The bremsstrahlung generated by RLC circuit

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Abstract

The bremsstrahlung is calculated in case that electron current is realized by the RLC circuit. We determine the bremsstrahlung energy caused by the uniform oscillation of the RLC circuit.

1 Introduction

An RLC circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel (Nillson et al. 2008). The circuit forms a harmonic oscillator for current.

The three circuit elements, R, L and C can be combined in a number of different topologies. All three elements in series or all three elements in parallel are the simplest in concept and the most straightforward to analysis. There are, however, other arrangements, some with practical importance in real circuits. One issue often encountered is the need to take into account inductor resistance. Inductors are typically constructed from coils of wire, the resistance of which is not usually desirable, but it often has a significant effect on the circuit.

2 Series RLC circuit

In the situation where we consider series RLC circuit, the three components are all in series with the voltage source $(R - L - C - v)$. The governing differential equation can be found by substituting into Kirchhoff's voltage law (KVL) the constitutive equation for each of the three elements. From KVL,

$$v_R + v_L + v_C = v(t), \tag{1}$$

where v_R, v_L, v_C are the voltages across R, L, C respectively and $v(t)$ is the time varying voltage from the source. Substituting the corresponding physical term, in eq. (1), we get the following integral differential equation:

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\tau=t} i(\tau)d\tau = v(t). \tag{2}$$

If we consider the more simple situation with $v = 0$, then instead of equation (2) we write

$$L\ddot{Q} + R\dot{Q} + Q/C = 0 \tag{3}$$

with stationary solution

$$Q = Ae^{-\frac{R}{2L}t}\sin(\omega t + \alpha), \tag{4}$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \tag{5}$$

The Thomson formula for the period of oscillations is when $R = 0$

$$T = 2\pi/\omega = 2\pi\sqrt{LC}. \tag{6}$$

3 Two RLC circuits with the mutual induction

In this configuration the components R_1, L_1, C_1 are inductive boned with R_2, L_2, C_2 configuration. The problem was published in the textbook by Landau et al. (1989) and the frequency of the inductive system was calculated in the form (for $R_1 = R_2 = 0$).

$$\omega_{1,2}^2 = \frac{L_1C_1 + L_2L_2 \pm [(L_1C_1 - L_2C_2)^2 + 4C_1C_2L_{12}^2]^{1/2}}{2C_1C_2(L_1L_2 - L_{12}^2)}. \tag{7}$$

After performing the Lorentz transformations of all known components in the last formula, we can eliminate the term of the mutual induction L_{12} (Pardy, 2017; Rohlf, 1994).

4 Lorentz-Dirac equation for a charge generating bremsstrahlung

It is well known that Lorentz-Dirac equation describes motion of a charged particle in electromagnetic field where also the bremsstrahlung force is involved in the equation.

The bremsstrahlung force is in the nonrelativistic limit expressed in the following form:

$$\mathbf{f} = \frac{2e^2}{3c^3}\ddot{\mathbf{v}}. \tag{8}$$

The force $\mathbf{f}$ is not active force and it means it cannot cause motion of electron. In other words the equation $\mathbf{f} = m\dot{\mathbf{v}}$ has no physical meaning. The bremsstrahlung force is meaningful only with addition of the active force. Then the bremsstrahlung force acts as the so called light friction.

The relativistic equation which involves the bremsstrahlung force is so called Lorentz-Dirac equation and it can be evidently written in the form (Landau et al., 1987):

$$mc\frac{dv_\mu}{d\tau} = \frac{e}{c}F_{\mu\nu}v^\nu + g_\mu, \tag{9}$$

where g^μ can expressed according to Landau et al. (1987) in the form:

$$g_\mu = \frac{2e^2}{3c}\left(\frac{d^2v_\mu}{d\tau^2} - v_\mu v^\nu \frac{d^2v_\nu}{d\tau^2}\right), \tag{10}$$

where the form of the bremsstrahlung term leads in the nonrelativistic limit to eq. (8), where u_μ is the four-velocity and the radiative term was approximated in the form (Landau et al., 1987):

$$g_\mu = \frac{2e^3}{3mc^3}\frac{\partial F_{\mu\nu}}{\partial x^\alpha}v^\nu v^\alpha - \frac{2e^4}{3m^2c^5}F_{\mu\alpha}F^{\beta\alpha}v_\beta + \frac{2e^4}{3m^2c^5}\left(F_{\alpha\beta}v^\beta\right)\left(F^{\alpha\gamma}v_\gamma\right)v_\mu. \tag{11}$$

It is possible to show that the space components of the 4-vector force g_μ is of the form (Landau et al., 1987)

$$\mathbf{f} = \frac{2e^3}{3mc^3}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left\{\left(\frac{\partial}{\partial t}+(\mathbf{v}\nabla)\right)\mathbf{E}+\frac{1}{c}\left[\mathbf{v}\left(\frac{\partial}{\partial t}+(\mathbf{v}\nabla)\right)\mathbf{H}\right]\right\}+$$

$$+\frac{2e^4}{3m^2c^4}\left\{\mathbf{E}\times\mathbf{H}+\frac{1}{c}\mathbf{H}\times(\mathbf{H}\times\mathbf{v})+\frac{1}{c}\mathbf{E}(\mathbf{vE})\right\}-$$

$$-\frac{2e^4}{3m^2c^5\left(1-\frac{v^2}{c^2}\right)}\mathbf{v}\left\{\left(\mathbf{E}+\frac{1}{c}(\mathbf{v}\times\mathbf{H})\right)^2-\frac{1}{c^2}(\mathbf{Ev})^2\right\}. \qquad (12)$$

