of special relativity one wishes to emphasize: it can be framed either in terms of the "error" of assuming absolute existence, i.e. that physical existence can be specified without reference to a given spacetime, or a "gap" in standard special relativity in that it lacks any reference to a physics-based concept of existence in spacetime.

Keywords Photon Existence Paradox · Ontic Equivalence Relation · Absolute Existence · Relative Existence · Areatime

PACS 03.30.+p · 14.70.Bh · *43.10.Mq

1 Introduction

Albert Einstein credited his discovery of the special theory of relativity to his reflections on a paradox, as can be seen by the following passage [1]:

> "...a paradox upon which I had already hit at the age of sixteen: If I pursue a beam of light with the velocity c (velocity of light in a vacuum),

Armin Nikkhah Shirazi
University of Michigan, Ann Arbor
Department of Physics
450 Church Street
Ann Arbor, MI 48109 Tel.: +734-474-1434
E-mail: armin@umich.edu

> I should observe such a beam of light as an electromagnetic field at rest though spatially oscillating. There seems to be no such thing, however, neither on the basis of experience nor according to Maxwell's equations. From the very beginning it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relative to the earth, was at rest. For how should the first observer know or be able to determine, that he is in a state of fast uniform motion? One sees in this paradox the germ of the special relativity theory is already contained."(p. 49-51)

Special relativity addresses Einstein's "lightbeam observer paradox" by postulating that c is independent of the motion of its source, as well as generalizing Galileo's relativity principle, applicable to the laws of mechanics, to all laws of physics. This paper will argue that in the process, it transforms Einstein's problem into a new paradox, which will be called the "photon existence paradox".

The statement of the paradox is simple: special relativity enjoins lightspeed objects from possessing certain fundamental properties one might naively think are necessary for physically existing objects. Had we not already known of the existence of such objects, we would likely have taken this enjoinment to imply that the existence of lightspeed objects is impossible; and yet, empirically, lightspeed objects such as photons do seem to exist.

Unlike other well-known "paradoxes" of special relativity, such as the twin paradox, this paradox has failed to gain attention among physicists in the century since the appearance of special relativity. It is probably fair to say that this is because the paradox can all too easily be dismissed as a problem not of physics but of metaphysics. However, a recently proved equivalence relation which resolves the paradox shows that there are definite physical consequences to its resolution which, at the time of this writing, will seem highly unfamiliar, and the consequences of which remain to be worked out.

2 The Existence of Lightspeed Objects as a Paradox

The reader is invited to imagine, as vividly as possible, how we would have interpreted special relativity if we were ignorant of the empirical evidence for the existence of lightspeed objects. It seems safe to suppose that we would have interpreted the idea that the speed of light cannot be reached from below quite differently; we would likely have taken it to mean that it is *impossible* for such objects to *exist*: every known object would be subluminal, and the impossibility of accelerating them to the speed of light, coupled with the observed absence of any luminal objects, would naturally invite this conclusion. In short, had we not already known of the existence of lightspeed objects, we would likely have mistaken an incidental observational fact for a *necessary* one.

Admittedly, this supposition cannot be tested because we never actually were in this situation: in modern physics, the first objects to be recognized to travel at the speed of light, the quanta of light, were proposed the same year as special relativity itself[1][2][3]. Consequently, we have recognized that lightspeed objects exist while, at the same time in an uneasy tension, the constraints of special relativity have forced us to deny that such objects can be assigned certain properties that naively would be expected to characterize every physically existing object. In the context of this discussion, two are particularly relevant:

– **It is impossible to assign to lightspeed objects a coordinate system origin in space.** The usual justification for this is that such an assignment presupposes a rest frame, and the existence of a rest frame for lightspeed objects directly contradicts the speed of light postulate. But consider how so very strange this must seem: if something exists, then it is utterly reasonable to suppose that we should be able to pin to it ("it" precisely indicating here that this object physically exists) the origin of a coordinate system in space, and yet special relativity tells us this cannot be done.
– **It is impossible to assign to lightspeed objects a finite observed duration of existence in spacetime.** This is a direct consequence of the fact that lightspeed objects are characterized by null intervals, and therefore by a zero proper time: since it is zero, if it were possible to assign a rest frame to lightspeed objects, we would find that the moment such an object comes into existence would in such a frame be the same as the moment it would go out of existence. The fact that it is not possible to assign such frames may call this analysis into question, but for objects characterized by timelike spacetime intervals, their proper time can unambiguously be interpreted as observed duration of existence in spacetime between given spacetime events. At any rate, rejecting this analysis leaves one with the position that not even the concept of "duration of existence in spacetime" can be applied to lightspeed objects. Whichever position one may hold, whether the observed duration of existence in spacetime of lightspeed objects is zero, or whether that very concept is inapplicable to them, consider again how so very strange this must seem.