Using the Lorentz equation we can express the second derivative in the form:

$$\frac{d^2v_\mu}{d\tau^2} = \frac{e}{mc^2}\partial_\alpha\left(F_{\mu\nu}\right)v^\alpha v^\nu + \frac{e^2}{mc^4}F_{\mu\nu}F^{\nu\alpha}v_\alpha. \qquad (13)$$

After insertion of the last equation in the Lorentz-Dirac equation, we get the Lorentz-Dirac equation in the final form:

$$mc\frac{dv_\mu}{d\tau} = \frac{e}{c}F_{\mu\nu}v^\nu +$$

$$+\frac{2e^3}{3mc^3}\left\{\partial_\alpha F_{\mu\nu}v^\nu v^\alpha - \frac{e}{mc^2}F_{\mu\alpha}F^{\nu\alpha}v_\nu + \frac{e}{mc^2}v_\mu F_{\alpha\beta}v^\beta F^{\alpha\gamma}v_\gamma\right\}, \qquad (14)$$

5 The bremsstrahlung generated by RLC circuit

If

$$v(t) = E_0\sin(\omega t), \qquad (15)$$

we can write the intensity of the current in the form

$$j = j_0\sin(\omega t+\alpha), \qquad (16)$$

where

$$j_0 = \frac{E_0}{\sqrt{R^2+\left(L\omega+\frac{1}{C\omega}\right)^2}}; \quad \tan\alpha = \frac{L\omega-\frac{1}{C\omega}}{R}, \qquad (17)$$

We can see the motion of RLC is the motion of the harmonic oscillator and it means that RLC produces bremsstrahlung of from harmonic oscillator.

In this case the adequate formula was derived in the following form for the average value of energy of radiation (Matveev, 1951; Sokolov et al., 1983):

$$W = \frac{Q^2\omega^2}{c^2}\sum_{\nu=1}^{\infty}\nu^2\int_0^2 \sin\theta d\theta \times$$

$$\tan^2\theta J_\nu(\nu\beta\cos\theta); \quad \beta = \frac{a\omega}{c}, \tag{18}$$

where Q is a effective charge and a is amplitude from eq. (17). Using formula (Watson, 1966)

$$\sum_{\nu=1}^{\infty}\nu^2 J_\nu^2(\nu\beta\cos\theta) = \frac{\beta^2\cos^2\theta(4+\beta^2\cos^2)}{16(1-\beta^2\cos^2)^{7/2}}, \tag{19}$$

we get

$$W = \frac{Q^2\omega^2\beta^2}{16c}\int_0^\pi \sin^2\theta d\theta = \frac{(4+\beta^2\cos^2\theta)}{(1-\beta^2\cos^2\theta)^{7/2}}. \tag{20}$$

After integration over angles, we get

$$W = \frac{Q^2\omega^2\beta^2}{21}\frac{(4-\beta^2)}{(1-\beta^2\cos^2)^{3/2}}. \tag{21}$$

6 The dipole and quadrupole radiation

In case of the relative small speeds electrons in the oscillator we can restrict our calculation to the first two term in equation (18). The first term gives the dipole radiation and the second term gives the quadrupole radiation. Considering only the small arguments in the Bessel functions, we get

$$J_1(x) = \frac{x}{2}\left(1-\frac{x^2}{8}\right); \quad J_2(x) = \frac{x^2}{8} \tag{22}$$

and for the probability of the radiation of the first harmonic we get

$$W_1 = \frac{Q^2\omega^2}{c^2}\int_0^\pi \sin\theta^2\theta\beta^2\cos^2\theta\left(1-\frac{1}{4}\cos^2\theta\right)d\theta =$$

$$\frac{Q^2\omega^2\beta^2}{3c^2}\left(1-\frac{1}{5}\beta^2\right) \approx \frac{Q^2a^2\omega^4}{3c^3}. \tag{23}$$

We get by the same way the quadrupole radiation formula as follows

$$W_2 = \frac{4}{15}\frac{Q^2\omega^2}{c}\beta^4 = \frac{4}{15}\frac{a^4Q^2\omega^6}{c^5}. \quad (24)$$

7 Discusion

The RLC circuit bremsstrahlung is very small for small RLC circuit. However, the intensity of radiation will be big for the giant circuits which can be documented by the giant Tesla projects and realization with electron discharge machine, transformer, and so on. The high intensity of the polychromatic radiation by the RLC circuit can be used in the experiments with the conversion of photons into gravitons. The article forms the preamble of the future investigation of relativistic electronic systems (Nillson et al. 2008) and it will be, no doubt, the integral part of such institutions as Bell Laboratories, NASA, CERN and so on.

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