Again, it is natural to suppose that had we not already known of the existence of lightspeed objects, these difficulties would have likely been interpreted as compelling theoretical evidence that it is impossible for such objects to exist. In this light, the empirical existence of objects like photons seems to pose a paradox.

It has been pointed out that there are three ways paradoxes can come about in physics: either due to "errors", "gaps" or "contradictions"[4]. The first of these refers to errors in assumptions or reasoning about a theory, such as misinterpreting the theory, misapplying it, or making mistakes in logical or mathematical deductions from it. These types of paradoxes can be resolved by recognizing and correcting the mistake. The second refers to instances in

[1] If one counts Newton's corpuscles of light, which had long fallen out of favor, then lightspeed objects predated special relativity by at least two centuries

which the theory has a flaw, but the flaw is not fatal to it. These types of paradoxes can be resolved by amending the theory so that the "gap" is bridged. The third refers to instances in which the theory contains a fatal flaw. Such paradoxes are perhaps the rarest of the three types and can only be resolved by replacing the theory with a new one. Special relativity replaced Newtonian theory because Einstein noticed a "contradiction".

The twin paradox and other such paradoxes of special relativity fall under the category of "errors". Under which of these three types does the "photon existence paradox" fall? Interestingly, the answer is not straightforward but depends on how one conceptualizes the problem. But Before this subtlety can be addressed, we need to discuss an equivalence relation which provides the resolution to the paradox.

3 The Ontic Equivalence Relation

Invariance and symmetry principles are pillars of modern physics, yet there are four such principles involved in spacetime transformations which, at the time of this writing, have not yet achieved explicit recognition, likely because they seem too self-evident. Their discussion in this section and the proof of the equivalence relation which follows from them is essentially a fuller discussion of what was in [5].

Special relativistic coordinate system transformations, namely rotations or translations in space and Lorentz boosts, as well as more general ones, namely conformal transformations and dilations, are taken for granted to leave the number of length dimensions of any transformed object unaffected. This reflects an invariance principle, the *invariance of absolute dimensionality.*

Similarly, such transformations leave unaffected any considerations about a transformed object's existence in spacetime, and this, too, reflects an invariance principle. If we assign the number 1 to any object that exists in spacetime, and the number 0 to any object that fails to exist in spacetime, and call these numbers *spacetime ontic values*, then the principle can be called the *invariance of spacetime ontic value.*

In order to ground the notion of "existence in spacetime" in physics, we need to be able to express it in terms of physics concepts. A postulate which does this (and is the only new assumption beyond standard special relativity) is a criterion according to which *an object exists in spacetime iff it is characterized by a timelike proper time.* As a postulate, it cannot be derived from anything else, but two plausibility arguments in its favor are that it is straightforward to conceptualize the proper time of timelike objects as their duration of existence in spacetime between given spacetime events, and that under the adoption of this postulate, spacetime ontic value is indeed a physical invariant. Henceforth, the discussion of special relativity will include this additional postulate unless the term "standard special relativity" is used.

Finally, any of the above spacetime transformations can only take us from one

$3+1$ spacetime slicing to another $3+1$ slicing, and this reflects two symmetry principles: that the dimensionality of physical space and of time is everywhere the same. These symmetry principles are the *homodimensionality of space* and the *homodimensionality of time*, respectively. When a spacetime is characterized by both, this is called the *isodimensionality of spacetime*.

It turns out that these four principles together, generalized to $n+1$ dimensional spacetimes, imply the proposition that existence in a spacetime is an equivalence relation by absolute dimensionality. This is the *ontic equivalence relation*. The proof of this is rather easy: an equivalence relation is defined by the properties of reflexivity, symmetry and transitivity, so to prove the proposition, we just need to show that these three conditions are satisfied.

Consider an $m-$dimensional object A and suppose that it exists in an $n+1$ dimensional region of spacetime in accordance with the four spacetime principles and special relativity. By assumption, the spacetime ontic value of A in the $n+1$ dimensional region is equal to one. What is m? It cannot be that $m > n$ because the region in which A exists cannot contain an object with more length dimensions than it has, and it cannot be that $m < n$ because that would imply that A is completely length contracted along at least one direction within the region, which by the special relativistic length contraction formula implies that it is not the case that $0 \leq \gamma < \infty$ and, consequently, that the spacetime ontic value of A in that region is zero, in contradiction to our starting assumption. So, we must have $m = n$. By the isodimensionality of spacetime, the $n+1$ dimensional region of spacetime in which A exists is, in fact, all of $n+1$-dimensional spacetime.

Given the above, reflexivity is just the proposition that A exists in the spacetime in which its proper time is timelike, so in which it exists. Now, consider an m-dimensional object B. By the same argument as just given for A, it must exist in an $m+1$ dimensional spacetime. To prove symmetry, suppose that B exists in the same spacetime as A. That means that the proper time of B is timelike in the spacetime in which the proper time of A is timelike, so the spacetime in which the proper time of A is timelike is the same as that in which the proper time of B is timelike, which is to say that A exists in the spacetime in which B exists. To prove transitivity, consider an $l-$dimensional object C. By the same argument as before, C must exist in an $l+1$-dimensional spacetime. Now, suppose that A exists in the same spacetime as B, and that B exists in the same spacetime as C. That means that the proper time time of A is timelike in the same spacetime in which the proper time of B is timelike, and the proper time of B is timelike in the same spacetime in which the proper time of C is timelike. It follows that the proper time of A is timelike in the same spacetime in which the proper time of C is timelike, so A exists in the same spacetime in which C exists.

Any equivalence relation on a set partitions that set into equivalence classes, which in this case means that the ontic equivalence relation partitions the set of all objects in the domain of physics into equivalence classes by the number of length dimensions that characterize them, such that each equivalence class exists only in a spacetime with the corresponding number of length dimen-

sions.
A striking implication of this is that lightspeed objects do not exist in our $3+1$ spacetime, but in a $2+1$ analog, since one of their sides is completely length contracted relative to three-dimensional space. Spaces with $2+1$ dimensions have been considered in theoretical physics, but to my knowledge only ever as toy models and mathematical abstractions with no claim to physical existence. So, in order to emphasize the equal footing of $2+1$ space with the repository of our existence, $3+1$ spacetime, as a physical space, I will call the former *areatime*.

4 Resolution of the Paradox

The implication of the ontic equivalence relation that lightspeed objects do not exist in spacetime but in areatime resolves the photon existence paradox. It allows us to understand how there can be empirical evidence for the existence of lightspeed objects even though special relativity does not permit the attribution of basic properties associated with their existence, which, it turns out, we should have been more careful to specify as *existence in spacetime*: it is impossible to assign to lightspeed objects the origin of a coordinate system in space because the coordinate system in which this is possible must belong to a 2-dimensional space[2]; it is impossible to assign to lightspeed objects a finite observed duration of existence in spacetime because, simply, they do not exist in spacetime but in areatime.
While the ontic equivalence relation resolves the photon existence paradox, there is still the question of which *type* of paradox it actually resolves.
Consider a resolution based on conceptualizing the paradox as the result of an “error”. In light of the ontic equivalence relation, the “error” here is the at present almost universally held tacit assumption that physical existence can be specified without reference to a given spacetime. I will call this the assumption of *absolute existence*. From the assumption of absolute existence it follows that existence *per se* implies existence in spacetime. The opposite notion is what I will call *relative existence*, that the existence of a physical object must always be specified relative to a given $n+1$-dimensional spacetime.
If special relativity contained indications that existence is absolute, then it would give rise to a situation in which a theory withholds some of the most fundamental characteristics of physical existence from objects for the existence of which there is compelling empirical evidence. This would rightly justify the perplexity associated with lightspeed objects, if not come close to a “contradiction”.
But, in fact, already standard special relativity gives indications that the assumption of absolute existence is false. For instance, one simple indication is

[2] which is presumably isomorphic to the null-plane propagating along the direction of motion of the lightspeed object

that the integral of any volume of space in the lightlike direction is zero. Conceptually, if we grant physical existence to lightspeed objects like photons and suppose absolute existence, then lightspeed objects exist in spacetime, and if lightspeed objects exist in spacetime, then it should be possible to obtain a finite spacetime volume by integrating a given spatial region over the classical spacetime path of such an object. Alas, this is impossible. So, the assumption of absolute existence is widely held not because of special relativity, but *in spite of it*: it is an "error".

However, as a matter of historical fact, such indications were for over a century totally insufficient to help us discover our mistaken assumption, and this opens the possibility to conceptualize the paradox as being due to a "gap".

If the "photon existence paradox" is conceptualized in this way, then the "gap" here refers to the absence of any direct reference to the concept of existence in spacetime in standard special relativity. The formulation of the ontic equivalence relation requires one additional assumption not contained in standard special relativity, namely a criterion for physical existence in spacetime. This criterion, which, recall, says that an object exists in spacetime iff it is characterized by a timelike spacetime interval, closes the "gap" because it permits the direct inference that lightspeed objects do not exist in spacetime, since they are not characterized by timelike spacetime intervals.

So, whether the paradox is due to an "error" or due to a "gap" just depends on which aspect of special relativity one wishes to emphasize: that its indirect mathematical indications that lightspeed objects do not exist in spacetime were there all along, but widely overlooked, or that it lacked a criterion for existence in spacetime by which one can determine this directly. The only definite statement pertaining to the type of paradox it poses is that it is *not* due to a "contradiction" in special relativity.

Now, even if it is conceded that lightspeed objects do not exist in spacetime, that by itself does not imply that they exist in areatime. After all, one could frame interactions involving, say, photons strictly in terms of anticorrelated changes in energy-momentum between interacting massive particles a distance ct apart, and thereby in principle eliminate photons entirely.

It turns out that standard special relativity gives indirect indications of the existence of lightspeed objects in areatime, too: as the speed of an object approaches the speed of light, its direction of motion and its time direction approach the lightlike direction. In the limit of c both directions can in effect be considered to have merged with each other along that direction. But that implies that spacetime, considered as a vector space, becomes linearly dependent, as what were two basis vectors below c now point in the same direction. This, in turn, implies that in such a vector space $3 + 1$ spacetime has one dimension too many. A reduction of spacetime by one dimension results either in a $3+0$ or a $2+1$ space. But we can exclude the former on the grounds that lightspeed objects have only two spacelike degrees of freedom, leaving $2 + 1$ dimensions.

Like the other indirect indications, this has been totally insufficient to help us recognize that lightspeed objects exist in areatime.

5 Conclusion

The passage quoted in the introduction continues as follows[1]:

> Today, everyone knows, of course, that all attempts to clarify this paradox satisfactorily were condemned to failure as long as the axiom of the absolute character of time...was rooted unrecognized in the unconscious. To recognize clearly this axiom and its arbitrary character already implies the essentials of the solution of the problem." (p.51)

The ontic equivalence relation is a relatively recent result and not yet widely known, but there can be little doubt that eventually it will become part of established physics, as the principles from which it is proved are already in use, even if due to their extreme obviousness they are "rooted unrecognized in our unconscious", and the proof itself is nearly trivial. Once the ontic equivalence relation is widely known and accepted, it is striking that this very passage can be applied to the photon existence paradox, provided that "time" is substituted by "existence".
Any fundamentally novel idea will at first seem highly unfamiliar, require reflection and a working out of its consequences before it can be wholly accepted. This was true for the early 20th century idea that there is no absolute time, and it is true for the early 21st century idea that there is no absolute existence. While the implications of the former have been largely worked out, this remains yet to be done for the latter.

Conflict of interest

The author declares that he has no conflict of interest.

References

1. Einstein, A.: Autobiographical Notes: Centennial Edition, Schilpp, A. (ed.)pp. 49-51. Open Court Publishing, Lasalle and Chicago (1979)
2. Einstein, A.: Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. Annalen der Physik **17**: 132-148 (1905)
3. Einstein, A.: Zur Electrodynamik bewegter Körper. Annalen der Physik **17**(10): 891-921 (1905)
4. Aharonov, Y., Rohrlich, D.: Quantum Paradoxes. pp. 1-9. Wiley-VCH Verlag, Weinheim(2005)
5. Nikkhah Shirazi, A.: Lorentz Transformations and Existence in Minkowski Spacetime, unpublished (2019), http://hdl.handle.net/2027.42/148314